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Model for Modified Holographic Ricci Dark Energy in Gravitation Theory of Branc Dicke

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Abstract

In this cosmological model, we have studied the spatially homogeneous and anisotropic Bianchi type and axially symmetric model filled with dark matter and dark energy in Brans-Dicke's [1] theory of gravitation. Here, we consider the modified holographic Ricci dark energy defined by Chen and Jing [21] as the suitable condition of dark energy. To obtain a solution we assumed the scale factor $a = [sinh(\gamma t)]^{\frac{1}{k}}$ used Mishra *et al*. [43]. We have solved field equations of Brans-Dicke theory of gravitation with the help of an axially symmetric anisotropic Bianchi-type space-time. We have determined the cosmological parameters, namely, EoS parameter, MHRDE density, matter density, skewness parameter, and BD scalar field. Here the various phenomena like the expanding universe, and shift from anisotropy to isotropy are observed in this model. A detailed physical discussion of these dynamical parameters are presented graphically. Some physical and geometrical behaviours of the models are also discussed and found to be in good agreement with the recent observations (OHD+JLA) datasets.

Keywords: Branc-Dicke theory, MHRDE, Cosmological parameters

Introduction

Brans-Dike [1] built up the scalar-tensor theories of gravitation, which joined Mach's rule into gravity. As indicated by this theory, the elements of gravity were characterized by the scalar field ϕ and the space- time was characterized by metric tensor. In BD theory, G is the inverse of the scalar field $\phi = (8\pi G^{-1})$. This scalar field couples to gravity with a coupling parameter ω. This parameter is very helpful for explaining the idea of the weak fields. Since BD theory assumes a significant job to tackle the missing matter, it gives the discretion of the accelerated expansion of the universe and discusses the idea of DP models.

The authors Singh and Agarwal [2,3], Reddy *et al*. [4,5], Socorro and Sabibo [6] and Jamila *et al*. [7] examined the cosmological model within the presence of Sa'ez-Ballester scalar-tensor theory of gravitation in various physical contexts. Spatially homogeneous cosmologies give an incredible thought in endeavours to understand the structure and properties of the space of the arrangements of Einstein field condition. Investigation of homogeneous and isotropic models are critical in understanding the early phases of the universe. In recent years, numerous authors like Reddy *et al.* [8], Pradhan *et al*. [9], Ali [10] explored Bianchi type-I cosmological model with time subordinate deceleration parameter q. Rao and Prasanthi [11] worked on Bianchi Type-I and III modified holographic Ricci dark energy in Saez-Blasster theory. Singh and kumar [12] analyzed the constant Bulk viscosity in F(R,T) with HRDE. Sarkar and Mahanta [13], Sarkar [14] and Reddy *et al.* [15] worked on Bianchi III V and V1.

Currently, there has been a great deal of enthusiasm for HDE models because these models are developed in a framed holographic principle, Sussind [16]. Many researchers [17-19] have acquired numerous properties (condition) of holographic Ricci Dark energy. Based on the space-time scalar curvature, the holographic Ricci dark energy model proposed by Granda and Oliveros [20] is relatively well adapted to the observational data. There are many advantages of this model, First the problem of fine tuning can be avoided. Also, this model does not presume the presence of event horizon, so the problem of causality can be avoided. The coincidence problem can also be solved effectively in this model. In this paper, we have considered the evolution of the universe in modified holographic Ricci Dark energy (MHRDE) and obtain the cosmological parameters. Chen and Jing [21] have developed a DE model and found that density of dark energy was the second-order derivative of Hubble parameter (H) and it is otherwise called modified holographic Ricci dark energy (MHRDE). This DE model is best fit to clarify the age issue of the old objects of the universe.

Li [22] utilized the fundamental construction of holographic rule inside the background of Quantum Gravity. Hsu [23] explored holographic dark energy and Gao *et al*. [24] proposed a holographic Ricci dark energy model, where the future event horizon is supplanted by the inverse of the Ricci scalar curvature. This model is known as the Ricci dark energy model (RDE). Mukherjee *et al.* [25] worked on holographic dark energy models with Hubble horizon as the infrared cut off. In the same context, Granda and Oliveroes [26] proposed the another HDE model. The expansion of (MHRDE) is given by $\rho_\Lambda = \frac{3}{8\pi G}(\xi H^2 + \eta \dot{H} + \zeta \ddot{H} H^{-1})$. Thus if we take the whole universe into account, then the vacuum energy related to the holographic principle may be viewed as DE, usually called holographic DE.

Ehsan [27] worked on NHDE with BD theory with the logarithmic scalar field investigating the observation (BAO+CMB+JLA) in the presence of interaction, where it is compatible. Milan and Singh [28] construct a BD theory with holographic dark energy with future event horizon and discussed the coincidence problem. Copeland [29] studied the inclusion of scalar fields into homogeneous cosmologies, where, it is a typical practice to study different scenarios, such as inflation, dark matter, and dark energy. However, since the early seventies, the problem exists in finding the appropriate source of matter and its corresponding Lagrangian to solve Ryan and Shepley [30] to a specific scenario.

