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Analysis of Porosity Effects on Unsteady Stretching Permeable Sheet

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Abstract

The aim of this paper is to analyze two-dimensional unsteady flow of a viscous incompressible fluid about a stagnation point on a permeable stretching sheet in presence of a time dependent free stream velocity. Fluid is considered in the porous media with radiation effect. The Rosseland approximation is used to model the radiative heat transfer. Using a time-dependent stream function, partial differential equations corresponding to the momentum and energy equations are converted into non-linear ordinary differential equations. The numerical solutions of these equations are obtained by using the Runge-Kutta Fehlberg method with the help of shooting technique. In the present work, the effect of porosity parameter, radiation parameter and suction parameter on flow and heat transfer characteristics are discussed. The skin-friction coefficient and the Nusselt number at the sheet are computed and discussed. The results reported in the paper are in good agreement with published work in literature by other researchers.

Keywords: Porous media, stretching sheet, thermal radiation, unsteady flow.

Introduction

Flow of an incompressible viscous fluid over a stretching surface is a classical problem in fluid dynamics and important in various processes. Polymer processing involve the strips drawing process. These strips are extruded from a die with some prescribed velocity which become stretched. The stretching surfaces undergo cooling or heating that causes surface velocity and temperature variations. The aforementioned issue has attracted many researchers in recent years due to its applications in many areas. Crane [1] reported an exact solution for the steady two-dimensional flow of a viscous and incompressible fluid stretching with a linear velocity. The pioneering work of Crane was subsequently extended by many authors to explore various aspects of the flow and heat transfer occurring in an infinite domain of the fluid surrounding the stretching sheet. Most of the work has been carried out on fluid at rest, but, in some practical applications, fluid can have some prescribed velocity. Mahapatra and Gupta [2] analyzed stagnation-point flow towards a stretching surface in the presence of free stream velocity. They reported that a boundary layer is formed when the stretching velocity is less than the free stream velocity. As the stretching velocity exceeds the free stream velocity, an inverted boundary layer is formed. Singh *et al*. [3-6] and Tomer *et al*. [7] reported the effect of porosity parameter and radiations on a stretching sheet for orthogonal flow and oblique flow respectively.

There are many situations where the flow and heat transfer are unsteady due to sudden stretching of a sheet. Pop and Na [8] analyzed the unsteady flow past a wall which starts impulsively to stretch from rest. They found that the unsteady flow would approach the steady flow situation after long passage of time. Heat transfer of an unsteady boundary layer flow over a stretching sheet has been studied by

Elbashbeshy and Bazid [9]. They reported that thermal boundary layer thickness and momentum boundary layer thickness decrease with the unsteadiness parameter. Ishak *et al*. [10] investigated boundary layer flow over a continuous stretching permeable surface. They found that the heat transfer rate at the surface increases with the unsteadiness parameter.

Flow through porous media plays an important role in many practical applications, such as ground water flows, extrusion of a polymer sheet from a dye, enhanced oil recovery processes and pollution movement. Attia [11] discussed the effect of porosity parameter on the velocity and thermal boundary layer. Elbashbeshy and Bazid [12] studied heat transfer in porous medium. Hayat *et al*. [13] used Homotopy Analysis Method (HAM) to give an analytic solution for flow through porous medium. All of the above authors discussed the Darcy flow. Pal and Modal [14] discussed non-Darcian flow in porous medium.

The radiation effect takes place at a high temperature. These radiation effects might play a significant role in controlling heat transfer process in the polymer processing industry. The quality of the final product depends to a great extends on heat controlling factors, and knowledge of radiative heat transfer in the system can lead to a desired product with sought characteristics. Free convection heat transfer with radiation effect over the isothermal stretching sheet and a flat sheet near the stagnation point has been investigated respectively by Ghaly and Elbarbary [15], and Pop *et al*. [16]. They found that boundary layer thickness increases with radiation. There have been very few attempts in literature to consider the effect of thermal radiation on the flow and heat transfer in a viscous fluid over an unsteady stretching surface. El-Aziz [17] studied the thermal radiation effects over an unsteady stretching sheet.

Fluid is considered in the presence of thermal radiation. The mechanical and thermal characteristic of such an unsteady process is investigated in the boundary layer approximation. This problem arises in a large class of industrial manufacturing processes, such as polymer extrusion, wire drawing, drawing of plastic sheet, coloring of fabrics etc. The reported results are in good agreement with the available published work in the literature.

