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MHD Visco-Elastic Fluid Flow over a Continuously Moving Vertical Surface with Chemical Reaction

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Abstract

An analysis has been developed to study the convective heat and mass transfer of a visco-elastic fluid flow over a continuously moving vertical surface with uniform suction in the presence of chemical reaction and heat generation. The flow is subjected to a transverse uniform magnetic field. The governing equations for this investigation are solved analytically. The effects of various physical parameters, like chemical reaction parameter, heat generation parameter, visco-elastic parameter, and magnetic parameter, on various momentum, heat, and mass transfer characteristics are discussed in detail, and shown graphically. The present visco-elastic fluid model works to suggest rheological liquids encountered in biotechnology (medical creams) and chemical engineering.

Keywords: Visco-elastic flow, MHD, chemical reaction, heat generation

Introduction

The study of laminar boundary layer flow of non-Newtonian fluids over a continuous moving solid surface has attracted several investigators [1,2] due to its wide range of applications, e.g., paper production, crystal growing, drawing of plastic sheets, manufacturing of foods, and slurry transporting. In view of these applications, an extensive range of mathematical models has been developed to simulate the diverse hydrodynamic behavior of these non-Newtonian fluids. In particular, different visco-elastic fluid models (like the Rivlin-Ericksen second order model, Oldroyd model, and Johnson-Seagalman model) have been presented by many investigators, in a diverse range of geometries, using various types of analytical and computational schemes (see for instance [3-6]).

Sakiadis [7,8] was the first to analyze the boundary layer flow over a continuous solid surface with constant speed. Later, the same work was extended by Erickson *et al*. [9] with heat and mass transfer. The flow of elastico-viscous fluid past impulsively started plate was discussed by [10,11]. Hayat *et al*. [12] analyzed the flow of visco-elastic fluid past an oscillating plate, in which he discussed the influence of suction/injection on velocity distribution. The study of non-Newtonian fluid flow through porous medium gained momentum, as some particular polymer solutions when injected into oil reservoirs attain better volumetric sweep efficiency in oil displacement mechanism, which is very important. Many authors have studied the flow of visco-elastic fluid past a moving vertical porous surface in the presence/absence of a magnetic field under different physical situations (see for instance [13-16]). Vajravelu and Roper [17] investigated the flow and heat transfer in a visco-elastic fluid over a stretching sheet with viscous dissipation and heat generation or absorption.

Chemical reactions usually accompany a large amount of exothermic and endothermic reactions. These characteristics can be easily seen in a lot of industrial processes. The reaction produced in a porous medium are extraordinarily common, such as the topic of PEM fuel cells modules and the pollution of underground water because of discharging the toxic substance, etc. Satyanarayana *et al*. [18,19] studied the effects of chemical reaction on various flow fields by using analytical methods. Chamkha [20] studied the problem of heat and mass transfer by the steady flow of an electrically conducting fluid on a uniformly moving vertical surface in the presence of a first-order chemical reaction.

To the best of our knowledge (from literature), the theoretical solution for convective heat and mass transfer of a chemically reacting and visco-elastic fluid flow over a continuously moving porous vertical surface with heat generation / absorption has remained unexplored. Therefore, the main objective of this paper is to extend the work of reference [21] in 3 directions: (i) to consider the MHD visco-elastic fluid (characterised by second-order fluid) (ii) to consider the heat source effect, (iii) to include the chemical reaction. An exact solution of the problem is given analytically. The effects of pertinent parameters on flow heat and mass transfer characteristics are studied in detail. This rheological model is very versatile and robust, and provides a relatively simple mathematical formulation which is easily incorporated into boundary layer theory for engineering applications [22,23].

