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Asymptotic Stability of a Delay-Difference Control System of Hopfield Neural Networks via Matrix Inequalities and Applications

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ABSTRACT

We have established a new sufficient condition for the asymptotic stability of a delay-difference control system of Hopfield neural networks in terms of certain matrix inequalities (based on a discrete analog of the Lyapunov second method). The result has been applied to obtain new stability conditions for some class of delay-difference control system such as delay-difference control system of Hopfield neural networks with multiple delays in terms of certain matrix inequalities.

Keywords: Hopfield neural networks, delay-difference control system, asymptotic stability, lyapunov function, matrix inequalities

INTRODUCTION

In recent decades, Hopfield neural networks have been extensively studied in many aspects and successfully applied to many fields such as pattern identifying, voice recognition, system controls, signal processing systems, static image treatment, and solving of nonlinear algebraic systems, etc. Such applications are based on the existence of equilibrium points, and the qualitative properties of the systems. In electronic implementation, time delays occur for many reasons such as circuit integration, switching delays of the amplifiers and communication delays, etc. Therefore, the study of the asymptotic stability of Hopfield neural networks with delays is of particular importance in the manufacture of high quality microelectronic Hopfield neural networks.

While stability analysis of continuous-time neural networks can employ the stability theory of differential systems [1], it is much harder to study the stability of discrete-time neural networks [2] with time delays [3] or impulses [4]. The techniques currently available in the literature for discrete-time systems are mostly based on the construction of the Lyapunov second method [5]. For the Lyapunov second method, it is well known that no general rule exists to guide the construction of a proper Lyapunov function for a given system. In fact, the construction of the Lyapunov function becomes a very difficult task.

In this paper, we consider a delay-difference control system of Hopfield neural networks of the form

$$v(k+1) = -Av(k) + BS(v(k-h)) + Cu(k) + f\ , \tag{1}$$

where $v(k) \in \Omega \subseteq \mathbb{R}^n$ is the neuron state vector, $h \geq 0$, $A = diag\{a_1, \ldots, a_n\}$, $a_i \geq 0$, $i = 1, 2, \ldots, n$ is the $n \times n$ constant relaxation matrix,
B is the $n \times n$ constant weight matrix,
C is $n \times m$ constant matrix, $u(k) \in \mathbb{R}^m$ is the control vector,
$f = (f_1, \ldots, f_n) \in \mathbb{R}^n$ is the constant external input vector and $S(z) = [s_1(z_1), \ldots, s_n(z_n)]^T$ with $s_i \in C^1\left[\mathbb{R}, (-1,1)\right]$ where s_i are the neuron activations and monotonically increase for each $i = 1, 2, \ldots, n$.

The asymptotic stability of the zero solution of the delay-differential system of Hopfield neural networks has been developed for several years. We refer to monographs by Burton [6] and Ye [7] and the references cited therein. Much less is known regarding the asymptotic stability of the zero solution of the delay-difference control system of Hopfield neural networks. Therefore, the purpose of this paper is to establish sufficient conditions for the asymptotic stability of the zero solution of (1) in terms of certain matrix inequalities.

Preliminaries

The following notations will be used throughout the paper. $\mathbb{R}^+$ denotes the set of all non-negative real numbers; $\mathbb{Z}^+$ denotes the set of all non-negative integers; $\mathbb{R}^n$ denotes the n-finite-dimensional Euclidean space with the Euclidean norm $\|.\|$ and the scalar product between x and y defined by $x^T y$; $\mathbb{R}^{n\times m}$ denotes the set of all $(n \times m)$-matrices; and A^T denotes the transposition of the matrix A.

Matrix $Q \in \mathbb{R}^{n\times n}$ is positive semidefinite $(Q \geq 0)$ if $x^T Qx \geq 0$, for all $x \in \mathbb{R}^n$. If $x^T Qx > 0$ ($x^T Qx < 0$, resp.) for any $x \neq 0$, then Q is positive (negative, resp.) definite and denoted by $Q > 0, (Q < 0,$ resp.). It is easy to verify that $Q > 0, (Q < 0,$ resp.) if

$$\exists \beta > 0: \ \ x^T Qx \geq \beta \|x\|^2, \forall x \in \mathbb{R}^n,$$
$$(\ \exists \beta > 0: \ x^T Qx \leq -\beta \|x\|^2, \forall x \in \mathbb{R}^n, \ \text{resp.})\ .$$

Fact 2.1 For any positive scalar ε and vectors x and y, the following inequality holds:

$$x^T y + y^T x \le \varepsilon x^T x + \varepsilon^{-1} y^T y.$$

Let us denote $V_\delta = \{x \in \mathbb{R}^n : \ \|x\| < \delta\}$.