Materials and methods

The mathematical model considered here consists of a viscous, incompressible, unsteady two-dimensional fluid flow on a permeable stretching sheet in porous medium. Fluid is considered in the presence of thermal radiation. A stretching sheet is placed in the plane $y=0$ and the $x-$axis is taken along the sheet as shown in **Figure 1**. The fluid occupies the upper half plane, i.e. $y>0$. The sheet has a uniform temperature T_∞ and moves with a non-uniform velocity $u_w(x,t)=\dfrac{cx}{1-\alpha t}$, where c and α are positive constants with dimension (time)$^{-1}$, c is the initial stretching rate and $\dfrac{c}{1-\alpha t}$ is the effective stretching rate which is increasing with time t (as discussed in El-Aziz [17]). The viscous dissipation, Joule heating and induced magnetic field are neglected. The governing equations of continuity, momentum and energy under the above assumptions are;

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{\partial U}{\partial x}+\frac{\partial U}{\partial t}+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\nu}{k*}(u-U) \tag{2}$$

and

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{K}{\rho\, c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho\, c_p}\frac{\partial q_r}{\partial y} \tag{3}$$

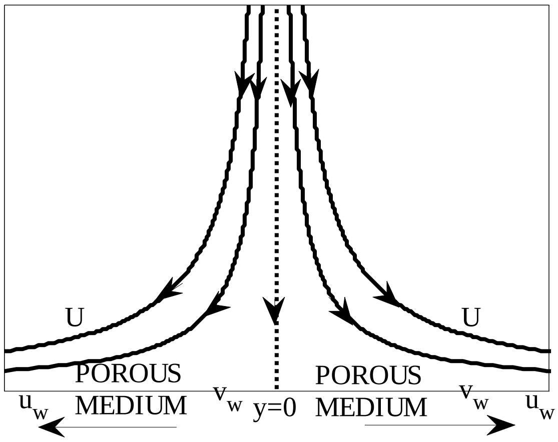

Figure 1 Physical model of the problem.

where u and v are velocity components along the x and y axes respectively, $U(x,t)=\dfrac{bx}{1-\alpha t}$ is the free stream velocity of fluid, b is a positive constant with dimension (time)$^{-1}$. ν is the kinematic viscosity, k^* is the permeability of saturated porous medium, T is the temperature, ρ is the density of the fluid, K is the thermal conductivity and c_p is the specific heat at a constant pressure.

Here, q_r is approximated by the Rosseland approximation, is expressed as;

$$q_r = -\frac{4\sigma_s}{3k}\frac{\partial T^4}{\partial y} \tag{4}$$

where k is the mean absorption coefficient, and σ_s is the Stefan-Boltzmann constant. It is assumed that the temperature difference within the flow is so small that T^4 can be expressed as a linear function of T_∞. This can be obtained by expanding T^4 using the Taylor series about T_∞ and neglecting the higher order terms. Thus,

$$T^4 = 4T_\infty^3\,T - 3T_\infty^4. \tag{5}$$

Therefore, using the above Eq. (4), change in radiative flux with respect to y is obtained as;

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma_s T_\infty^3}{3k}\frac{\partial^2 T}{\partial y^2}. \tag{6}$$

Boundary conditions for the given model are;

$$\left.\begin{array}{l} u = u_w(x,t),\ v = v_w(t)\ \ and\ T = T_w(x,t)\ \ at\ \ y = 0 \\ u = U(x,t) \qquad\quad and \qquad T \to T_\infty \quad as\ \ y \to \infty \end{array}\right\} \tag{7}$$

where $v_W(t) = -v_O \dfrac{1}{\sqrt{1-\alpha t}}$ is the velocity of suction $(v_o > 0)$ at the wall, of the fluid. As discussed in El-Aziz [17], $T_w(x,t) = T_\infty + \dfrac{T_o \operatorname{Re}_x (1-\alpha t)^{1/2}}{2}$ is the wall temperature, where $\operatorname{Re}_x = \dfrac{u_w x}{\nu}$ is the local Reynolds number based on the stretching velocity $u_w(x)$, T_o is a reference temperature such that $0 \le T_o \le T_w$. The expression for $u_w(x,t)$, $T_w(x,t)$, $U(x,t)$ and $v_W(t)$ are valid only for time $t < \alpha^{-1}$ unless α become zero.

