A Smoke Filling Time Function in Compartment by Dimensional Analysis

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Abstract

A function for illustrating the smoke filling time and related factors in compartment rooms due to various fire movements by dimensional analysis proposed by this study. Dimensional analysis, a numerically expressed physical quantity concept, is applied in developing a function with constants left. Factors effecting smoke filling time are listed for analysis. These factors are height of the compartment room (H), ambient density (ρ_a), gravity acceleration (g), and ambient temperature (T_a). The factors have been grouped into dimensionless groups. There are 7 Pi groups; these are reduced into the form of a function and can be consolidated in the form of a compact equation through further experiments.

Keywords: Smoke filling time, fire compartment, dimensional analysis

Introduction

In fire incidents, the most significant harmful matters are smoke, a visible product of fire, and invisible bodies from the burning material, which produces combustion products such as toxic gas. Smoke may hostilely affect all of the burning room's occupants and also any fire response personal. The composition of smoke consists of solid particles and gases evolved when a material undergoes pyrolysis, including the amount of air entrained into the flame [1]. The time taken for smoke to fill a compartment influences the fire safety designs of buildings, because it is a common practice that the performance of fire safety evacuation safety is evaluated by means of comparing the smoke filling time with the available safe egress time (ASET). In predicting smoke filling time, 2-zone smoke models are widely used, as it is the most simplistic method of determining fire models, by dividing the enclosure into 2 control volumes called upper zone and lower zone, or hot layer and cold layer. The method uses the mass conservation equation and energy conservation equation as a basis to solve the model [2]. The smoke filling time without opening area equations are developed numerically by using such a model as a foundation. The equation of smoke filling time will be used as the basis to derive a new compact function that is more suitable for compartment fire with an opening area. Dimensional analysis is used as a method to simplify the function of smoke filling time, which is dependent on a number of parameters. This research aims to show a result of dimensional analysis as a function of time that can be obtained facilely from a wide range of opening area in compartment fires.

Smoke filling

In a compartment fire, the combustion product will fill the room. Cooper [3] defined development and growth of the upper layer as 'smoke filling'. The gasses in the ceiling and wall jets redistribute themselves across the upper volume of the room [3]. Finally, a relatively quiet, raised temperature of uniform smoke layer thickness is arranged below the continuing ceiling-jet flow activity. The flows generated by the ceiling jet-wall interaction are submerged as the thickness of this layer grows. A distinctive material interface which separates the lower ambient air from the upper, heated, smoke-laden gases defines the boundary of the layer. With longer time, the level of the smoke layer interface decreases, while the temperature and smoke density of the upper layer increases. The smoke filling process is an essential feature of any zone-type compartment fire model.

Smoke filling time

In order to plan fire safety strategies, such as Available Safe Egress Time (ASET), the length of the time interval between fire detection or successful alarm and the commencement of life safety hazard, and the Require Safe Egress Time (RSET), based on fire safety standard, smoke filling time calculations are essential for models for predicting the smoke filling process in a room of fire [3].

Smoke filling time calculations give results for the time it takes to fill a compartment with smoke. In order to calculate smoke filling time, examination is undertaken of a volume composed of a single room, where a fire of uniform heat release rate will produce smoke to rise and form a horizontal layer of hot gas, which will make the smoke in the room divide into 2 apparent layers. The assumption of a certain temperature of 2 layers, the upper hot gas layer and the lower ambient layer, are T_g and T_a [2]. Zukoski [4] assumed that the energy release rate resulted in a pressure increase, because of the thermal expansion of the gases and because air was pressed out via an opening based on a small opening leakage, which was located either at the floor level or at the ceiling level. In other words, there is no mass flow into the room, and either cold gases or hot gases of temperature constitute the mass flow out. There are 2 cases that can be considered for calculating smoke filling time. The first one is small leakage areas at floor level, and the second one is small leakage areas at ceiling level.