In the same context, Bali and Jain [31,32] have worked on Bianchi type 1 non-static magnetized barotropic perfect fluid and the Bianchi type V magnetized string dust cosmological model in General Relativity. They have also investigated the Bianchi type III non-static magnetized cosmological model for perfect fluid distribution in general relativity [33].

In the present paper, inspired by the above discussion, we consider the spatially homogeneous and anisotropic Bianchi space-time universe within the framework of modified holographic Ricci dark energy in the BD gravity theory. The aim of this work is to study the modified holographic Ricci dark energy under BD theory of gravity.

The above discussion shows that our models are in good agreement with the recent scenario of modern cosmology. From the top of the investigations, we studied the time evolution in cosmological parameters in Bianchi type MHRDE. The plan of our paper is to discuss the field equations and deals with the solution of the problem. Kinematical and physical parameters of the model and their physical discussion are described by the graphical representations of the present model. Finally, the last segment contains our final remarks.

Metric and field equations

BD theory presents a scalar field ϕ, in addition to the metric tensor fields g_{ij}, which plays an important role of universe. The combined scalar and tensor field equation of BD theory is given by;

$$G_{ij} = -8\pi\phi^{-1}T_{ij} - \omega\phi^{-2}(\phi_{,i}\phi_{,j} - \tfrac{1}{2}g_{ij}\phi_{,k}\phi'^{k}) - \phi^{-1}(\phi_{i;j} - g_{ij}\phi'^{k}_{;k}) \tag{1}$$

and

$$\phi^{\prime k}_{;k} = \frac{8\pi}{(2\omega+3)} T \tag{2}$$

$G_{ij} = R_{ij} - \frac{1}{2} R g_{ij}$ is an Einstein tensor, R is the scalar curvature, ω denotes BD coupling parameter and T_{ij} is the stress energy tensor of matter. Now we introduce the energy conservation equation like as;

$$T^{ij}_{,j} = 0 \tag{3}$$

Comma and semicolon denote partial and covariant differentiation.

Axially symmetric space times are cosmologically important and play a vital role in the study of the universe on a scale that does not neglect anisotropy and inhomogeneity [34]. Marder [35] obtained a space time axially symmetric representing the material distribution. Kilinc [36] demonstrated that cosmological models axially symmetrical contributed significantly to the interpretation of early stages.

In this part, we write anisotropic axially symmetric metric space equation and the BD field equations in the presence of anisotropic matter distribution;

$$ds^2 = dt^2 - U^2 dx^2 - V^2(dy^2 + dz^2) \tag{4}$$

where $U(t)$ and $V(t)$ are function of t. In this proposed model, the BD field Eqs. (1) - (3) can also be written as the form;

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi\phi^{-1}(T'_{ij} + \overline{T}_{ij}) - \omega\phi^{-2}(\phi_{,i}\phi_{,j} - \frac{1}{2} g_{ij}\phi_{,k}\phi^{\prime k}) - \phi^{-1}(\phi_{i;j} - g_{ij}\phi^{\prime k}_{;k}) \tag{5}$$

Where T'_{ij} is the the energy-momentum tensor for matter and $\overline{T_{ij}}$ is the energy-momentum tensor for MHRDE which defined and used Naidu [37] as;

$$T'_{ij} = \rho_m u_i u_j, i, j = 1,2,3,4 \tag{6}$$

$$\overline{T_{ij}} = (\rho_\Lambda + \rho_m) u_i u_j - \rho_\Lambda g_{ij} \tag{7}$$

Here ρ_m is the matter energy density and ρ_Λ is the energy density of the modified holographic Ricci dark energy. Now for parameterizing we have from Eq. (7);

$$\overline{T_i^j} = diag[-1, \omega_1, \omega_2, \omega_3]\rho_\Lambda = diag[-1, \omega, (\omega + \delta), \omega(+\alpha)]\rho_\Lambda \tag{8}$$

where ω have to be used as EoS parameter given by;

$$\omega\rho_\Lambda = p_\Lambda \tag{9}$$

Here ω_1, ω_2, ω_3 are Equation of state parameters along the $x - y$ and z axis. For the simplification, assuming $\omega_1 = \omega$. The skewness parameter δ and α are the deviations from ω on the y and z axes, respectively.
Also, we have considered axial symmetricity;

$$\alpha = \delta \tag{10}$$

The wave equation satisfies the scalar field equation;

$$\phi_{;k}^{;k} = \frac{8\pi}{(2\omega+3)}(T' + \overline{T}) \tag{11}$$

and the energy conservation equation are also written in the form;

$$(T'_{ij} + \overline{T_{ij}})_{;j} = 0 \tag{12}$$

For solving coordinate system and Eqs. (5) - (12) by using metric (4) then we get the field equations as;