Lemma 2.1 [8] The zero solution of the difference system is asymptotically stable if there exists a positive definite function $V(x) : \mathbb{R}^n \to \mathbb{R}^+$ such that

$$\exists \beta > 0 : \Delta V(x(k)) = V(x(k+1)) - V(x(k)) \le -\beta \|x(k)\|^2,$$

along the solution of the system. In case the above condition holds for all $x(k) \in V_\delta$, we say that the zero solution is locally asymptotically stable.

We present the following technical lemmas, which will be used in the proof of our main result.

Lemma 2.2 [9] For any constant symmetric matrix $M \in \mathbb{R}^{n\times n}$, $M = M^T > 0$, scalar $s \in \mathbb{Z}^+ / \{0\}$, vector function $W : [0, s] \to \mathbb{R}^n$, we have

$$s\sum_{i=0}^{s-1}(w^T(i)Mw(i)) \ge \left(\sum_{i=0}^{s-1} w(i)\right)^T M \left(\sum_{i=0}^{s-1} w(i)\right).$$

Main results

In this section, we consider the sufficient conditions for asymptotic stability of the zero solution v^* of (1) in terms of certain matrix inequalities. Without loss of generality, we can assume that $v^* = 0, S(0) = 0$ and f =0 (for otherwise, we let $x = v - v^*$ and define $S(x) = S(x + v^*) - S(v^*))$.

The new form of (1) is now given by

$$x(k+1) = -Ax(k) + BS(x(k-h)) + Cu(k). \qquad (2)$$

This is a basic requirement for the controller design. Now, we are interested designing a feedback controller for the system (2) as

$$u(k) = Kx(k),$$

where K is $n \times m$ constant control gain matrix.

The new form of (2) is now given by

$$x(k+1) = -Ax(k) + BS(x(k-h)) + CKx(k). \qquad (3)$$

Throughout this paper we assume that the neuron activations $s_i(x_i)$, $i = 1, 2, \ldots, n$ are bounded and monotonically nondecreasing on $\mathbb{R}$, and $s_i(x_i)$ is Lipschitz continuous, that is, there exist constants $l_i > 0$, $i = 1, 2, ..., n$ such that

$$|s_i(v_1) - s_i(v_2)| \le l_i |v_1 - v_2|, \qquad \forall v_1, v_2 \in \mathbb{R}. \tag{4}$$

By condition (4), $s_i(x_i)$ satisfy

$$|s_i(x_i)| \le l_i |x_i|, \; i = 1, 2, ..., n. \tag{5}$$

Theorem 3.1 *The zero solution of the delay-difference control system (3) is asymptotically stable if there exist symmetric positive definite matrices* $P, G, W, L = diag[l_1, \ldots, l_n] > 0$ *and* $\varepsilon > 0$ *satisfying the following matrix inequalities of the form*

$$\psi = \begin{pmatrix} (1,1) & 0 & 0 \\ 0 & (2,2) & 0 \\ 0 & 0 & (3,3) \end{pmatrix} < 0, \tag{6}$$

where

$$(1,1) = APA - APCK - K^T C^T PA - C^T K^T PC - P + hG + W$$
$$+ \varepsilon APBB^T PA + \varepsilon_1 K^T C^T PBB^T PCK,$$
$$(2,2) = \varepsilon^{-1} LL + \varepsilon_1^{-1} LL + LB^T PBL - W,$$
$$(3,3) = -hG.$$

Proof Consider the Lyapunov function $V(y(k)) = V_1(y(k)) + V_2(y(k)) + V_3(y(k))$, where

$$V_1(y(k)) = x^T(k) P x(k),$$
$$V_2(y(k)) = \sum_{i=k-h+1}^{k} (h - k + i) x^T(i) G x(i),$$
$$V_3(y(k)) = \sum_{i=k-h+1}^{k} x^T(i) W x(i),$$

P, G, and W being symmetric positive definite solutions of (6) and $y(k) = [x(k), x(k-h)]$.