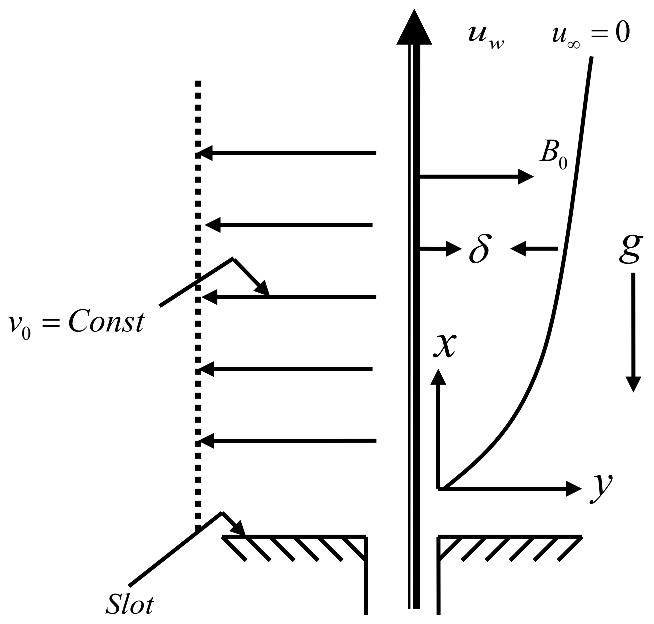

Figure 1 Physical model of the problem.

Mathematical formational of the problem

We consider the steady boundary layer convective flow of an electrically conducting, chemically reacting visco-elastic (second ordered fluid) fluid on a continuously moving porous vertical surface with heat generation/absorption. Here, at y = 0, the porous vertical plate moving vertically with a uniform velocity u_w in a fluid and heat is supplied from the plate to the fluid at a uniform rate, in the presence of a uniform transverse magnetic field of strength B_0. The x-axis is taken in the upward moving porous vertical plate and the y-axis is normal to it. The transverse applied magnetic field is assumed to be very small, so that the induced magnetic field is negligible. The concentration of the diffusing species in the binary mixture is assumed to be very small in comparison with the other chemical species which are present and, hence, the Soret and Dufour effects are negligible. The physical model of the problem is shown in **Figure 1**. Under the above assumptions, the appropriate governing equations of Continuity, Momentum, Energy, and Diffusion are given by;

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\begin{aligned} u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y} &= g\beta(T'-T'_\infty)+g\beta^*(C'-C'_\infty)+\frac{\mu_1}{\rho}\frac{\partial^2 u}{\partial y^2} \\ &+\frac{\mu_2}{\rho}\left(u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\right)-\frac{\sigma B_0^2 u}{\rho}-\frac{\mu u}{\rho K'} \end{aligned} \tag{2}$$

$$u\frac{\partial T'}{\partial x}+v\frac{\partial T'}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y^2}+\frac{\mu_2}{\rho C_p}u\frac{\partial^2 u}{\partial x\partial y}\frac{\partial u}{\partial y}-\frac{Q_0}{\rho C_p}(T'-T'_\infty) \tag{3}$$

$$u\frac{\partial C'}{\partial x}+v\frac{\partial C'}{\partial y}=D\frac{\partial^2 C'}{\partial y^2}-K_l C' \tag{4}$$

The initial and boundary conditions are;

$$\begin{aligned} &u=u_w,\ v=-v_0=Constant<0,\ \frac{\partial T}{\partial y}=\frac{-q}{K},\ \frac{\partial C'}{\partial y}=\frac{-J}{D} \quad at\ \ y=0 \\ &u\to 0,\ T'\to T'_\infty,\ C'\to C'_\infty \quad as\ \ y\to\infty \end{aligned} \tag{5}$$

Introducing the following non-dimensional quantities;

$$\begin{aligned} &Y=\frac{yv_0}{v_1}\quad Pr=\frac{\mu C_p}{k}\quad Kr=\frac{K_l\nu}{v_0^2}\quad Sc=\frac{\nu}{D}\quad U=\frac{u}{u_w} \\ &\phi=\frac{\nu Q_0}{\rho C_p v_0^2}\quad \alpha=\frac{\mu_2\rho v_0^2}{\mu_1^2}\quad K=\frac{K' v_0^2}{\nu^2}\quad M=\frac{\sigma_1 B_0^2 v_1}{\rho v_0^2} \\ &Gr=\frac{v_1 g\beta}{u_w v_0^2}\left(\frac{qv_1}{kv_0}\right)\quad T=(T'-T_\infty)\left(\frac{kv_0}{qv_1}\right)\quad C=(C'-C_\infty)\left(\frac{kv_0}{Jv_1}\right) \end{aligned} \tag{6}$$