First, small leakage areas at floor level requires a differential equation for smoke filling time, achieved by combining the plume mass flow rate, conversion of the conservation of mass, and a simple result of the conservation of mass integration. This differential equation for smoke filling time as is shown below.

$$\frac{dz}{dt}\rho_a S + \frac{Q}{c_p T_a} + 0.21\left(\frac{\rho_a^2 g}{c_p T_a}\right)^{1/3} \dot{Q}^{\ 1/3} z^{5/3} = 0. \tag{1}$$

where z is smoke clear height
t is time
ρ_a is air density at ambient temperature
S is floor area
Q is heat
c_p is specific heat capacity
T_a is lower ambient layer temperature
g is gravity acceleration
$\dot{Q}$ is heat release rate

In spite of making this applicable to several various geometries and diverse heat release rate, a differential equation for smoke filling time in dimensionless function must be used. This equation is composed by dimensionless height, dimensionless heat release rate, and dimensionless time as the following;

$$\frac{dy}{d\tau} + \dot{Q}^* + 0.21\left(\dot{Q}^*\right)^{1/3} y^{5/3} = 0. \tag{2}$$

where y is dimensionless height of smoke layer interface above floor
τ is dimensionless time

Second, small leakage areas at ceiling level produce more simply. This is because of chosen control volume as the lower layer and disentangle mass balance. Moreover, the only mass leaving the lower layer flows through the plume [2]. This equation can be written as following;

$$\frac{d}{dt}(\rho_a z S) + \dot{m_p} = 0. \tag{3}$$

where m_p is plume mass flow rate

The above equation is able to be simplified as a dimensionless structure as the following;

$$\frac{dy}{d\tau} + 0.21\left(\dot{Q}^*\right)^{1/3} y^{5/3} = 0. \tag{4}$$

Mowrer [5] reviewed and extended fire modeling concepts related to enclosure smoke filling developed by Zukoski [4] and Zukoski *et al*. [6]. Mowrer [5] recast the mass-based analysis of Zukoski in terms of the volumetric flow rates typically used for ventilation system design. A comparison of hand calculation based on a global analysis with numerical smoke filling calculations was done using a spreadsheet template. According to the result of the comparison, it was suggested that there was little difference in conditions predicted between these 2 calculation methods. Moreover, the hand calculations were appropriate for initial fire hazard analyses.

Dimensional analysis

Dimensional analysis is a process for eliminating extraneous information from a relation between quantities [7]. Dimensional analysis is often unconsciously used by practically everyone who deals with physics problems, and sometimes offers an aid in the solution of physically based differential equations to provide a guide to experimental planning and to the correlation of data. It may be most importantly used as a means of developing the ability to generalize experience and to apply knowledge for engineers. Dimensional analysis is concerned with the nature of the relationship between various quantities which enter a physical problem [7]. In the system of mechanics, generally, there are 3 factors, space, time and mass. Each of 3 is taken as being distinct from the others and as necessary for the examination and specification of the sequences of mechanical occurrences [8]. It must be known that there is only one relationship existing between a certain number of physical quantities before the dimensional analysis can be applied, there are no pertinent quantities which have been omitted, and no extraneous quantities included. The group of all physical quantities in such a relationship can be expressed in the form of the following function;

$$\emptyset(q_1, q_2, q_3, \ldots, q_n) = 0. \tag{5}$$

where $\emptyset$ is a function group
q is physical quantity

The process of dimensional analysis can group the original quantities into 'dimensionless groups' to form a new relationship to;

$$\emptyset(\prod_1, \prod_2, \prod_3, \ldots, \prod_n) = 0. \tag{6}$$

where $\prod$ is dimensionless Pi

For dimensional analysis in the engineering field, the relationship of physical quantity from observation and experiment is analyzed in the dimensionless term by using mathematical theories. There are 2 methods: Rayleigh's method and Buckingham's $\prod$ (Pi) method.