Introducing the stream function $\psi(x,y)$ as defined by $u = \dfrac{\partial \psi}{\partial y}$, $v = -\dfrac{\partial \psi}{\partial x}$, the dimensionless temperature $\theta = \dfrac{T - T_\infty}{T_w - T_\infty}$ and the similarity variable $\eta = \sqrt{\dfrac{c}{(1-\alpha t)\nu}}\, y$, $\psi = \sqrt{\dfrac{\nu c}{(1-\alpha t)}}\, x F(\eta)$ and $T = T_\infty + T_o \left[\dfrac{c\, x^2}{2\nu}\right](1-\alpha t)^{-3/2}\theta(\eta)$.

With the help of above relations, the governing Eqs. (2) and (3) finally reduces to;

$$F'''(\eta) + F(\eta)F''(\eta) - F'(\eta)^2 - h\left(F'(\eta) + \frac{\eta}{2}F''(\eta)\right) - P(F'(\eta) - \lambda) + h\lambda + \lambda^2 = 0 \tag{8}$$

$$\theta''(\eta)(3R+4) - 3R\Pr\left(\frac{h}{2}\big(3\theta(\eta) + \eta\theta'(\eta)\big) + 2F'(\eta)\theta(\eta) - F(\eta)\theta'(\eta)\right) = 0 \tag{9}$$

where $h = \dfrac{\alpha}{c}$ is the unsteadiness parameter, $P = \dfrac{\nu}{ck^*}(1-\alpha t)$ is porosity parameter, $R = \dfrac{kK}{4\sigma_s T_\infty^3}$ is radiation parameter, $\Pr = \dfrac{\mu c_p}{K}$ is the Prandtl number, and $\lambda = \dfrac{b}{c}$ is the ratio of free stream velocity parameter to the stretching velocity parameter.

The corresponding boundary conditions are;

$$\left.\begin{aligned} &F(0) = s, \quad F'(0) = 1 \ \textit{and}\ \theta(0) = 1 \ \ \textit{at}\ \eta = 0 \\ &F'(\infty) \to \lambda, \ \textit{and} \quad \theta(\infty) \to 0 \quad \textit{as}\ \eta \to \infty \end{aligned}\right\} \tag{10}$$

where $s = \dfrac{v_o}{\sqrt{\nu c}}$ is the suction parameter ($s > 0$ corresponds to suction).

The physical quantities of interest are the skin friction and Nusselt number. The skin-friction coefficient at the sheet is given by;

$$C_f = 2\operatorname{Re}^{-1/2} F''(0) \tag{11}$$

where $\operatorname{Re} = \dfrac{u_w x}{\nu}$ is the local Reynolds number. The rate of heat transfer in terms of the Nusselt number at the sheets is given by;

$$Nu = -\operatorname{Re}^{1/2}\theta'(0). \tag{12}$$

Results and discussion

In the absence of an analytical solution of this problem, numerical solution is an obvious choice. Thus, the governing boundary layer and thermal boundary layer Eqs. (8) and (9) with boundary conditions (10) are solved using the Runge-Kutta Fehlberg method with shooting technique. Different values of porosity parameter P, radiation parameter R and suction parameter s taking step size 0.001 are used for numerical simulation. In the numerical simulation, step-size 0.002 and 0.003 were also checked, and values of $F''(0)$ and $\theta'(0)$ were found in each case correct up to 6 decimal places. To assess the accuracy of the present method, comparison with previously reported data available in the literature has been made. It is clear from **Table 1** that the numerical values of $F''(0)$ in the present paper for different values of λ, in the absence of P, h and s, are in good agreement with the results published in Mahapatra and Gupta [2] and Pop *et al*. [16]. To further validate the approach used in the paper, comparison of the local Nusselt number for different values of the unsteadiness parameter has been carried out and shown in **Table 2**, which are in good agreement with El-Aziz [17]. From **Tables 1** and **2**, it is clear that the scheme used in this paper is stable and accurate.

Table 1 Comparison of the values of $F''(0)$ for different values of stretching parameter λ.