$$\frac{\ddot{U}}{U} + \frac{\ddot{V}}{V} + \frac{\dot{U}}{U}\frac{\dot{V}}{V} - \frac{w}{2}\frac{\dot{\phi}^2}{\phi^2} - \frac{\ddot{\phi}}{\phi} - \frac{\dot{\phi}}{\phi}\left(\frac{\dot{U}}{U} + \frac{\dot{V}}{V}\right) = -8\pi\phi^{-1}(\omega + \delta)\rho_\Lambda \tag{13}$$

$$2\frac{\ddot{V}}{V} + \frac{\dot{V}^2}{V^2} - \frac{w}{2}\frac{\dot{\phi}^2}{\phi^2} - \frac{\ddot{\phi}}{\phi} - 2\frac{\dot{\phi}}{\phi}\frac{\dot{V}}{V} = -8\pi\phi^{-1}\omega\rho_\Lambda \tag{14}$$

$$2\frac{\dot{U}}{U}\frac{\dot{V}}{V} + \frac{\dot{V}^2}{V^2} + \frac{w}{2}\frac{\dot{\phi}^2}{\phi^2} - \frac{\dot{\phi}}{\phi}\left(\frac{\dot{U}}{U} + 2\frac{\dot{V}}{V}\right) = -8\pi\phi^{-1}(\rho_\Lambda + \rho_m) \tag{15}$$

$$\ddot{\phi}^2 + \dot{\phi}\left(\frac{\dot{U}}{U} + 2\frac{\dot{V}}{V}\right) = 8\pi(3 + 2w)^{-1}(\rho_\Lambda + \rho_m - (3\omega + 2\delta) \tag{16}$$

The conservation equations takes the form;

$$\dot{\rho_m} + \dot{\rho_\Lambda} + \left(\frac{\dot{U}}{U} + 2\frac{\dot{V}}{V}\right)\rho_m + \left(\frac{\dot{U}}{U} + 2\frac{\dot{V}}{V}\right)(1 + \omega)\rho_\Lambda + 2\rho_\Lambda\delta\left(\frac{\dot{V}}{V}\right) = 0 \tag{17}$$

In this part, the field Eqs. (13) - (17) having seven unknowns U,V, ϕ, ρ_m, ρ_Λ, δ and ω. Now we solve these equations and find the solution by using the assumed condition.

First, we assume that shear scale σ^2 is proportional to the scale expansion θ, which gives Collins *et al*. [38].

The condition leads to $\frac{\dot{U}}{U} = m\,\frac{\dot{V}}{V}$;

$$U = c\,V^m \tag{18}$$

here c is the constant.

$$U = V^m \tag{19}$$

Throne [39] has explained this assumptive condition. In the literature, many authors like Pradhan [40], Jotania *et al*. [41], Akarsu [42] have assumed this condition for the development of physically viable cosmological models.

Second, we considered the varied deceleration parameter (q) explained by Mishra *et al*. [43] is given by;

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\left(\frac{\dot{H}+H^2}{H^2}\right) = u(t) \tag{20}$$

here (a) is the average scale factor of the universe. The purpose of select this type of DP is behind that the universe exhibits phase transition by the observations SNe Ia and CMB. For simplify the above the Eq. (20) using some suitable condition, we get the scale factor obtained as Mishra *et al.* [43];

$$\frac{\ddot{a}}{a} + u\frac{\dot{a}^2}{a^2} = 0 \tag{21}$$

solving Eq. (21) and assuming $u = u(a)$ also we consider $u = u(t) = u(a(t))$, noted a is the time dependent function. Then we get;

$$\int e^{\int \frac{u}{a} da}\, da = t + d \tag{22}$$

here d is the integrating constant and u is variable, then select $\int \frac{u}{a} da$ in such type, it should be integrable, hence we consider;

$$\int \frac{u}{a} da = ln\ \phi(a) \tag{23}$$

which does not affect the nature of generality of the solution. Now we get from Eqs. (22) and (23);

$$\int \phi(a)\, da = t + e \tag{24}$$

The choice of $\phi(a)$, in Eq. (24) is quite arbitrary. Universe is consistent for the physical viable model we choose;

$$\phi(a) = \frac{ka^{k-1}}{\gamma\sqrt{1+a^{2k}}} \tag{25}$$

here γ and $k > 0$ are arbitrary constants. In this case neglecting the integration constant e, then we get the exact solution as;

$$a(t) = [sinh(\gamma t)]^{1/k} \tag{26}$$

where γ is an arbitrary constant and k is a positive constant. Mishra *et al.* [43] and Maurya *et al.* [44] worked on this average scale factor and have analyzed some cosmological parameters. Now from Eqs. (19) and (26), we have to obtain the metric potentials as;

$$U = [sinh(\gamma t)]^{\frac{3m}{k(m+2)}} \tag{27}$$

$$V = [sinh(\gamma t)]^{\frac{3}{k(m+2)}} \tag{28}$$

so that the metric Eq. (4) can be written as;

$$dS^2 = dt^2 - \left([sinh(\gamma t)]^{\frac{6m}{k(m+2)}} \right) dx^2 - \left([sinh(\gamma t)]^{\frac{6}{k(m+2)}} \right) (\, dy^2 + dz^2\,) \tag{29}$$