Rayleigh's method is the analysis of a functional relationship of some variables in the form of an exponential equation. By determining ɸ as any function varied by function $ɸ^1$, $ɸ^2$, $ɸ^3$,..., $ɸ^n$, the relationship can be written as the equation below.

$$ɸ = f(ɸ_1, ɸ_2, ɸ_3, \ldots, ɸ_n) \quad (7)$$

where C is a dimensionless constant and a, b, c,..., m are arbitrary exponents. The above equation can be written in the form below.

$$ɸ = C(ɸ_1^a, ɸ_2^b, ɸ_3^c, \ldots, ɸ_n^m). \quad (8)$$

Buckingham's ∏ method relies on the empirical data grouped into terms of ∏ which is varied by n variables of term ∏. ∏ equals the number of variable n minus the m fundamental dimensions. For this experiment setup case, there are n variables;

$$f(ɸ_1, ɸ_2, ɸ_3, \ldots, ɸ_n) = 0. \quad (9)$$

These variables can be written as the below function.

$$f(\prod_1, \prod_2, \prod_3, \ldots, \prod_{n-m}) = 0. \quad (10)$$

Finding a parameter of group ∏ by choosing the fundamental dimensions m of ɸ which has a different value appearing in m and choosing such dimensions as repeating variables if $ɸ^1$, $ɸ^2$, $ɸ^3$ are variables consisting of MLT fundamental dimensions, the ∏ groups are;

$$\prod_1 = ɸ_1^{x_1}, ɸ_2^{y_1}, ɸ_3^{z_1}, ɸ_4 \quad (11)$$

$$\prod_2 = ɸ_1^{x_2}, ɸ_2^{y_2}, ɸ_3^{z_2}, ɸ_5 \quad (12)$$

$$\prod_{n-m} = ɸ_1^{x_{n-m}}, ɸ_2^{y_{n-m}}, ɸ_3^{z_{n-m}}, ɸ_{n-m} \quad (13)$$

The derived equation is an exponential equation which is able to be calculated in ∏ term until the value becomes dimensionless by determining a set of exponents of the MLT fundamental dimensions as zero.

Dimensional analysis is widely recognized in fire research and is regarded as a vitally useful tool. Due to art of the dimensional analysis, Thomas [9] discussed various fire safety engineering questions stimulated during his lecturers and literature. There were 9 examples given. The first one is the use of dimensional analysis. The second one is the combination of dimensionless groups as a result of physical arguments. The third one is their use in the formulation of a solution of a differential equation with its boundary or initial conditions. The fourth one is choosing one of 2 alternative formulations of a dimensionless variable RT/E or $/RT^2(T - T_0)$. The fifth one is the consequence of the difference between dependent and independent variables. The sixth one is their use in evaluating a quantity. The seventh one is the structure of a formula as a result of physical considerations such as bent over plumes and flame lengths. The eighth one is the choice and significance of a characteristic length for inclusion in a dimensionless variable k_g/v_a in $v_a x/k_g$ and l in the Delichatsios-Saito length. The ninth one is the analysis of measurements of quantities claimed to be a part of a dimensionless variable. All 9 examples Thomas [9] discussed enable how dimensional analysis work as a magic art in fire science research.