Then the difference of $V(y(k))$ along trajectory of the solution of (3) is given by $\Delta V(y(k)) = \Delta V_1(y(k)) + \Delta V_2(y(k)) + \Delta V_3(y(k))$, where

$$\begin{aligned}
\Delta V_1(y(k)) &= V_1(x(k+1)) - V_1(x(k)) \\
&= [-Ax(k) + BS(x(k-h)) + CKx(k)]^T \\
&\quad \times P[-Ax(k) + BS(x(k-h)) + CKx(k)] \\
&\quad -x^T(k)Px(k) \\
&= x^T(k)[APA - APCK - K^T C^T PA \\
&\quad -C^T K^T PC - P]x(k) \\
&\quad -x^T(k)APBS(x(k-h)) \\
&\quad -S^T(x(k-h))B^T PAx(k) \\
&\quad +x^T(k)K^T C^T PBS(x(k-h)) \\
&\quad +S^T(x(k-h))B^T PCKx(k) \\
&\quad +S^T(x(k-h))B^T PBS(x(k-h)),
\end{aligned}$$

$$\Delta V_2(y(k)) = \Delta\left(\sum_{i=k-h+1}^{k} (h-k+i)x^T(i)Gx(i)\right) = hx^T(k)Gx(k) - \sum_{i=k-h+1}^{k} x^T(i)Gx(i),$$

$$\Delta V_3(y(k)) = \Delta\left(\sum_{i=k-h+1}^{k} x^T(i)Wx(i)\right) = x^T(k)Wx(k) - x^T(k-h)Wx(k-h), \qquad (7)$$

where (5) and **Fact 2.1** are utilized in (7), respectively.
Note that

$$\begin{aligned}
-x^T(k)APBS(x(k-h)) - S^T(x(k-h))B^T PAx(k) &\le \varepsilon x^T(k)APBB^T PAx(k) \\
&\quad +\varepsilon^{-1} S^T(x(k-h))S(x(k-h)), \\
x^T(k)K^T C^T PBS(x(k-h)) + S^T(x(k-h))B^T PCKx(k) &\le \varepsilon_1 x^T(k)K^T C^T PBB^T PCKx(k) \\
&\quad +\varepsilon_1^{-1} S^T(x(k-h))S(x(k-h)),
\end{aligned}$$

$$S^T(x(k-h))B^T PBS(x(k-h)) \le x^T(k-h)LB^T PBLx(k-h),$$

$$\varepsilon^{-1} S^T(x(k-h))S(x(k-h)) \le \varepsilon^{-1} x^T(k-h)LLx(k-h),$$

$$\varepsilon_1^{-1} S^T(x(k-h))S(x(k-h)) \le \varepsilon_1^{-1} x^T(k-h)LLx(k-h),$$

hence

$$\begin{aligned}
\Delta V_1 &\le x^T(k)[APA - APCK - K^T C^T PA - C^T K^T PC - P]x(k) \\
&\quad +\varepsilon x^T(k)APBB^T PAx(k) + \varepsilon_1 x^T(k)K^T C^T PBB^T PCKx(k) \\
&\quad +\varepsilon^{-1} x^T(k-h)LLx(k-h) + \varepsilon_1^{-1} x^T(k-h)LLx(k-h) \\
&\quad +x^T(k-h)LB^T PBLx(k-h)
\end{aligned}$$

Then we have

$$\begin{aligned}\Delta V \le\ & x^T(k)[APA - APCK - K^TC^TPA - C^TK^TPC - P + hG + W \\ & +\varepsilon APBB^TPA + \varepsilon_1 K^TC^TPBB^TPCK]x(k) \\ & + x^T(k-h)[\varepsilon^{-1}LL + \varepsilon_1^{-1}LL + LB^TPBL - W]x(k-h) \\ & - \sum_{i=k-h}^{k-1} x^T(i)Gx(i).\end{aligned}$$

Using **Lemma 2.2**, we obtain

$$\sum_{i=k-h+1}^{k} x^T(i)Gx(i) \ge \left(\frac{1}{h}\sum_{i=k-h+1}^{k} x(i)\right)^T (hG)\left(\frac{1}{h}\sum_{i=k-h+1}^{k} x(i)\right).$$