Third we defined "modified holographic Ricci dark energy" (ϕ plays the role of G^{-1} in BD theory) defined by;

$$\rho_\Lambda = \frac{3\phi}{8\pi}(\xi H^2 + \eta \dot{H} + \zeta \ddot{H} H^{-1}) \tag{30}$$

We use a power law between the average scale factor and BD scalar field of the universe because the field equations are highly non-linear, which is in the form of;

$$\phi = \phi_0 a^n \tag{31}$$

also from Eq. (26) and Eq. (31) the BD scalar field in the model is given by;

$$\phi = \phi_0 [sinh(\gamma t)]^{n/k} \tag{32}$$

where ϕ_0, γ, k and n are positive constants.

Cosmological parameter of the model

In this section, we deal with the cosmological parameters of the universe, which plays a vital role in cosmology. We have a tendency to additionally discuss of the physical behaviour of those parameters' victimization graphical illustration.

The volume of the universe is;

$$V = (AB^2) = a^3(t) = [sinh(\gamma t)]^{3/k} \tag{33}$$

The Hubble parameter is;

$$H = \frac{\gamma}{k} coth(\gamma t) \tag{34}$$

The shear scalar is;

$$\sigma^2 = 3\left(\frac{m-1}{m+2}\right)^2 \frac{\gamma^2}{(k^2)} coth^2(\gamma t) \tag{35}$$

The scalar expansion in the universe is;

$$\theta = 3H = \frac{3\gamma}{k} coth(\gamma t) \tag{36}$$

The Anisotropy parameter is;

$$A_h = 2\left(\frac{m-1}{m+2}\right)^2 \tag{37}$$

Using Eq. (34), the deceleration parameter is obtained as;

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) = k\,(1 - tanh^2 \gamma\, t) - 1 \tag{38}$$

here γ and k are positive constants. The present estimation of q depicts whether the universe is accelerates or decelerates. If $q > 0$, the universe is in the acceleration phase. As per Eq. (38) we found that if $\gamma t < tanh^{-1}\left(\frac{(k-1)}{k}\right)^{\frac{1}{2}}$ then $q > 0$, and if $\gamma t > tanh^{-1}\left(\frac{(k-1)}{k}\right)^{\frac{1}{2}}$ at that point $q < 0$. Ongoing perceptions, clarify that our universe is accelerating. We additionally speaks to as DP lies in the scope of $-1 < q < 0$. For the present universe $(t_o = 12.36Gyr)$ with $(q_o = -0.52)$ Amirshashchi *et al*. [45] and Yu, Ratna [46] in light of (OHD+JLA) Data. Further, DP (q) as an element of the redshift parameter $z = -1 + \frac{a_0}{a}$ where a_0 is the present estimation of the scale factor, at $z = 0$, is given by;

$$q(z) = k - 1 - k\left[tanh\left(sinh^{-1}\sqrt{\frac{k-1-q_0}{(q_0+1)(z+1)^{2k}}}\right)\right]^2 \tag{39}$$

$$\gamma = \frac{1}{t_0} tanh^{-1}\left[1 - \left(\frac{1+q_0}{k}\right)\right]^{\frac{1}{2}} \tag{40}$$

here H_0 and t_0 is the present value of Hubble's parameter and present age of universe. We consider the two cases based on (OHD+JLA) data;

$$\gamma = \frac{1}{12.36} tanh^{-1}\left[1 - \left(\frac{0.48}{k}\right)\right]^{\frac{1}{2}} \tag{41}$$

Table 1 Values of (k, γ), for different observed values of DP (q_0) and MHRDE.

k	$\boldsymbol{\gamma}$	$\boldsymbol{q_0}$	$\boldsymbol{\rho_\Lambda}$	$\boldsymbol{\rho_m}$
0.25	0.061	-ve	+ve	+ve
0.50	0.0164	-ve	+ve	+ve
0.75	0.0560	-ve	+ve	+ve
1.0	0.0736	-ve	+ve	-ve
1.25	0.0855	+ to -	+ve	-ve
1.50	0.094	+ to -	+ve	-ve

These two values of (k, γ) are the best fit with latest observation (OHD+JLA) and have been used these values to draw all figures in the paper. From Eq. (30) and Eq. (34), we obtained the energy density of the modified holographic dark energy as;

$$\rho_\Lambda = \left[\frac{3\phi_0}{8\pi}(\sinh\gamma t)^{\frac{n}{k}}\gamma^2\left(\frac{\xi}{k^2}\coth^2\gamma t + \left(2\varsigma - \frac{\eta}{k}\right)\cos ech^2\gamma t\right)\right] \tag{42}$$

