WALAILAK JOURNAL
http://wjst.wu.ac.th Applied Mathematics

Slip Flow of a Maxwell Fluid Past a Stretching Sheet

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Received: 12 December 2012, Revised: 2 May 2013, Accepted: 13 June 2013

Abstract

The slip flow rate of a non-Newtonian fluid (Maxwell model) past a stretching sheet is investigated in this paper. The slip condition for the Maxwell fluid is formulated and presented for the first time. The governing nonlinear partial differential equations and boundary conditions are transformed to nonlinear ordinary differential equations and boundary conditions using the well established similarity transformations for a stretching flow. For the numerical solution of the nonlinear problem we first linearize it using quasilinearization. Then the boundary value problem was transformed into 2 initial value problems by employing the method of superposition. The initial value problems were then integrated using a fourth order Runge-Kutta method. The influence of the slip parameter on the velocity components and skin friction coefficient is analyzed through graphical results. The results are valid for all the values of the slip parameter ranging from zero (no-slip) to infinity (full slip). It is also found that numerical results exist for the values less or equal to one of the dimensionless relaxation time parameter.

Keywords: Slip condition, Maxwell fluid, stretching flow, quasilinearization, numerical solution

Introduction

Generally, the problems for Newtonian and non-Newtonian fluid flows are solved under the assumption that the fluid sticks to the boundary. This is known as the famous no-slip boundary condition. This no-slip boundary condition is not valid for many non-Newtonian fluids and flow situations. For example if the fluid is a polymer solution, emulsion, suspension or foam the no-slip boundary condition is not appropriate and one needs to replace it with the slip boundary condition. On the other hand the no-slip boundary condition is required for a certain amount of roughness and for sufficiently smooth surface this condition is inadequate and one needs to incorporate the slip boundary condition which states that the tangential component of velocity at the surface is proportional to the wall shear stress. Navier [1] and Maxwell [2] independently proposed this linear slip boundary condition. The issue of the slip boundary condition is discussed in detail by Beavers and Joseph [3] and for more details readers are referred to this article. A literature survey indicates that a number of flow problems of viscous fluids have been analyzed using the Navier slip boundary condition [4-8].

The study of viscous and non-Newtonian fluid flows over a stretching sheet is important in the context of the applications of these flows in industrial processes such as extrusion of polymer sheets, coating of thin films and in chemical engineering. The literature is abundant for the stretching flows for Newtonian and non-Newtonian fluids incorporating additional features like heat transfer analysis, magnetic field, porous plate, porous medium, chemical reaction, mass transfer, nonlinear stretching velocity, steady and unsteady flows. However, there are very few articles related to stretching flows with partial slip. The slip flow of a viscous fluid over a stretching sheet was examined for a numerical solution by Wang [9]. The problem considered by Wang is solved for the exact solution by Anderson [10]. Ariel [11] discussed the slip flow of an axisymmetric flow over a stretching sheet and obtained a numerical solution of the problem. Sajid *et al*. [12] analyzed the unsteady flow of a viscous fluid with partial slip through a porous medium. The partial slip condition is replaced by a general slip boundary condition in a recent article by Sajid *et al*. [13]. In the above studies the considered fluid is viscous. However, the literature regarding the effects of slip on stretching flow of non-Newtonian fluids is scarce and there are very few articles available in the literature. Ariel *et al*. [14] analyzed the slip effects on the stretching flow of a Walters' B fluid and obtained an exact solution of the problem. The heat transfer analysis for the slip flow of a second grade fluid is discussed by Hayat *et al*. [15] for the series solution using the homotopy analysis method. Sahoo [16] examined the partial slip on axisymmetric flow of an electrically conducting viscoelastic fluid. More recently, Sahoo [17] provided the numerical solution for the slip effects of the axisymmetric flow of a second grade fluid over a radially stretching sheet.