Kuwana *et al*. [10] used dimensional analysis to understand the effect of flow circulation on an increase in flame height. According to a fire whirl in an open space which caused devastating damage in Hifukusho-ato, Tokyo, after the Great Kanto Earthquake in 1921, Kuwana *et al*. [10] conducted 1/1000th scale-model experiments in a large, low-speed open-loop wind tunnel to understand the generation mechanism of the open-space fire whirls. A critical lateral wind velocity that generated intense fire whirls was found in analyzing the experiments. Various data, including scale-model experiments by other researchers and real urban fire whirls, developed and validated a scaling law that predicts critical wind velocity. The results of numerical simulations by other researchers supported the simple analysis. Shi *et al*. [11] studied the centerline temperature varying characteristics of a spill plume free developing out of a wide-vent cabin by carrying out full scale experiments in a cabin model located in a full-scale experimental atrium apparatus, and performing Computational Fluid Dynamics (CFD) simulations by large eddy model Fire Dynamics Simulator (FDS) developed by NIST, to validate the applicability of the double-part prediction model (MEDP) proposed on the centerline temperature of the spill plume free development. Shi *et al*. [11] found that spill plume development in a large atrium due to a cabin fire could be divided into three typical regions, the horizontal curved section at the cabin door, the linear plume regime at the near-field, and the axisymmetric plume regime at the far-field. From the horizontal curved regime at the door, the nearfield 2-dimensional linear plume regime, through to the far-field axisymmetric plume regime, the physical development process of the spill plume was analyzed. This identified the coupling correlation existing between the linear virtual origin, the critical transition height from linear to axisymmetric regime, the axisymmetric virtual origin, and other parameters, such as the spill plume depth at the door, the mass and heat flow rate outside the door, the door width and height. The basic mathematic equation regarding each part's temperature was derived. They gave the virtual origin position of a linear plume regime and an axisymmetric plume regime and proposed the critical transition height of a spill plume from a 2-dimensional linear plume to an axisymmetric plume regime. After this, an MEDP model for the spill plume centerline temperature was provided. Four groups of full-scale experiments, together with Large Eddy Simulations (LES), were conducted in order to study the flow characteristics of the spill plume in cabins and further validate the proposed centerline temperature model. The comparison showed that the proposed model was capable of well predicting the centerline temperature of the spill plume, further validating the applicability.

Method and derived function

The starting procedure of calculating smoke filling time by use of the method of dimensional analysis is to select the important factors affecting smoke filling time. These are the initial conditions of the environment of a compartment which may affect smoke filling time when a fire occurs. The first condition that the researcher is concerned about is the room size. The room size consists of the height of the compartment and floor area of a compartment, which is the width of the compartment floor multiplied by the length of the compartment floor. In this case, the room shape is a rectangular shape. The second condition is the size of the fire. This is in the form of the heat release rate in kilowatts. The third condition is the location of the opening. For this experiment setup, there is an opening in the compartment room. There are 2 locations for the opening in this research: a wall opening and a ceiling opening. The fourth condition is the size of the opening, which is the height and width of the opening. The fifth one concerns environmental properties, such the related environmental properties of gravity acceleration due to mass, air pressure, hot gas pressure, air temperature, and hot gas temperature.

According to the literature review and the conditions the researcher listed, the potential factors are listed as the following;

- Height of the compartment room (H)
- Floor area of the compartment room (A)
- Heat release rate of the heat source ($\dot{Q}$)
- Distance between ceiling and window (d_s)
- Gravity acceleration (g)
- Height from the floor to the neutral plane (h)
- Ambient temperature (T_a)
- Ambient density (ρ_a)
- Specific heat capacity (c_p)

- Area of an opening (A_{open})
- Filling time (t)

These factors can be categorized by fundamental dimensions (MLT system) in **Table 1**. As mentioned above, there are 2 methods of dimensional analysis: Rayleigh's method and Buckingham's ∏ (Pi) method. However, Rayleigh's method is limited if there is more than 1 non-dimensional group [12].

There are 11 variables with 4 dimensions (M, L, T and Ɵ). According to the statement of the ∏ theorem of Buckingham [13], if k is the minimum number of primary quantities necessary to express the dimensions of the q in the below equation;

$$\emptyset(q_1, q_2, q_3, \dots, q_n) = 0 \quad (14)$$

then

$$n - m = k \quad (15)$$

Table 1 Factors categorized by fundamental dimensions.

Factor	Unit	Dimension
Height of the compartment room (H)	m	L
Floor area of the compartment room (A)	m^2	L^2
Heat release rate of the heat source ($\dot{Q}$)	kW	ML^2T^{-3}
Distance between ceiling and window (d_s)	m	L
Gravity acceleration (g)	$\frac{m}{t^2}$	LT^{-2}
Height from the floor to the neutral plane (h)	m	L
Ambient temperature (T_a)	K	Ɵ
Ambient density (ρ_a)	$\frac{kg}{m^3}$	ML^{-3}
Specific heat capacity (c_p)	$\frac{kJ}{kgK}$	$L^2T^{-2}Ɵ^1$
Area of an opening (A_{open})	m^2	L
Filling time (t)	s	T

Therefore, 11 variables minus 4 dimensions equal 7. There are 7 Pi groups. Four variables are selected as repeating variables, as mentioned in previous chapter. Due to 2 conditions for solving the equation [14],

1) Each of the fundamental dimensions must appear in at least one of the m variables.