By using Eqs. (15), (27), (28), (42) as obtained matter density as;

$$\rho_m = \left[\frac{3\phi_0}{8\pi}(\sinh\gamma t)^{\frac{n}{k}}\frac{\gamma^2}{k^2}\left(\begin{array}{l}\frac{18m+9}{(m+2)^2}\coth^2\gamma t + \frac{\omega n^2}{2}\cot h^2\gamma t - (3n+3\xi)\coth^2\gamma t + (-3\eta k + 6\zeta k^2) \\ \qquad + (3\eta k - 6\zeta k^2)\coth^2\gamma t\end{array}\right)\right] \tag{43}$$

from Eqs. (13), (14), (27), (42) the skewness parameters are given by;

$$\delta=\frac{-\frac{1}{3}\left(\frac{9\gamma^2(m^2-m-2)}{k^2(m+2)^2}\coth^2\gamma t-\frac{3\gamma^2(m+1)}{k\,(m+2)}\cos ech^2\gamma t+\frac{6n\gamma^2}{k^2(m+2)}\coth^2\gamma t-\frac{3n\gamma(m+1)}{k^2(m+2)}\coth^2\gamma t\right)}{\gamma^2\left(\frac{\xi}{k^2}\coth^2\gamma t+\left(2\varsigma-\frac{\eta}{k}\right)\cos ech^2\gamma t\right)} \tag{44}$$

from Eqs. (14), (27), (28), (38) we get the Eos parameters are given by;

$$\omega=\frac{-\frac{1}{3}\left(\frac{27\gamma^2}{k^2(m+2)^2}\coth^2\gamma t+\frac{\gamma^2}{k}\left(n+\frac{6}{m+2}\right)\cos ech^2\gamma t-\frac{(2+\omega)n^2\gamma^2}{2k^2}\coth^2\gamma t-\frac{6n\gamma}{k^2(m+2)}\coth^2\gamma t\right)}{\gamma^2\left(\frac{\xi}{k^2}\coth^2\gamma t+\left(2\varsigma-\frac{\eta}{k}\right)\cos ech^2\gamma t\right)} \tag{45}$$

Results and discussion

Figure 1 demonstrates the variation of deceleration parameter q with cosmic time (t) according to Eq.38 and referenced decisions of the estimations of the constants. These graphs determine for different values k and γ From **Figures 1(a)** and **(b)** shows if we select $k \leq 1$, our model is developing just in the accelerating stage as indicated by most recent information planck data (2018), this model is the best fit for these quantities. Also it was observed that for $k \geq 1$, the energy reaches to negative quantities. So the model is only valid for $k < 1$. In this scenario, we did not study for early universe but always for accelerating universe at present phase (**Table 1**). From **Figure 1(b)** corresponding Eq. (39) depicts the variation of DP versus redshift z. We analyzed here that our model is advancing just in an accelerating stage < 0 . So we take two arrangement of significant worth like $k = 0.50$ and $\gamma = 0.0164$ (case I) and (caseII) $k = 0.75$ and $\gamma = 0.0560$, this esteem is determined from Eq. (41) utilizing ($q_0 = 0.48$ and $t_0 = 12.36$) which are from the joint OHD+JLA dataset utilized Amirhashchi *et al.* [45].

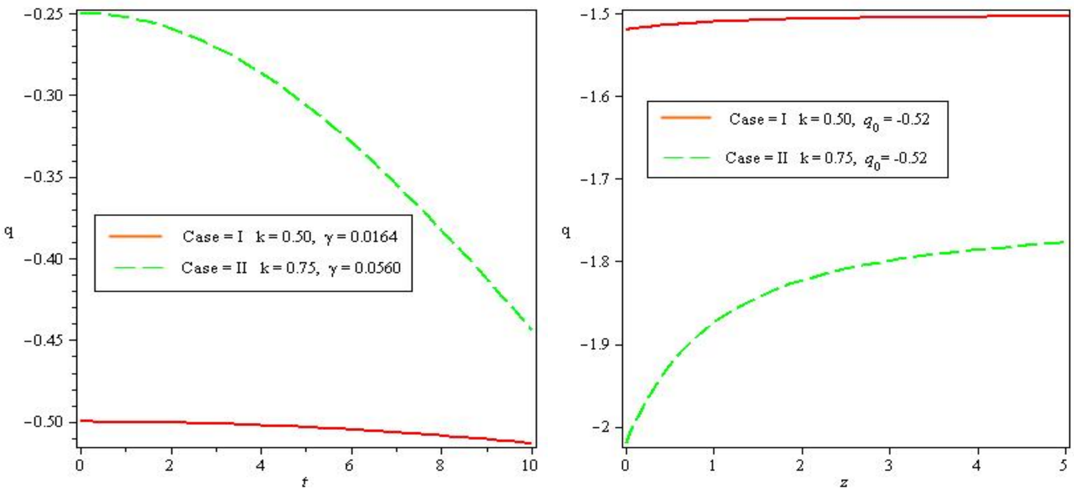

Figure 1 (a) Plot of Decelerating parameter (q) versus t, (b) Plot of Decelerating parameter (q) versus z.