From the above inequality it follows that:

$$\begin{aligned}\Delta V \le\ & x^T(k)[APA - APCK - K^TC^TPA - C^TK^TPC - P + hG + W \\ & +\varepsilon APBB^TPA + \varepsilon_1 K^TC^TPBB^TPCK]x(k) \\ & + x^T(k-h)[\varepsilon^{-1}LL + \varepsilon_1^{-1}LL + LB^TPBL - W]x(k-h) \\ & - \left(\frac{1}{h}\sum_{i=k-h+1}^{k} x(i)\right)^T (hG)\left(\frac{1}{h}\sum_{i=k-h+1}^{k} x(i)\right) \\ & = \left(x^T(k), x^T(k-h), \Big(\frac{1}{h}\sum_{i=k-h+1}^{k} x(i)\Big)^T\right)\begin{pmatrix}(1,1) & 0 & 0 \\ 0 & (2,2) & 0 \\ 0 & 0 & (3,3)\end{pmatrix} \\ & \times \begin{pmatrix} x(k) \\ x(k-h) \\ \Big(\frac{1}{h}\sum_{i=k-h+1}^{k} x(i)\Big)\end{pmatrix} \\ & = y^T(k)\psi y(k),\end{aligned}$$

where

$$\begin{aligned}(1,1) =\ & APA - APCK - K^TC^TPA - C^TK^TPC - P + hG + W \\ & +\varepsilon APBB^TPA + \varepsilon_1 K^TC^TPBB^TPCK, \\ (2,2) =\ & \varepsilon^{-1}LL + \varepsilon_1^{-1}LL + LB^TPBL - W, \\ (3,3) =\ & -hG,\end{aligned}$$

$$y(k) = \begin{pmatrix} x(k) \\ x(k-h) \\ (\frac{1}{h}\sum_{i=k-h+1}^{k} x(i)) \end{pmatrix}$$

By the condition (6), $\Delta V(y(k))$ is negative definite, namely there is a number $\beta > 0$ such that $\Delta V(y(k)) \le -\beta \|y(k)\|^2$, and hence, the asymptotic stability of the system immediately follows from **Lemma 2.1**. This completes the proof.

Remark 3.1 Theorem 3.1 gives a sufficient condition for the asymptotic stability of the delay-difference system (2) via matrix inequalities. These conditions are described in terms of certain diagonal matrix inequalities, which can be realized by using the linear matrix inequality algorithm proposed in [10]. In the case of Hu and Wang [11] these conditions describe asymptotic stability of the delay-difference system of Hopfield neural networks via matrix inequalities in terms of certain symmetric matrix inequalities, which can be realized by using the Schur complement lemma and linear matrix inequality algorithm proposed in [10].

Example 3.1 Let us consider a delay-difference control system of Hopfield neural networks (3), given by the system

$$x(k+1) = -Ax(k) + BS(x(k-h)) + CKx(k),$$

where the matrices are

$$A = \begin{pmatrix} 3.5 & 0 \\ 0 & 1.8 \end{pmatrix},\ B = \begin{pmatrix} 0.5588 & -0.0824 \\ 0.0706 & 0.7412 \end{pmatrix},\ s_i(x_i) = \frac{2}{\pi}\tan^{-1}(x_i), i = 1,2,\ C = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

$$K = \begin{pmatrix} 15.2369 & 6.9865 \end{pmatrix},\ \varepsilon = 0.5 \text{ and } h = 1.$$

Using the LMI Toolbox in MATLAB, we found that the LMIs in **Theorem 3.1** are feasible and

$P = \begin{pmatrix} 0.2795 & 0.0722 \\ 0.0722 & 0.5725 \end{pmatrix}$, $W = \begin{pmatrix} 0.9018 & 0.3625 \\ 0.3625 & 1.0628 \end{pmatrix}$, $L = \begin{pmatrix} 2.7293 & 0 \\ 0 & 2.8806 \end{pmatrix}$ are a set of solutions to the LMIs (6).

Therefore, we have

$$\psi = \begin{pmatrix} -0.0245 & 0 \\ 0 & -0.1896 \end{pmatrix}$$

The eigenvalues are -0.0245 and -0.1896, respectively. This implies the matrix $\psi < 0$. It follows from Lemma 2.1 that the zero solution of the delay-difference control system of Hopfield neural networks is asymptotically stable.

Applications

In this section, we apply the main result of this paper, which provides a sufficient condition for the asymptotic stability of the delay-difference control system of Hopfield neural networks with multiple delays in terms of certain matrix inequalities.