We make use of the assumptions that the velocity, temperature and concentration fields are independent of the distance parallel to the surface as given in Schlichting [24], and Boussinesq's approximation. Eqs. (2), (3), and (4) are reduced to the following non-dimensional form;

$$\alpha\frac{d^3U}{dY^3}-\frac{d^2U}{dY^2}-\frac{dU}{dY}+(M+\frac{1}{K})U=GrT+GmC \tag{7}$$

$$\frac{d^2T}{dY^2} + Pr\frac{dT}{dY} - Pr\phi T = 0 \tag{8}$$

$$\frac{\partial^2 C}{\partial y^2} + Sc\frac{\partial C}{\partial y} - KrScC = 0 \tag{9}$$

The corresponding initial and boundary conditions reduces to;

$$\begin{aligned} &U = 1, \ \frac{\partial T}{\partial Y} = -1, \ \frac{\partial C}{\partial Y} = -1 \quad at \quad Y = 0 \\ &U \to 0, \ T \to 0, \ C \to 0 \quad as \quad Y \to \infty \end{aligned} \tag{10}$$

Solving (8) and (9) with conditions (10), we get;

$$T(Y) = \frac{e^{-A_1 Y}}{A_1} \tag{11}$$

$$C(Y) = \frac{e^{-A_2 Y}}{A_2} \tag{12}$$

To solve (7), we note that $\alpha < 1$ and, hence, we can assume that;

$$U(Y) = U_0(Y) + \alpha U_1(Y) + \alpha^2 U_2(Y) + \dots \tag{13}$$

Substituting (13) in (7) and equating the coefficients of α^0, α^1, and neglecting those of α^2, we get;

$$U_0'' + U_0' - (M + 1/K)U_0 = -GrT - GmC \tag{14}$$

$$U_0''' - U_1'' - U_1' + (M + 1/K)U_1 = 0 \tag{15}$$

The corresponding initial and boundary conditions are;

$$\begin{aligned} &U_0 = 1, \ U_1 = 0, \ \frac{\partial T}{\partial Y} = -1, \ \frac{\partial C}{\partial Y} = -1 \quad at \quad Y = 0 \\ &U_0 \to 0, \ U_1 \to 0, \ T \to 0, \ C \to 0 \quad as \quad Y \to \infty \end{aligned} \tag{16}$$

Solving Eqs. (14) and (15), under conditions (16) and then substituting in (13), we get;

$$U = (A_7 + \alpha A_{10})e^{-A_2 Y} + (A_8 + \alpha A_{11})e^{-A_4 Y} + (A_9 + \alpha A_{13})e^{-A_6 Y} \tag{17}$$

Skin friction coefficient

The skin-friction is an important physical parameter for this type of boundary layer flow. Knowing the velocity field, the skin-friction at the plate can be obtained, which in non-dimensional form is given by;

$$\tau = \frac{\tau'}{\rho u_w v_0} = -\left(\frac{dU}{dY}\right)_{Y=0} + \alpha\left(\frac{d^2U}{dY^2}\right)_{Y=0}$$

$$\begin{aligned}\tau = A_1A_4 + A_2A_5 + A_3A_6 + \alpha\{A_1A_7 + A_2A_8 + A_3A_{10} + \\ A_1^2A_4 + A_2^2A_5 + A_3^2A_6\} + \alpha^2\{A_1^2A_7 + A_2^2A_8 + A_3^2A_{10}\}\end{aligned} \tag{18}$$

where $A_1 = \frac{\Pr + \sqrt{\Pr^2 + 4\phi \Pr}}{2}$ $A_2 = \frac{Sc + \sqrt{Sc^2 + 4KrSc}}{2}$ $A_3 = \frac{1 + \sqrt{1 + 4(M + 1/K)}}{2}$

$$A_4 = \frac{-G\mathrm{r}}{A_1(A_1^2 - A_1 - (M + 1/K))} \quad A_5 = \frac{-Gm}{A_2(A_2^2 - A_2 - (M + 1/K))} \quad A_6 = 1 - A_4 - A_5$$