Figure 2(a) The expression for Hubble parameter (H) and volume (V) are given in Eqs. (33) and (34). We depict the variation of (H) and (V) with respect to t in **Figures 2(a)** and **2(b)**, respectively. From these figures, we observed that (H) is a positive decreasing function of time t, whereas the volume is increasing with time.

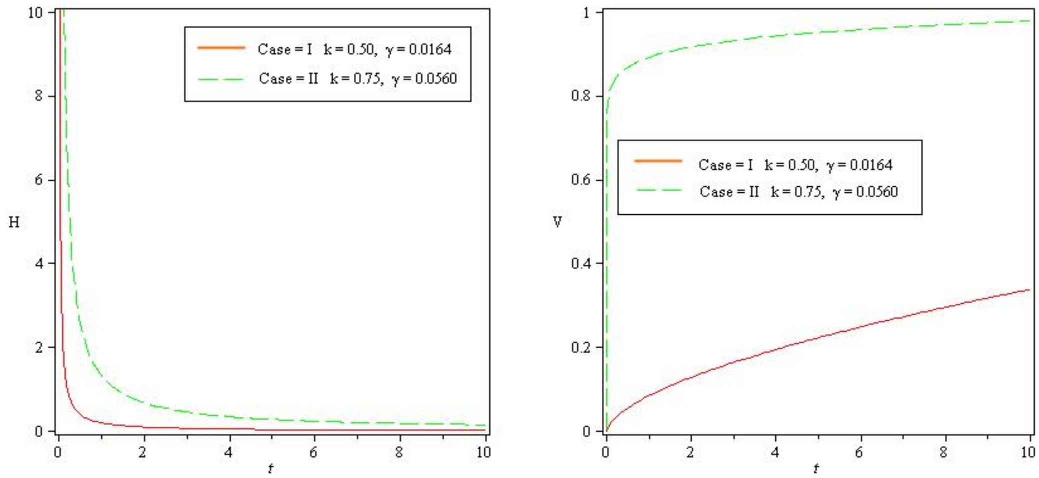

Figure 2 (a) Plot of Hubble parameter (H) versus t, (b) Plot of Volume (V) versus t.

Figure 3(a) corresponding to the Eq. (42), portray the energy density (ρ_Λ) with cosmic time t for two cases. It is seen that (ρ_Λ) stays positive during the cosmic evolution. Here, we additionally noticed that $t \to 0$, $\rho_\Lambda \to \infty$. This showed that in early universe, the density was very high. The ρ_Λ is a positive decreasing function of time and it approaches near zero as $t \to \infty$. It is worth mentioning that (ρ_Λ) in case I, is a fast decreasing in comparison to case II. In **Figure 3(b)**, we have plotted matter density in terms of cosmic time t. Eq. (43) corresponding to matter density (ρ_m) it is a decreasing function of time and it is remaining positive during the cosmic evaluation. First we can see in the figure, it decreased sharply, then gradually, and in the present epoch it approached a small positive value at the present epoch. Here (ρ_m) and (ρ_Λ) tends to 0 as $t \to \infty$. Since density decreases indicate volume increase, which is consistent with well-established theory and thus our model is expanding.

Figure 4 demonstrates the BD scalar field ϕ versus time (t), which relates to the Eq. (32), Now we have find that this scalar field increases with time for both cases.

Figure 5 delineates the idea of the skewness parameter with t for various estimations of k and γ in every one of the two cases. Here we see that the skewness parameter is positive and increases as the BD field increases.

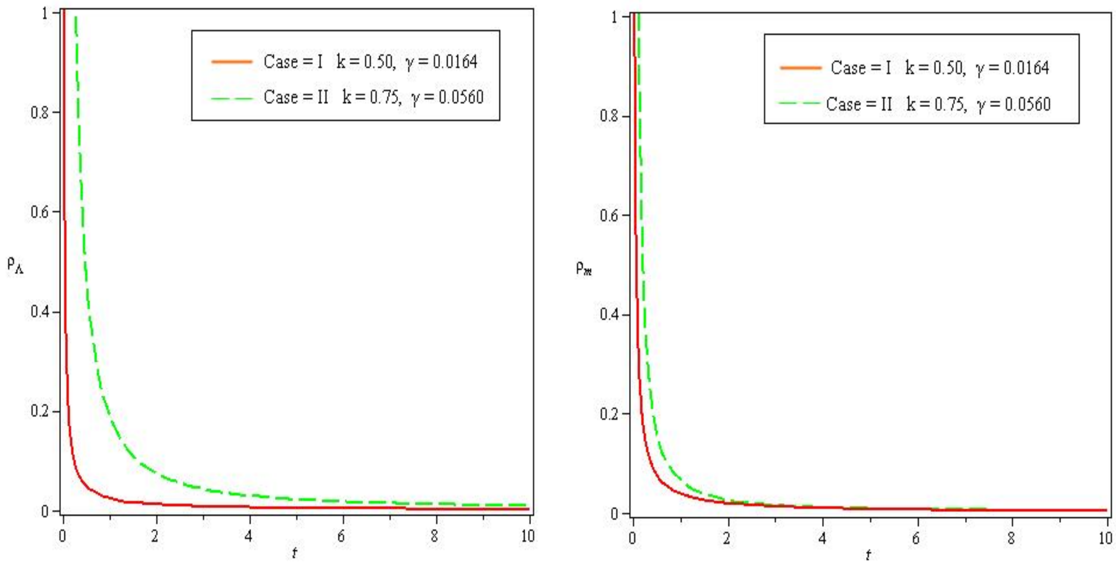

Figure 3 Plot of Energy density (a) MHRDE (ρ_{Λ}) versus t, (b) matter (ρ_m) versus t.