2) It must not be possible to form a dimensionless group from one of the variables within a recurring set. A recurring set is a group of variables forming a dimensionless group.

The researcher decided to choose the following variables;

- Height of the compartment room (H)
- Ambient density (ρ_a)
- Gravity acceleration (g)
- Ambient temperature (T_a)

The seven Pi groups are able to be written as the below.

$$\prod_1 = t\sqrt{\frac{g}{H}} \tag{16}$$

$$\prod_2 = \frac{A}{H^2} \tag{17}$$

$$\prod_3 = \frac{\dot{Q}}{\rho_a\sqrt{g^3H^7}} \tag{18}$$

$$\prod_4 = \frac{d_s}{H} \tag{19}$$

$$\prod_5 = \frac{h}{H} \tag{20}$$

$$\prod_6 = \frac{A_{open}}{H^2} \tag{21}$$

$$\prod_7 = \frac{c_pT_a}{gH} \tag{22}$$

According to the 7 $\prod$ groups, a function with groups of dimensionless variables can be written as the following;

$$t\sqrt{\frac{g}{H}} = f\left(\frac{A}{H^2}, \frac{\dot{Q}}{\rho_a\sqrt{g^3H^7}}, \frac{d_s}{H}, \frac{h}{H}, \frac{A_{open}}{H^2}, \frac{c_pT_a}{gH}\right). \tag{23}$$

However, this function is able to be reduced by grouping the 3rd and the 7th $\prod$ groups of the above function.

$$t\sqrt{\frac{g}{H}} = f\left(\frac{A}{H^2}, \dot{Q}^*, \frac{h}{H}, \frac{A_{open}}{H^2}, \frac{d_s}{H}\right) \tag{24}$$

where [1,4,14] $\dot{Q}^* = \frac{\dot{Q}}{\rho_a c_p T_a \sqrt{g} H^{\frac{5}{3}}}$ (25)

In order to generalize the parameters, 30 compartment room scenarios were simulated by the Consolidated Models for Fire Growth and Smoke Transport (CFAST) program. The experiment conditions: the fixed values of window size, window location, fuel (methane), ambient temperature, and ambient density, are shown in **Table 2** below.

Table 2 Fixed initial factor values.

Initial factor	**Value**	**Unit**
Fuel: Methane (cp)	2.3	$kJkgK$
Ambient temperature (Ta)	293.15	K
Ambient density (ρa)	1.2	kgm3
Window size	1	m2
Window location (d_s)	0.5	M

Simulations of such conditions were run. According to the fixed values of Window Size and Window Location (ds), the dimensional analysis function above was consolidated as the below function.

$$t\sqrt{\frac{g}{H}} = f\left(\frac{A}{H^2}, \dot{Q}^*, \frac{h}{H}\right) \quad (26)$$

Then, this function can be reduced again as;

$$t\sqrt{\frac{g}{H}} = \frac{A}{H^2} f_1\left(\dot{Q}^*, \frac{h}{H}\right) \quad (27)$$

and

$$t\sqrt{\frac{g}{H}}\frac{H^2}{A} = f_1\left(\dot{Q}^*, \frac{h}{H}\right) \quad (28)$$

where $\tau = t\sqrt{\frac{g}{H}}\frac{H^2}{A}$ from Zukoski [4] and Karlsson and Quintere [2]. In summary, the final correlation function can be simplified to;

$$\tau = f_1\left(\dot{Q}^*, \frac{h}{H}\right) \quad (29)$$

The values acquired from the CFAST simulation were the height of the neutral plane and the filling time. Multiple simulation cases were designed for more frequent results of testing. Various room sizes and heat release rates were the non-constant variables. The simulations gave the different simulation results seen in the Appendix. The filling time derived from the simulation results was the time when the layer height value met its minimum value by the opening level.