$$A_7 = \frac{-A_1^3A_4}{A_1^2 - A_1 - (M + 1/K)} \quad A_8 = \frac{-A_2^3A_5}{A_2^2 - A_2 - (M + 1/K)} \quad A_9 = \frac{-A_3^3A_6}{A_3^2 - A_3 - (M + 1/K)}$$

$$A_{10} = -(A_7 + A_8)$$

Results and discussions

The formulation of the problem, which accounts for the effects of MHD visco-elastic fluid over a continuously moving vertical surface with porous medium and chemical reaction, has been performed in the preceding sections. The governing equations of the flow field were solved analytically and the expressions for the velocity, temperature, concentration, and skin-friction were obtained. In order to get physical insight into the problem, the velocity, temperature, and concentration fields have been discussed by assigning numerical values of magnetic parameter M, Schmidt number Sc, visco-elastic parameter α, and heat generation/absorption parameter ϕ.

In order to verify the accuracy of the present work, particular results are compared with those available in the literature through **Table 1**. The numerical values of skin-friction obtained in the present case are compared with those of Kumar *et al*. [21] for different values of magnetic parameter *M* and fixed values of the remaining parameters. It is clearly seen that there is an excellent agreement between the respective research of our study and those of Kumar *et al*. [21]. This has established confidence in the analytical results reported in this article.

Figures 2 and **3** display the effect of chemical reaction parameter Kr on the velocity distribution and the species concentration profiles, respectively. As expected, the presence of a chemical reaction significantly affects the concentration profiles as well as the velocity profiles. It should be mentioned that the studied case is for a destructive chemical reaction *Kr* > *O*. In fact, as Kr increases, considerable reduction in the velocity profiles is predicted, and the concentration values decrease. Here, the velocity of non-Newtonian fluid is greater than the Newtonian fluid, due to the viscosity effects of fluid. The Newtonian fluid has less friction with the plate surface as compared to non-Newtonian fluid.

Figures 4 and **5** show the effect of Schmidt number Sc on the velocity and concentration profiles, respectively. As the Schmidt number increases, the velocity and concentration distributions decrease. This causes the concentration buoyancy effects to decrease, yielding a reduction in the fluid velocity.

The graphs of the velocity and temperature profiles for different values of heat source/sink parameter ϕ are shown graphically in the **Figures 6** and **7** respectively. Physically speaking, the presence of a heat source/sink effects has the tendency to reduce the fluid temperature. This is due to the fact that, when heat is absorbed, the buoyancy force decreases, which retards the flow rate and thereby gives rise to a decrease in the velocity and temperature profiles. These behaviors are obvious from the graphs.

Figure 8 depicts the influence of the visco-elastic parameter α on the velocity profiles. Obviously, increasing the visco-elastic parameter α causes a slight increase in the velocity profiles in the early stages of the velocity growth. The effect of increasing values of visco-elastic parameter is to reduce the horizontal velocity and thereby reduces the boundary layer thickness and, hence, induces an increase in the absolute value of the velocity gradient at the surface, i.e., the thickness of the boundary layer is much larger for higher values of the visco-elastic parameter .

Figure 9 illustrates the effect of the magnetic parameter *M* on the velocity profiles in the boundary layer. It is noticed from the figure that the velocity profiles decrease with increasing values of Hartman number M in the boundary layer. Application of a transverse magnetic field to an electrically conducting fluid gives rise to a resistive-type force called the Lorentz force. This force has the tendency to slow down the motion of the fluid in the boundary layer. Also, we notice that the velocity of the non-Newtonian fluid is greater than that of Newtonian fluid, as mentioned in [25].

Table 1 Effect of *M* on skin friction for Pr = 0.88, Gr = 2.5, K = 8.7, Gm = 6.68, Kr = 6.67, Sc = 6.0.

