Editorial: Special Issue on Analytical and Approximate Solutions for Numerical Problems

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Though methods and algorithms in numerical analysis are not new, they have become increasingly popular with the development of high speed computing capabilities. Indeed, the ready availability of high speed modern digital computers and easy-to-employ powerful software packages has had a major impact on science, engineering education and practice in the recent past. Researchers in the past had to depend on analytical skills to solve significant engineering problems but, nowadays, researchers have access to tremendous amount of computation power under their fingertips, and they mostly require understanding the physical nature of the problem and interpreting the results. For some problems, several approximate analytical solutions already exist for simple cases but finding new solution to complex problems by designing and developing novel techniques and algorithms are indeed a great challenging task to give approximate solutions and sufficient accuracy especially for engineering purposes. In particular, it is frequently assumed that deriving an analytical solution for any problem is simpler than obtaining a numerical solution for the same problem. But in most of the cases relationships between numerical and analytical solutions complexities are exactly opposite to each other. In addition, analytical solutions are limited to relatively simple problems while numerical ones can be obtained for complex realistic situations. Indeed, analytical solutions are very useful for testing (benchmarking) numerical codes and for understanding principal physical controls of complex processes that are modeled numerically. During the recent past, in order to overcome some numerical difficulties a variety of numerical approaches were introduced, such as the finite difference methods (FDM), the finite element methods (FEM), and other alternative methods. Numerical methods typically include material on such topics as computer precision, root finding techniques, solving systems of equations, differentiation and integration, and interpolation and approximation to name a few.

In case of analytical investigations one has to employ related formula to obtain the analysis results. However, many systems possess complex functionality which is hard to track the system behavior by formula. In such cases, the end user has to simulate the system and analyze it, in some well-defined situation to estimate its behavior. It is known, that, numerical solutions are the result of a differential equation solved by employing suitable numerical methods, where the exact solution is not given, rather an approximate solution is given instead. It is to be noted that, analytic solutions arise from analytic methods, where the exact form of the solution is given. Analytical methods are dividing a system logically into basic parts and reasoning or action from a perception of the parts and interrelations of the subject. But, in contrary, simulation is a technique of conducting experiments using models of a system to figure out the behavior at different environments.

It is known that there are 3 different types of methods such as experimental, analytical and numerical in order to solve field problems. In case of experimental type solving field problems are expensive, time consuming, and usually do not allow much flexibility in parameter variation. Analytical methods are the most rigorous ones, providing exact solutions, but they become hard to use for complex problems. In contrast, numerical methods have become popular with the development of computing capabilities, and although they give approximate solutions, they have sufficient accuracy for solving engineering problems in real time. The most satisfactory solution of a field problem is an exact mathematical one. However, in many practical cases such an analytical solution cannot be obtained and the end user must resort to numerical approximate solution, analytical solutions are useful in checking solutions obtained from numerical methods. But, every numerical method involves an analytic simplification to the point where it is easy to apply numerical method.

Analytical models are mathematical models that have a closed form solution, i.e. the solution to the equations employed to describe changes in a system can be expressed as a mathematical analytic function. In contrast, the numerical models are mathematical models that employ some sort of numerical time-stepping procedure to obtain the models behavior over time. The mathematical solution is represented by a generated table and/or graph. It is to be noted that analytical method and numerical method is to serve for different purposes. Analytical method is to understand the mechanism and physical effects through the model problem. Moreover, it is useful to validate the numerical method. The numerical method is primarily to solve complex problem, physically or geometrically. It is pertinent to pin point out that the "numerical approach" is not automatically equivalent to the "approach with use of computer", although the end user usually employ numerical approach to find the solution with the help of modern digital computers. This is due to the high computer performance incomparable to abilities of human brain. Numerical approach enables solution of a complex problem with a great number (but) of very simple operations. The major advantage of numerical methods over analytical methods is that they are easily implementable with modern day computers and provide solutions in quick time as compared to analytical methods. However, the advantages of numerical methods are also conditional. The numerical solution method selected must have the important properties of consistency, stability, convergence, conservation, boundedness, realisability and most importantly accuracy. In particular, the numerical solutions of fluid flow and heat transfer problems are only approximate solutions. The systematic errors due to modeling, discretisation and iteration must be minimum (set as a condition). Suppose, if an exact solution is possible analytically and if the requirement is an exact solution then there is no advantage of numerical method.

The main intention of this special issue is to provide an analytical and approximate solution for some numerical problems of ordinary differential equations (ODEs). Furthermore, it focuses on constructing new numerical methods and algorithms for ODEs including mathematical analysis and its behaviour. Particularly, with a study of well-posedness initial value problems and boundary value problems for a first/second order differential equations and system of such equations are also carried out for better understanding. Sometimes, it is not easy to be a good numerical analyst, since, as in other branches of mathematics, because it takes considerable skill and experience to be able to leap nimbly to and fro from the general and abstract to the particular and practical. Moreover, numerical analysis is well motivated and employs many important mathematical concepts.

There are numerous research opportunities which are open and available in the field of numerical analysis, thus end user can easily find topics for research and venues for presenting their results. Indeed, researchers can gain sufficient knowledge to be able to identify some area of study, and as they become more and more familiar with different algorithms, they can easily develop applications for research problems in numerous fields. Researchers were able to explore in greater detail some topic that interested them and to see first-hand the importance of computation and analysis in real world applications. Some of the applications of numerical methods are computer graphics (root finding, interpolation, curve fitting, optimization, ODE solver, PDE solver, finite element method) such as physics-based animation, geometry modelling; computer vision (optimization, curve fitting, linear equations), such as stereo vision, shape from shading; machine learning (curve fitting, linear equations, function approximation, such as pattern recognition, neural network; simulation for prototyping (ODE solver, PDE solver, optimization, numerical integration, interpolation, finite element method such as circuit design, mechanical design, CAD/CAM; generate motion based on physical laws (e.g., Newton's laws, Fluid dynamics); simulated physical phenomena such as gravity, momentum, collision, friction, fluid flow (liquid, gas, turbulence), flexibility, elasticity, fracture and so on. Some of the application software which really helps to solve real time problems through numerical methods is MATLAB, MATHEMATICA, Maple and MathCAD, etc.

As a guest editor of this special issue, I would like to take this opportunity to acknowledge the effort of many reviewers, editorial board members and editor-in-chief who shape the content of this volume perfect.