We consider the delay-difference control system of Hopfield neural networks with multiple delays of the form

$$v(k+1) = -Av(k) + \sum_{i=1}^{m} B_i S(v(k-h_i)) + Cu(k) + f\,, \tag{8}$$

where $v(k) \in \Omega \subseteq \mathbb{R}^n$ is the neuron state vector, $0 \le h_1 \le \ldots \le h_m$, $A = diag\{a_1, \ldots, a_n\}$, $a_i \ge 0$, $i = 1,2,...,n$ is the $n \times n$ constant relaxation matrix, B_i, $i = 1,2,...,n$ are the $n \times n$ constant weight matrices, C is $n \times m$ constant matrix, $u(k) \in \mathbb{R}^m$ is the control vector, $f = (f_1, \ldots, f_n) \in \mathbb{R}^n$ is the constant external input vector and $S(z) = [s_1(z_1), \ldots, s_n(z_n)]^T$ with $s_i \in C^1[\mathbb{R}, (-1,1)]$ where s_i is the neuron activations and monotonically increasing for each $i = 1,2,...,n$.

We consider the sufficient conditions for asymptotic stability of the zero solution v^* of (8) in terms of certain matrix inequalities. Without loss of generality, we can assume that $v^* = 0, S(0) = 0$ and $f = 0$ (for otherwise, we let $x = v - v^*$ and define $S(x) = S(x + v^*) - S(v^*))$.

The new form of (8) is now given by

$$x(k+1) = -Ax(k) + \sum_{i=1}^{m} B_i S(x(k-h_i)) + Cu(k) \tag{9}$$

This is a basic requirement for controller design. Now, we are interested in designing a feedback controller for the system (9) as

$$u(k) = Kx(k)\,,$$

where K is an $n \times m$ constant control gain matrix.

The new form of (9) is now given by

$$x(k+1) = -Ax(k) + \sum_{i=1}^{m} B_i S(x(k-h_i)) + CKx(k) \tag{10}$$

Theorem 4.1 *The zero solution of the delay-difference system of Hopfield neural networks with multiple delays (10) is asymptotically stable if there exist symmetric positive definite matrices P, G_i, W_i, $L = diag[l_1, \ldots, l_n] > 0$, $i = 1,2,...,m$ and $\varepsilon, \varepsilon_1 > 0$ satisfying the following matrix inequalities of the form*

$$\psi=\begin{bmatrix} (0,0) & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & (1,1) & (1,2) & \cdots & (1,m) & 0 & 0 & 0 & \cdots & 0 \\ 0 & (2,1) & (2,2) & \cdots & (2,m) & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & (m,1) & (m,2) & \cdots & (m,m) & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & (m+1,m+1) & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & (m+2,m+2) & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & (2m,2m) \end{bmatrix}<0 \tag{11}$$

where

$$(0,0)=APA-APCK-K^TC^TPA-C^TK^TPC-P+\sum_{i=1}^{m}(h_iG_i+W_i),$$

$$+\varepsilon\sum_{i=1}^{m}\sum_{j=1}^{m}APB_iB_j^{\ T}PA+\varepsilon_1\sum_{i=1}^{m}\sum_{j=1}^{m}K^TC^TPB_iB_j^{\ T}PCK$$

$$(1,1)=\varepsilon^{-1}LL+\varepsilon_1^{-1}LL+LB_1^{\ T}PB_1L-W_1 ,$$

$$(1,2)=\varepsilon^{-1}LL+\varepsilon_1^{-1}LL+LB_1^{\ T}PB_2L ,$$

$$(1,m)=\varepsilon^{-1}LL+\varepsilon_1^{-1}LL+LB_1^{\ T}PB_mL ,$$

$$(2,1)=\varepsilon^{-1}LL+\varepsilon_1^{-1}LL+LB_2^{\ T}PB_1L ,$$

$$(2,2)=\varepsilon^{-1}LL+\varepsilon_1^{-1}LL+LB_2^{\ T}PB_2L-W_2 ,$$

$$(2,m)=\varepsilon^{-1}LL+\varepsilon_1^{-1}LL+LB_2^{\ T}PB_mL ,$$

$$(m,1)=\varepsilon^{-1}LL+\varepsilon_1^{-1}LL+LB_m^{\ T}PB_1L ,$$

$$(m,2)=\varepsilon^{-1}LL+\varepsilon_1^{-1}LL+LB_m^{\ T}PB_2L ,$$

$$(m,m)=\varepsilon^{-1}LL+\varepsilon_1^{-1}LL+LB_m^{\ T}PB_mL-W_m ,$$

$$(m+1,m+1)=-h_1G_1 ,$$

$$(m+2,m+2)=-h_2G_2 ,$$

$$(2m,2m)=-h_mG_m .$$

Proof Consider the Lyapunov function $V(y(k))=V_1(y(k))+V_2(y(k))+V_3(y(k))$, where