	M = 1.0	**M = 2.0**	**M = 3.0**	**M = 4.0**
Rajesh Kumar [20] ϕ = 0.0, K = ∞, Gm = 0.0, Kr = 0.0, α = 0	1.366	1.449	1.525	1.595
Present study	1.3661	1.444	1.516	1.584

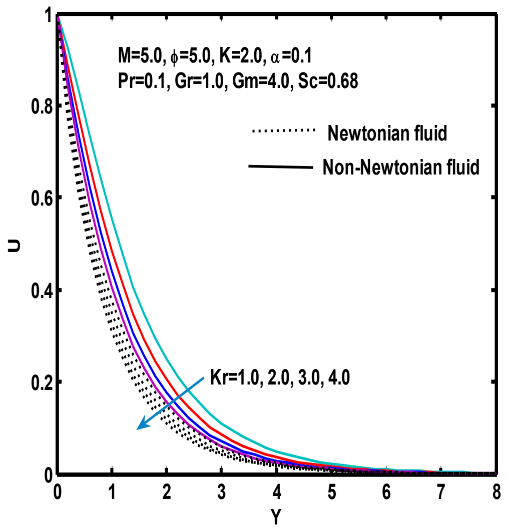

Figure 2 Effect of Kr on velocity.

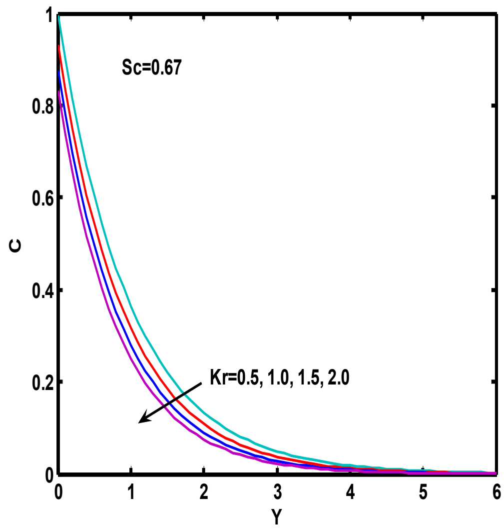

Figure 3 Effect of Kr on concentration.

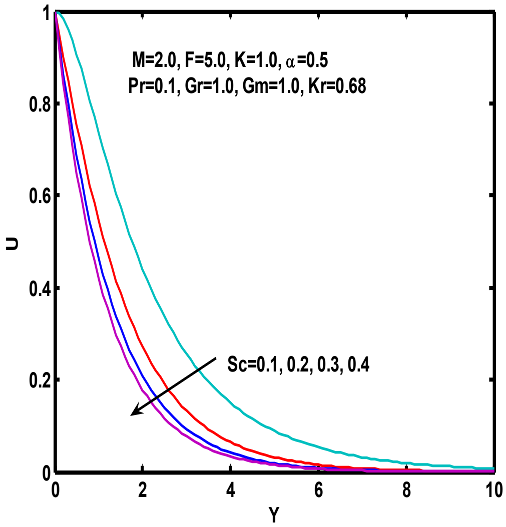

Figure 4 Effect of Sc on velocity.

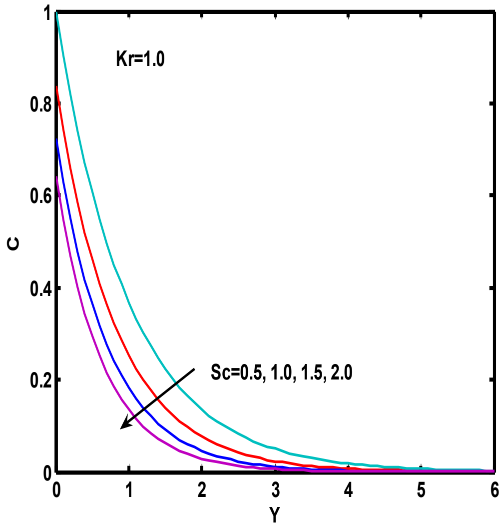

Figure 5 Effect of Sc on concentration.