Regression analysis

Regression analysis was introduced as a tool for governing correlation functions. The data from the simulations and the simplified functions from dimensional analysis with unknown constants were selected in order to see the relationships of dimensionless groups by information trend.

From correlation function (29), $\tau = f_1\left(\dot{Q}^*, \frac{h}{H}\right)$, there are 3 Pi groups, ∏1, ∏2, and ∏3, which would be considered to have dimensionless relationships. ∏1 is dimensionless time (τ), ∏2 is dimensionless heat release rate ($\dot{Q}^*$) and ∏3 is dimensionless group of $\frac{h}{H}$.

The correlation between τ and $\dot{Q}^*$ in correlate with $\frac{h}{H}$ at several values is in the form of a parabola function as the below

$$\tau = C_1{\dot{Q}^*}^2 + C_2\dot{Q}^* + C_3 \quad (30)$$

Where $C_1, C_2, C_3 = Constant$, with only the constant term, C_3, varies by the dimensionless variable $\frac{h}{H}$. However, the filling time (t) versus neutral plane height (h) shows that $t \propto h^2$ and also neutral plane height (h) is linearly related to different heat release rate ($\dot{Q}$) as shown in **Figure 1** and **2** below.

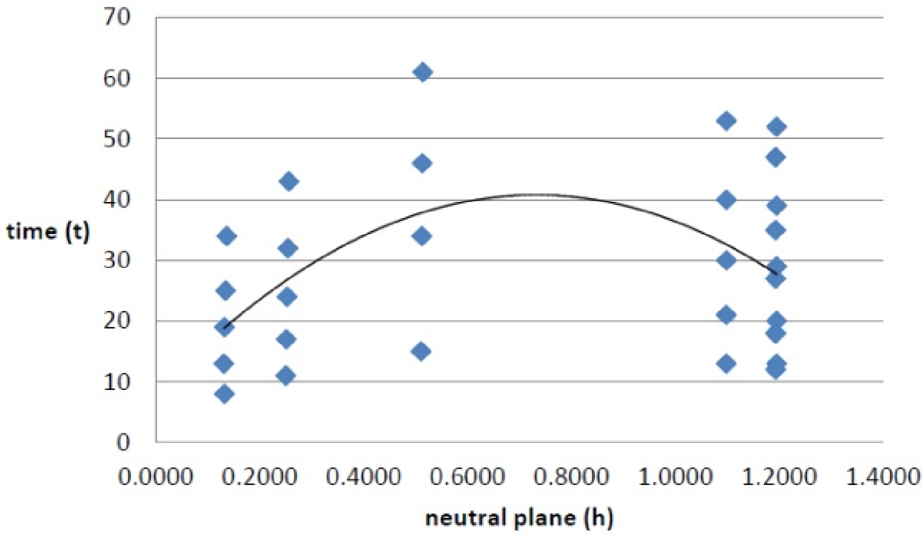

Figure 1 Time vs Neutral plane.

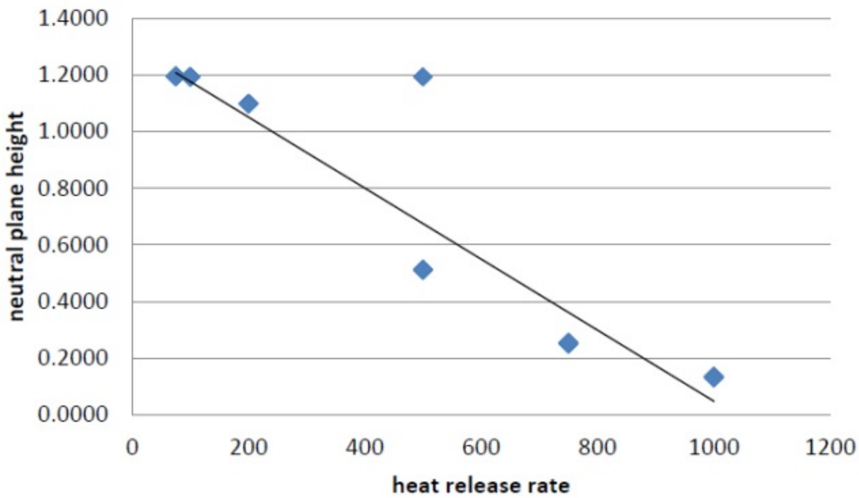

Figure 2 Neutral plane vs Heat release rate.