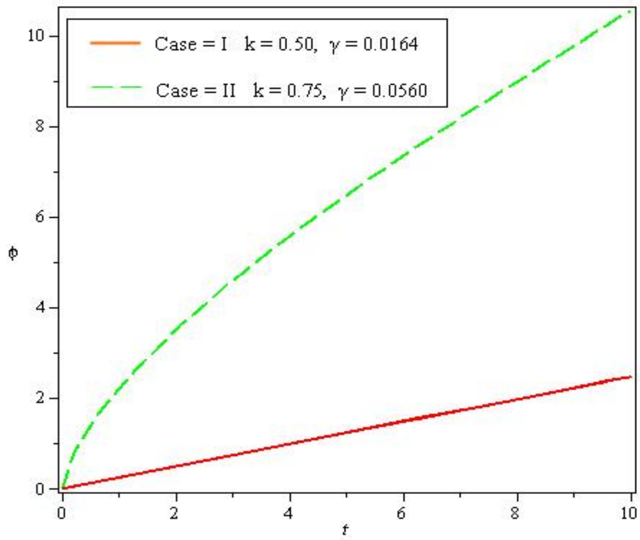

Figure 4 Plot scalar field (ϕ) versus (t).

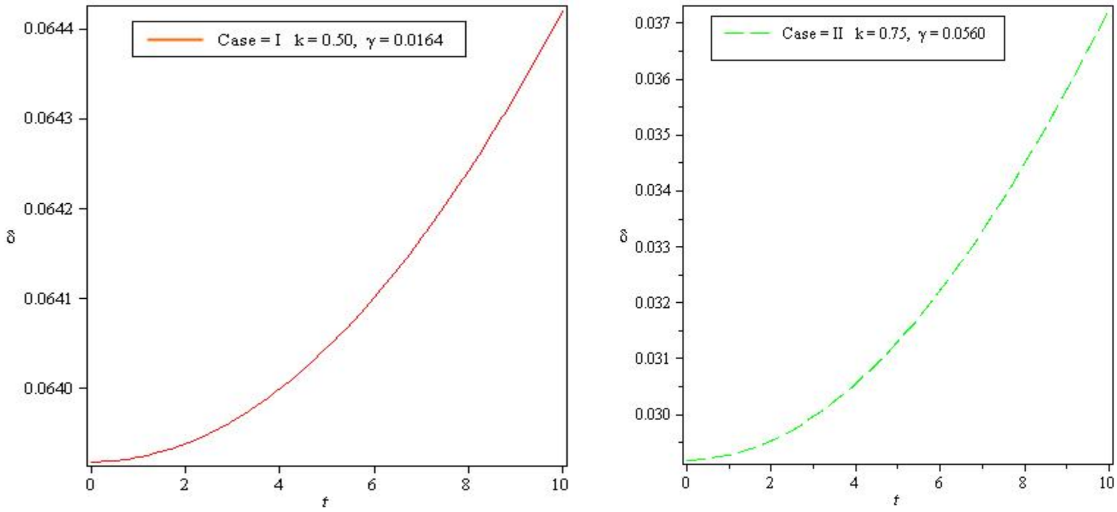

Figure 5 Plot of Skewness parameter (δ) versus (t).

Figure 6 escribe the behaviour of EoS parameter with cosmic time. The dark energy EoS parameter could be considered as a constant parameter $(\omega = -1)$ described by cosmological constant or a scalar fields. As interval $(-1 < \omega < -1/3)$ is called quintessence region, phantom region indicated by $(\omega < -1)$. In our model from graphical representation and the estimation of (OHD+JLA) informational collection, we found that EoS parameter starts from very high phantom $(\omega << -1)$ and tends to cross the phantom divide line (PDL) $(\omega = -1)$. If we take $k \leq 1$, we examine this outcome for expecting two cases for $k = 0.5$ and $k = 0.75$. From the above analysis, we found that our models in case I is dominated by phantom dark energy and in case II the EoS parameter lies in quintessence and phantom region. While for the joint (OHD+JLA) dataset, the evaluated estimation of the EoS parameter only varied in phantom region. Here ω is not exactly –1. There is a phantom region, which evolves with negative and its range is in decent concurrence with huge scale structure information Komatsu *et al*. [47]. For JLA data ,the dark energy EoS parameter lies between quintessence and phantom regions $(-1.36 < \omega < -0.84)$. Whereas for OHD and OHD+JLA data the EoS parameter only varies in phantom region { $(-1.54 < \omega < -1.404)$ according to (OHD) data and for JLA EoS parameter $(-1.46 < \omega < -1.08)$}. Here the graph clearly noticed that our model support to phantom dark energy scenario. This data set is the joint agreement of planck (2015) and WMAP collaboration [45]. From this data, it was found that EoS parameter support to phantom region and these values are best fit for this data. Our MHRDE model describes that the EOS parameter is a decreasing function of time epoch at a constant value.