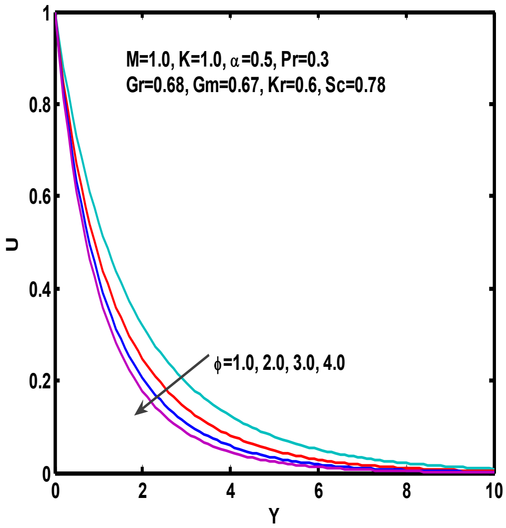

Figure 6 Effect of ϕ on velocity.

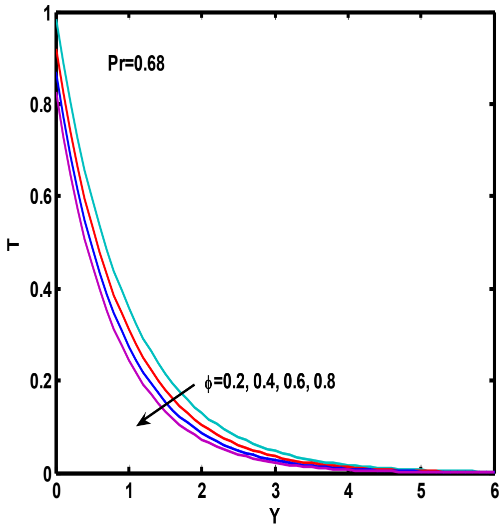

Figure 7 Effect of ϕ on temperature.

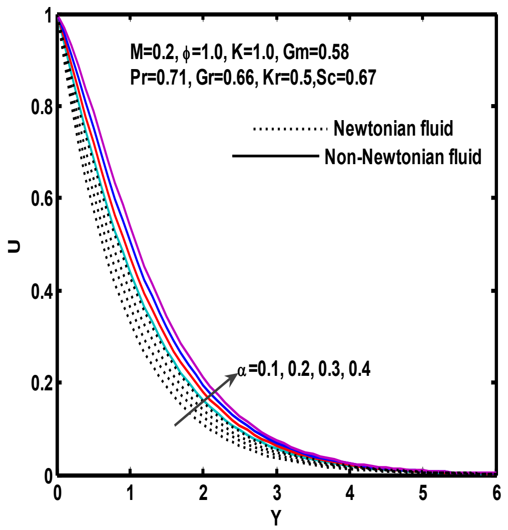

Figure 8 Effect of α on velocity.

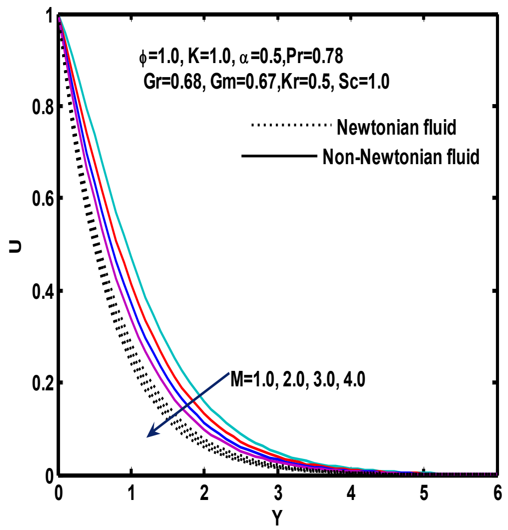

Figure 9 Effect of M on velocity.

Conclusions

We have presented results for the flow field of a fluid, which is called the second order fluid or the fluid of grade 2, on a continuously moving vertical porous surface with uniform suction in the presence of chemical reaction and heat generation. A systematic study on the effects of visco-elastic parameter and other physical parameters controlling the flow heat and mass transfer characteristics has been carried out. The following conclusions can be drawn from the computed results:

1) The effect of visco-elastic parameter is to decrease velocity distribution in the boundary layer. A similar effect for magnetic field is seen in the velocity profiles.

2) With different values of heat source parameter ϕ, it is observed that the velocity and temperature decreases with increase in heat source parameter.

3) A comparison of the second-grade fluid with the viscous case shows that, in the case of second-grade fluid, for different values of M, Kr, α velocity is greater than that of the viscous case.

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