In all the above studies regarding the slip flow of non-Newtonian fluids only differential type non-Newtonian fluids are taken into consideration. The reason for this choice is that the constitutive equations of differential type fluids are in explicit form in terms of stress components and therefore the shear stress can be easily computed in terms of the velocity components. However, the constitutive relationships for rate type fluids are implicit and one cannot have the expression of shear stress in terms of velocity components. This fact is the major cause of the lack of literature on the two dimensional flow of rate type fluids. The simplest subclass of rate type fluids is the Maxwell fluid and one can find a number of articles regarding the stretching flows of Maxwell fluid [18-25] and references there in. However, for Oldroyd-B fluids there are only two studies regarding the flow over a stretching sheet [26-27]. To the best of our knowledge there is no single article available in the literature in which the effects of the slip boundary condition are analyzed for the stretching flow of a rate type fluid. Keeping this fact in mind the problem of slip flow of a Maxwell fluid is considered in this paper. The slip condition is developed by eliminating the shear stress from the Navier slip boundary condition. The details of the paper are as follows: The slip condition is developed in section 2. The mathematical formulation of the problem is also presented in the same section. The numerical solution of the governing nonlinear ordinary differential equation subject to nonlinear boundary condition is obtained first by linearizing the governing equation using quasilinearization. Then the developed linear boundary value problem is converted to initial value problem using the method of superposition [28]. The solution of the initial value problems is then developed by the fourth order Runge-Kutta method. The details of the numerical procedure are given in section 3. The graphical results and their discussions are presented in section 4. Section 5 is devoted to some concluding remarks.

Problem formulation

Consider the steady, incompressible, two-dimensional, laminar flow of a Maxwell fluid past a stretching sheet. For the mathematical modeling we take a Cartesian coordinate system such that the x-axis is along the flow direction and the y-axis is normal to it. The flow is governed by the continuity equation and equation of motion given by;

$$\nabla \cdot \mathbf{V} = 0, \tag{1}$$

$$\rho \mathbf{a} = \nabla \cdot \mathbf{T}, \tag{2}$$

where $\mathbf{V}$ is the velocity vector, $\mathbf{T}$ is the Cauchy stress tensor and $\mathbf{a}$ is acceleration vector given by;

$$\mathbf{a} = \frac{d\mathbf{V}}{dt} = \frac{\partial \mathbf{V}}{\partial t} + \left(\mathbf{V} \cdot \nabla\right)\mathbf{V}. \tag{3}$$

The Cauchy stress tensor for a Maxwell fluid is;

$$\mathbf{T} = -p\mathbf{I} + \mathbf{S}, \tag{4}$$

where the extra stress tensor $\mathbf{S}$ satisfies the following implicit relation;

$$\mathbf{S} + \lambda_1 \frac{D\mathbf{S}}{Dt} = \mu \mathbf{A}_1, \tag{5}$$

in which λ_1 is the relaxation time, μ is the dynamic viscosity and $\mathbf{A_1}$ is the first Rivlin-Ericksen tensor defined as;

$$\mathbf{A}_1 = \mathbf{L} + \mathbf{L}^T, \quad \mathbf{L} = \nabla \mathbf{V}, \tag{6}$$

and for a 2 rank tensor **S**, a vector **b** and a scalar function ϕ we respectively have;

$$\frac{DS}{Dt} = \frac{\partial S}{\partial t} + \left(V \cdot \nabla\right) S - LS - SL^T, \tag{7}$$

$$\frac{Db}{Dt} = \frac{\partial b}{\partial t} + \left(V \cdot \nabla\right) b - Lb. \tag{8}$$

$$\frac{D\phi}{Dt} = \frac{\partial \phi}{\partial t} + \left(V \cdot \nabla\right) \phi. \tag{9}$$

Using Eq. (4) one can write;

$$\rho a = -\nabla p + \nabla \cdot S, \tag{10}$$

where ρ is the fluid density and p is the pressure. Our purpose is known to eliminate $\mathbf{S}$ between Eqs. (5) and (10). Applying $\left(1 + \lambda_1 D / Dt\right)$ onto Eq. (10), we have;

$$\rho\left(\mathbf{a} + \lambda_1 \frac{D\mathbf{a}}{Dt}\right) = -\left(1 + \lambda_1 \frac{D}{Dt}\right) \nabla\, p + \left(1 + \lambda_1 \frac{D}{Dt}\right)\left(\nabla \cdot \mathbf{S}\right), \tag{11}$$

Following Harris [29] we use;

$$\frac{D}{Dt}\left(\nabla \cdot\right) = \nabla \cdot \left(\frac{D}{Dt}\right). \tag{12}$$