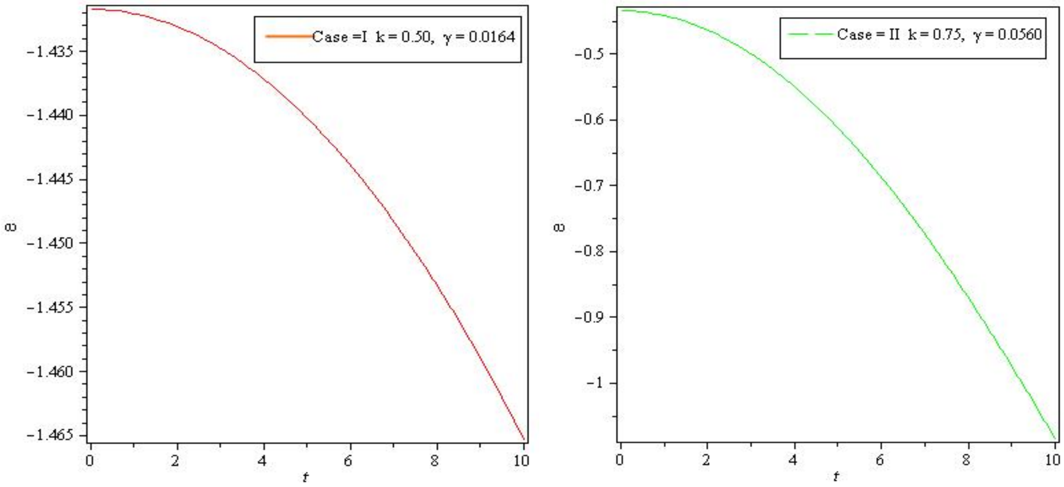

Figure 6 Plot of EoS parameter (ω) versus t.

Conclusions

In the present paper, we have examined cosmological models of Bianchi type space time within the framework of Brans-Dicke scalar theory of gravitation. This model is dependent on the two different values of k and corresponding value of γ. Here we assuming the scale factor as $a(t) = [sinh(\gamma t)]^{\frac{1}{k}}$ .This model used two different observational data, observational measurements of the Hubble parameter (OHD) and Joint Light-curve Analysis (JLA). The physical interpretation of the cosmological parameter represented through a graph with MHRDE and their determined solution is summarized below.

- We have presented the DP (q) as a function of t and discussed its significance. In case I $k = 0.5$ and $\gamma = 0.0164$, we have combined the information of cosmological model in an accelerating stage only. In case II for $k = 0.75$ and $\gamma = 0.0560$ likewise, we obtained a cosmological model from the early accelerating stage. We are accepting the constant $\phi_0 = 15$, $m = 0.99$, $n = 0.1$ $\xi = 0.2$, $\eta = 0.48$ and $\chi = 0.5$ used Aditya and Reddy [48]. Deceleration parameter (q) is believed to be evaluated much better to the joint (HOD+JLA) regarding any individual dataset. These parameters are in great understanding with those acquired Farooq *et al.* [49] and Busa *et al.* [50].
- The expression for the spatial volume (V) is given in Eq. (33). It can be seen that the spatial volume is zero at t = 0 and increases with an increase in time t, which shows consistency with the concept of an expanding universe.
- The expansion for the scalar expansion θ, mean Hubble parameter (H), the mean anisotropic parameter A_h and the shear scalar σ^2, for the model are respectively given by Eqs. (34) - (37).
- Throughout the growth, the two densities are seen to be positive, maintaining a constant at the present age. It can be seen that as time increases, energy densities of matter and MHRDE both decrease. The underlying physical result is that the decrease in MHRDE and matter-energy density with an increase in cosmic time contributes to the expanding universe.
- Energy densities are not influenced by the BD scalar field, but rather matter density increases with the BD scalar field and has no impact in the present age. Hubble parameter decreases with an increase in time. The behavior of spatial volume, which confines to increase the cosmic time, tends to be infinite and vanishes at $t = 0$ demonstrated in figures. The graph indicates that the BD scalar field and the skewness parameter is constantly positive and increases with time.

- Our MHRDE model describes that EOS parameter is a decreasing function of time epoch a constant value. Our model explains the various physical and Kinematical phenomena of the universe, which is also consistent with recent observations.
- In this model, we have successfully clarified the two fundamental problems in cosmology fine-tuning and problems of coincidence. Our model explains successfully the latest scenario of cosmic accelerated growth.

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