Thus, the dimensionless correlation function can be rewritten to be

$$\tau\left(\frac{H}{h}\right)^2 = f(\dot{Q}^*) \tag{31}$$

The above correlation function (31) can be plotted as **Figure 3**.

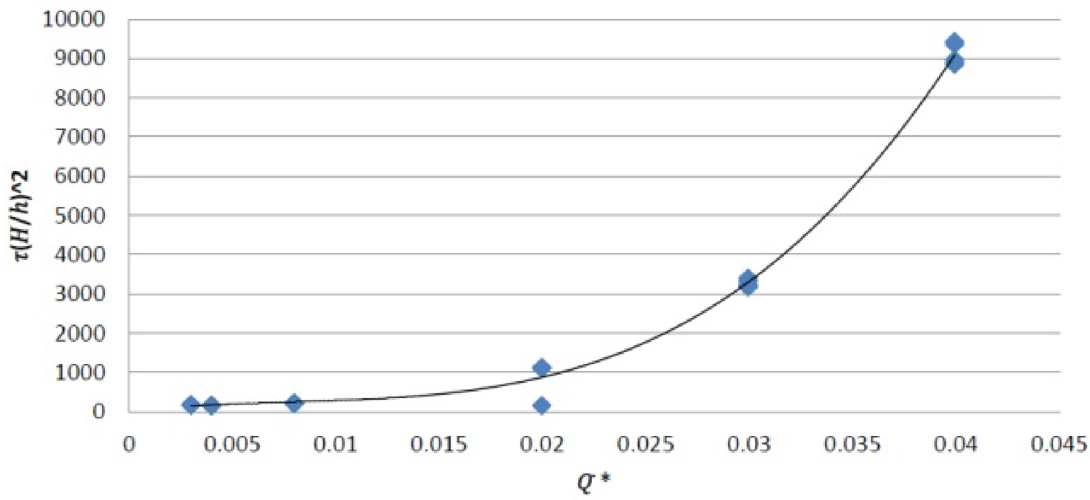

Figure 3 Dimensionless time group vs. Dimensionless heat release rate.

From **Figure 3**, the correlation equation can be rewritten as

$$\tau\left(\frac{H}{h}\right)^2 = constant \times (\dot{Q}^*)^n \tag{32}$$

by replacing with $\tau = t\sqrt{\frac{g}{H}\frac{H^2}{A}}$ [2,4] and $\dot{Q}^* = \frac{\dot{Q}}{\rho_a c_p T_a \sqrt{g} H^{\frac{5}{3}}}$ [1,4] in the above equation, hence

$$t\sqrt{\frac{g}{H}\frac{H^2}{A}}\left(\frac{H}{h}\right)^2 = constant \times \left(\frac{\dot{Q}}{\rho_a c_p T_a \sqrt{g} H^{\frac{5}{3}}}\right)^n \tag{33}$$

by substituting fixed initial factor values from **Table 2**, the smoke filling time with an opening of 1 m^2 at 0.5 m from ceiling can be finalized as in following:

$$t = constant \times 0.505\left(\frac{\dot{Q}}{2424H^{\frac{5}{2}}}\right)^n \frac{Ah^2}{H^4} \tag{34}$$

Conclusions and recommendation

The output of this research paper is a function of smoke filling time group with constants. Furthermore, such function was consolidated by a regression analysis integrated with data from CFAST simulations. Dimensional analysis is vague in many research areas. and there are also some limitations for this analysis. For this study methodological approach, Buckingham's ∏ method is implemented. Therefore, the limitations of Buckingham's ∏ method will be provided.