λ	**Value of $F''(0)$**		
	Pop *et al*. [16]	**Mahapatra and Gupta [2]**	**Present result**
0.1	−0.9694	−0.9694	−0.969658
0.2	−0.9181	−0.9181	−0.918169
0.5	−0.6673	−0.6673	−0.667271
2.0	2.0174	2.0175	2.017393
3.0	4.7290	4.7293	4.729037

Table 2 Comparison of Local Nusselt number for different values of unsteadiness parameter h, Prandtl number $\Pr$.

h	**Value of $-\theta'(0)$**		
	Pr	**El-Aziz [17]**	**Present result**
0.8	0.01	0.13810	0.13775
	0.1	0.45170	0.45168
	1.0	1.67280	1.67277
	10	5.70503	5.70502
1.2	0.01	0.16580	0.16621
	0.1	0.50870	0.50871
	1.0	1.81800	1.81806
	10	6.12067	6.12073

Table 3 Values of $F''(0)$ and $-\theta'(0)$ for different values of stretching parameter λ, porosity parameter P when Prandtl number $\Pr = 0.71$, unsteadiness parameter $h = 0.8$, suction parameter $s = 0$ and radiation parameter $R = 1$.

P	$\lambda = 0.5$		$\lambda = 2$	
	$F''(0)$	$-\theta'(0)$	$F''(0)$	$-\theta'(0)$
0	−0.76689	0.94443	2.15300	1.20026
1	−0.91538	0.93505	2.37273	1.20688
2	−1.04299	0.92807	2.57400	1.21248
3	−1.15661	0.92256	2.76091	1.21732

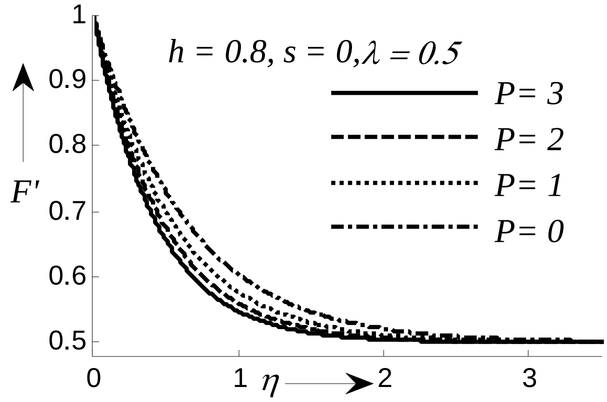

Figure 2 Velocity profile $F'(\eta)$ versus η for different values P when $s = 0$, $h = 0.8$, $\lambda = 0.5$.

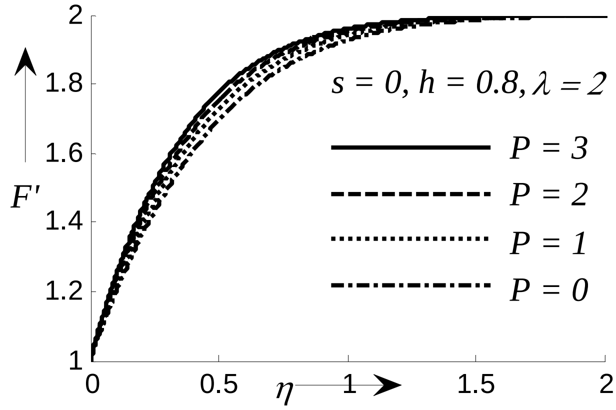

Figure 3 Velocity profile $F'(\eta)$ versus η for different values P when $s = 0$, $h = 0.8$, $\lambda = 2$.

Figure 2 and **3** present the velocity profiles for various values of the porosity parameter for $\lambda = 0.5$ and 2, respectively. The effect of the porosity parameter on the velocity profile depends on the stretching parameter. **Figure 2** shows that the boundary layer thickness decreases considerably as the porosity parameter increases for the stretching parameter less than one. For $\lambda < 1$, the free stream velocity less than the stretching velocity and as the porosity parameter increases it implies an increase in pressure and straining motion near the stagnation point which results in a thinning of the velocity boundary layer. Whereas for $\lambda > 1$, the velocity boundary layer thickness increases as the porosity parameter increases. This is due to the inverted boundary layer formed for $\lambda > 1$. Though the porosity parameter does not directly enter into the energy equation, it actually affects the velocity distribution and therefore affects the temperature profile indirectly. It is worth mentioning here that in the absence of porous medium, the velocity profile is identically equal to the temperature profile i.e. $F'(\eta) = \theta(\eta)$ for the Prandtl number equal to 1.

Table 4 Values of $-\theta'(0)$ for different values of stretching parameter λ, porosity parameter P, radiation parameter R when Prandtl number $\Pr = 0.71$, unsteadiness parameter $h = 0.8$ and suction parameter $s = 0$.