$$V_1(y(k))=x^T(k)Px(k),$$

$$V_2(y(k)) = \sum_{i=1}^{m} \sum_{j=k-h_i+1}^{k} (h-k+i)x^T(j)G_i x(j),$$

$$V_3(y(k)) = \sum_{i=1}^{m} \sum_{j=k-h_i+1}^{k} x^T(j)W_i x(j),$$

P, G_i, and W_i, $i = 1, 2, ..., m$ being symmetric positive definite solutions of (11) and $y(k) = \left[x(k), x(k-h_1), \ldots, x(k-h_m)\right]$.
Then the difference of $V(y(k))$ along a trajectory of the solution of (10) is given by

$$\Delta V(y(k)) = \Delta V_1(y(k)) + \Delta V_2(y(k)) + \Delta V_3(y(k)),$$

where

$$\begin{aligned}
\Delta V_1(y(k)) &= V_1(x(k+1)) - V_1(x(k)) \\
&= [-Ax(k) + \sum_{i=1}^{m} B_i S(x(k-h_i)) + CKx(k)]^T P[-Ax(k) + \sum_{i=1}^{m} B_i S(x(k-h_i)) + CKx(k)] \\
&\quad - x^T(k)Px(k) \\
&= x^T(k)[APA - APCK - K^T C^T PA - C^T K^T PC - P]x(k) \\
&\quad - \sum_{i=1}^{m} x^T(k)APB_i S(x(k-h_i)) - \sum_{i=1}^{m} S^T(x(k-h_i))B_i^T PAx(k) \\
&\quad + \sum_{i=1}^{m} x^T(k)K^T C^T PB_i S(x(k-h_i)) + S^T(x(k-h_i))B_i^T PCKx(k) \\
&\quad + \sum_{i=1}^{m}\sum_{j=1}^{m} S^T(x(k-h_i))B_i^T PB_j S(x(k-h_j)),
\end{aligned}$$

$$\Delta V_2(y(k)) = \Delta\left(\sum_{i=1}^{m} \sum_{j=k-h_i+1}^{k} (h_i - k + j)x^T(j)G_i x(j) \right) = \sum_{i=1}^{m} h_i x^T(k)G_i x(k) - \sum_{i=1}^{m} \sum_{j=k-h_i+1}^{k} x^T(j)G_i x(j),$$

$$\Delta V_3(y(k)) = \Delta\left(\sum_{i=1}^{m} \sum_{j=k-h_i+1}^{k} x^T(j)W_i x(j) \right) = \sum_{i=1}^{m} x^T(k)W_i x(k) - \sum_{i=1}^{m} x^T(k-h_i)W_i x(k-h_i).$$

The rest of the proof is similar to that of **Theorem 3.1** need hold.

CONCLUSIONS

In this paper, we have established a sufficient condition for the asymptotic stability of a delay-difference control system of Hopfield neural networks in terms of certain matrix inequalities based on a discrete analog of the Lyapunov second method. The result has been applied to obtain new stability conditions for some class of delay-difference control system such as delay-difference control system of Hopfield neural networks with multiple delays in terms of certain matrix inequalities.

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บทคัดย่อ

เกรียงไกร ราชกิจ

การมีเสถียรภาพเชิงเส้นกำกับของของระบบเชิงผลต่างของแบบจำลองข่ายงานระบบประสาทโฮปฟิลด์แบบควบคุมในพจน์ของอสมการเมตริกซ์

เราได้สร้างเงื่อนไขใหม่ที่เพียงพอสำหรับการมีเสถียรภาพเชิงเส้นกำกับของระบบเชิงผลต่างของแบบจำลองข่ายงานระบบประสาทโฮปฟิลด์แบบควบคุมในพจน์ของอสมการเมตริกซ์ โดยใช้วิยุตของวิธีไลยาปูนอฟ และผลลัทธ์ถูกประยุกต์ได้เงื่อนไขใหม่สำหรับการมีเสถียรภาพเชิงเส้นกำกับของแบบจำลองข่ายงานระบบประสาทโฮปฟิลด์แบบควบคุมแบบเป็นกลางที่มีเวลาล่าช้าหลายตัวในพจน์ของอสมการเมตริกซ์

ภาควิชาคณิตศาสตร์ คณะวิทยาศาสตร์ มหาวิทยาลัยแม่โจ้ อำเภอสันทราย จังหวัดเชียงใหม่ 50290