The overall limitations of the dimensional analysis for both Rayleigh's method and Buckingham's ∏ method are following [15,16];

1) Dimensional analysis does not have information on dimensionless constants.

2) Sometimes, the factors deriving the relation connecting 2 or more physical quantities are hard to know.

3) Implementing the dimensional analysis correctly needs a concrete background of physical experience and judgment.

Numrich [17] stated that there are the limitations in applying the ∏ Theorem when the relationship among quantities involved is unknown, when it is not clear that all the relevant quantities have been identified, or when some quantities have been introduced that are not relevant to the problem.

Theoretically, dimensional analysis is likely able to solve any problem of any system. However, there are still several limitations in terms of usage. For example, if a quantity is dependent on trigonometric or exponential functions, dimensional analysis cannot be used [18]. The recommendation for this issue is to be cautious in implementing dimensional analysis within certain circumstances.

As mentioned before, the equation derived from the dimensional analysis is not properly consolidated; therefore, further research should be conducted. Computational Modeling by numerous Consolidated Models should be able to revise the equation by simulating results obtained by generating the ∏ groups' values.

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APPENDICES

Appendix A: List of variables, symbols and description.

Symbol	Description	Symbol	Description
T_g	Upper hot gas layer temperature	m	Number of quantities
T_a	Lower ambient layer temperature	k	The minimum number of primary quantities necessary to express the dimensions of the q
z	Smoke clear height	M	Dimension of mass
t	Time	L	Dimension of length
ρ_a	Air density at ambient temperature	T	Dimension of time
S	Floor area	Θ	Dimension of temperature
c_p	Specific heat capacity	H	Height of the room
Q	Heat	A	Area of the room
τ	Dimensionless time	d_s	Opening Location, Distance between ceiling and Opening
g, g	Gravity acceleration	h	Height
z	Smoke clear height	A_{open}	Area of opening
y	Dimensionless height of smoke layer interface above floor	n	Number of quantities
$\dot{m}_p$	Plume mass flow rate	Π	Dimensionless Pi group
$\emptyset$	function group	ϕ	Dimensionless function group
q	Group of physical quantities	C	Dimensionless constant
RT/E	dimensionless temperature	l	Dimensionless length

Appendix B: The value of Pi groups for 30 cases.

Case	∏1	∏2	∏3
	τ	$\dot{Q}^*$	$\frac{h}{H}$
Case 1	39.8933	0.002995	0.478108
Case 2	36.36825	0.003993	0.477292
Case 3	40.23721	0.007986	0.439348
Case 4	35.65636	0.019966	0.477304
Case 5	25.79308	0.039932	0.053635
Case 6	34.04687	0.029949	0.099978
Case 7	46.77146	0.019966	0.20478
Case 8	33.27308	0.029949	0.102405
Case 9	36.11032	0.003993	0.477288
Case 10	39.61818	0.002995	0.478112
Case 11	37.14204	0.003993	0.477292
Case 12	41.26894	0.007986	0.43938
Case 13	33.01515	0.029949	0.101093
Case 14	26.13699	0.039932	0.052858
Case 15	40.23721	0.002995	0.478116
Case 16	41.011	0.007986	0.439456
Case 17	47.20134	0.019966	0.205619
Case 18	26.30895	0.039932	0.054543
Case 19	40.23721	0.002995	0.478084
Case 20	37.14204	0.003993	0.477276
Case 21	46.42755	0.019966	0.20414
Case 22	24.76136	0.039932	0.05277
Case 23	35.65636	0.003993	0.477304
Case 24	41.59909	0.007986	0.439356
Case 25	33.67545	0.029949	0.100434
Case 26	25.75182	0.039932	0.052264
Case 27	40.23721	0.002995	0.478108
Case 28	41.26894	0.007986	0.439416
Case 29	47.45928	0.019966	0.205159
Case 30	33.01515	0.029949	0.10171