R	$\lambda = 0.5$		$\lambda = 2$	
	$P = 0$	$P = 2$	$P = 0$	$P = 2$
1	0.94443	0.92807	1.15505	1.21248
3	1.21599	1.19567	1.50157	1.51728
6	1.32774	1.30599	1.62343	1.64047
10	1.38151	1.35912	1.68173	1.69939

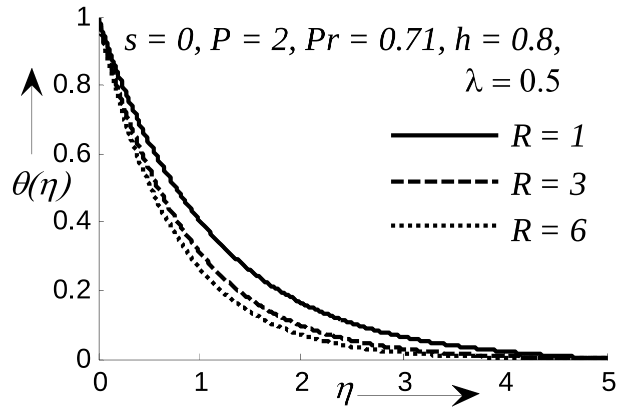

Figure 4 Temperature profile $\theta(\eta)$ versus η for different values R when $P = 1, h = 0.8,$ $\Pr = 0.71, s = 0$ and $\lambda = 0.5$.

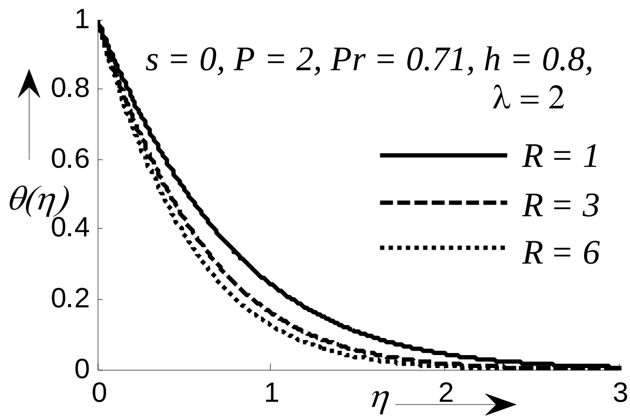

Figure 5 Temperature profile $\theta(\eta)$ versus η for different values R when $P = 1, h = 0.8,$ $\Pr = 0.71, s = 0$ and $\lambda = 2$.

It is observed from **Figure 4** and **5** that the increase of the radiation parameter R leads to a decrease of the temperature profile for all values of λ . This result can be explained by the fact that a increase in the value of R for a given value of T_∞ means a increase in the Rosseland radiation absorptivity k. The divergence of the radiative heat flux $\partial q_r / \partial y$ decreases as k increases, which in turn decreases the rate of radiative heat transferred to the fluid, and hence the fluid temperature decreases. In view of this explanation the effect of radiation become more significant as $R \to \infty,$ and can be neglected when $R \to 0.$ Further, it is seen from figures that for small values of $R,$ the thermal boundary layer thickness are small. Therefore, higher values of radiation parameters imply a higher surface heat flux. The skin friction and Nusselt numbers increase with the radiation parameter as shown in **Table 4**.

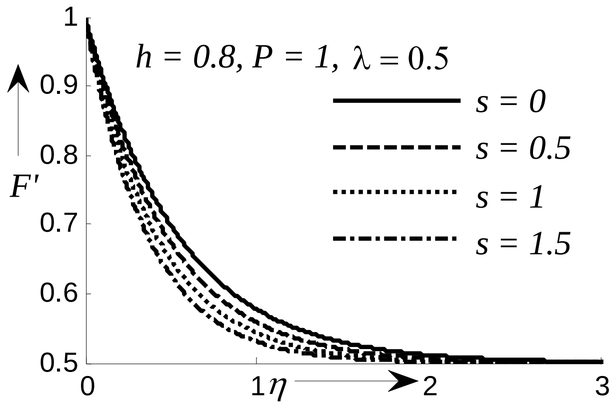

Figure 6 Velocity profile $F'(\eta)$ versus η for different values s when $P = 1, h = 0.8, \lambda = 0.5.$

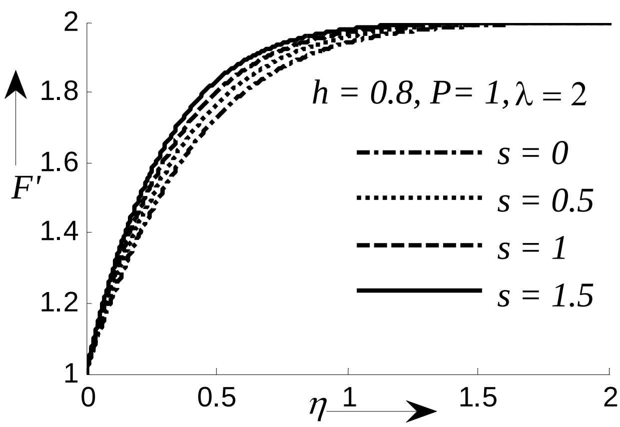

Figure 7 Velocity profile $F'(\eta)$ versus η for different values s when $P = 1, h = 0.8, \lambda = 2.$

Now we concentrate on the velocity and temperature distribution for the variation of the suction parameter s in the presence of the unsteadiness parameter $h.$ It is observed from **Figure 6** that with an increase in suction parameter $s\,(s > 0),$ fluid velocity is found to decrease for $\lambda < 1,$ that is, suction causes a decrease in the velocity of the fluid in the boundary layer region. This effect acts to decrease the wall shear stress. An increase in suction causes progressive thinning of the boundary layer. For $\lambda > 1$, it is observed from **Figure 7** that with an increase in s, the fluid velocity increases, which is due to the fact that an inverted boundary is formed. **Figure 8** and **9** exhibit that the temperature $\theta(\eta)$ in the boundary layer also decreases with the increase $s\,(s > 0)$. Thermal boundary layer thickness decrease with the increase in suction parameter, which leads to an increase in the rate of heat transfer.

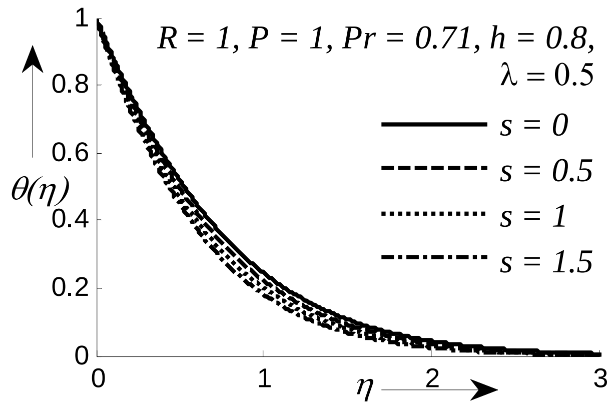

Figure 8 Temperature profile $\theta(\eta)$ versus η for different values s when $P=1$, $R=1$, $\Pr = 0.71$, $h = 0.8$ and $\lambda = 0.5$.

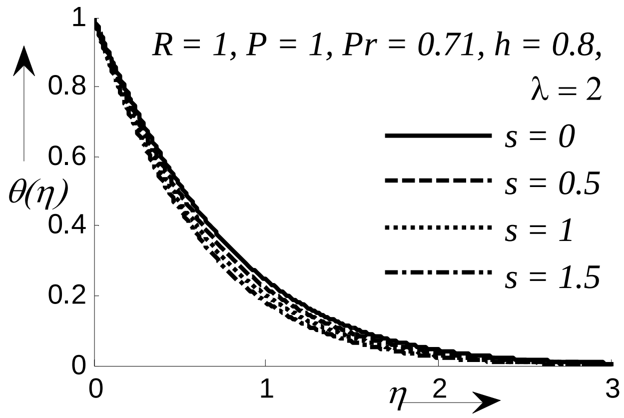

Figure 9 Temperature profile $\theta(\eta)$ versus η for different values s when $P=1$, $R=1$, $\Pr = 0.71$, $h = 0.8$ and $\lambda = 2$.

Conclusions

Two-dimensional viscous, incompressible, unsteady flow on a permeable stretching sheet in a porous medium in the presence of a time dependent free stream velocity has been investigated. A numerical solution for the governing equations has been obtained, which allows the computation of the flow and heat transfer characteristics for various values of the porosity parameter, unsteadiness parameter, radiation parameter, suction parameter and stretching parameter. The results show that;

1) The effect of the porosity parameter on the velocity profile depends on the stretching parameter.

2) The temperature profile decreases with the increase of the radiation parameter. However, the Nusselt number and skin friction number increases with the radiation parameter.

3) The velocity profile decreases as the suction parameter increases with the stretching parameter less than one, and it increases when the stretching parameter is greater than one.

4) The temperature profile decreases as the suction parameter increases.

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