Eq. (11) thus gives;

$$\begin{aligned}\rho\left(\mathbf{a}+\lambda_1\frac{D\mathbf{a}}{Dt}\right)&=-\left(1+\lambda_1\frac{D}{Dt}\right)\nabla p+\nabla\cdot\left(1+\lambda_1\frac{D}{Dt}\right)\mathbf{S}\\&=-\left(1+\lambda_1\frac{D}{Dt}\right)\nabla p+\mu\nabla\cdot\mathbf{A}_1.\end{aligned}\tag{13}$$

For a two-dimensional flow having velocity $\mathbf{V}=\left[u(x,y),v(x,y)\right]$ one gets in the absence of pressure gradient the following equations in component form;

$$\rho\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\lambda_1\left\{u^2\frac{\partial^2 u}{\partial x^2}+v^2\frac{\partial^2 u}{\partial y^2}+2uv\frac{\partial^2 u}{\partial x\partial y}\right\}\right]=\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right),\tag{14}$$

$$\rho\left[u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+\lambda_1\left\{u^2\frac{\partial^2 v}{\partial x^2}+v^2\frac{\partial^2 v}{\partial y^2}+2uv\frac{\partial^2 v}{\partial x\partial y}\right\}\right]=\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right),\tag{15}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0.\tag{16}$$

Using the boundary layer approximations [30];

$$u=O(1),\quad v=O(\delta),\quad x=O(1),\quad y=O(\delta)\tag{17}$$

the flow is governed by Eq. (16) and;

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\lambda_1\left[u^2\frac{\partial^2 u}{\partial x^2}+v^2\frac{\partial^2 u}{\partial y^2}+2uv\frac{\partial^2 u}{\partial x\partial y}\right]=\nu\frac{\partial^2 u}{\partial y^2}.\tag{18}$$

The appropriate boundary conditions for the stretching flow with partial slip at the wall are;

$$\begin{aligned}&u-ax=\frac{\beta}{\mu}S_{xy},\ v=0\quad\text{at}\quad y=0,\\&u\to 0\quad\text{as}\quad y\to\infty,\end{aligned}\tag{19}$$

where $a>0$ is the constant stretching rate having dimensions reciprocal of time and β is the slip length. The problem here is that we do not have an explicit expression for the extra stress tensor component S_{xy} and to get the condition in the form of velocity components we need to eliminate S_{xy} from Eq. (19). For this we apply the operator $\left(1+\lambda_1 D/Dt\right)$ on first part of Eq. (19) and get;

$$\begin{aligned}\left(1+\lambda_1\frac{D}{Dt}\right)(u-ax)&=\frac{\beta}{\mu}\left(1+\lambda_1\frac{D}{Dt}\right)S_{xy}\\&=\beta\left(A_1\right)_{xy}.\end{aligned}\tag{20}$$

Using Eq. (9) and (17) one can simplify Eq. (20) to give;

$$u - ax + \lambda_1 \left[u \frac{\partial u}{\partial x} - au + v \frac{\partial u}{\partial y} \right] = \beta \frac{\partial u}{\partial y} \quad \text{at} \quad y = 0. \tag{21}$$

It is pointed out here that Eq. (21) for the slip flow of a Maxwell fluid is given for the first time in the literature. Introducing the following similarity variables;

$$u = axf'(\eta), \qquad v = -\sqrt{av}\, f(\eta), \qquad \eta = \sqrt{\frac{a}{v}}\, y, \tag{22}$$

here prime denotes derivative with respect to η. Using transformations Eq. (22), the continuity Eq. (16) is identically satisfied and Eq. (18) and boundary conditions (22) and $\left(19, (ii, iii)\right)$ reduces to;

$$f''' - f'^2 + ff'' + \lambda\left(2ff'f'' - f^2 f'''\right) = 0, \tag{23}$$

$$\begin{aligned} &f = 0, \quad f' = 1 - \lambda\left\{(f')^2 - f'\right\} + \gamma f'' \quad \text{at} \quad \eta = 0, \\ &f' \to 0 \quad \text{as} \quad \eta \to \infty, \end{aligned} \tag{24}$$

where $\lambda = \lambda_1 a$ is the dimensionless relaxation time and $\gamma = \beta\sqrt{a/v}$ is the dimensionless velocity slip parameter.

Numerical solution procedure

For the numerical solution of the problem given by nonlinear Eq. (23) and nonlinear boundary conditions (24), we first linearize Eq. (23) using quasilinearization and found the linear equation at the $(k+1)^{\text{th}}$ step in the following form;

$$Af_{k+1}''' + Bf_{k+1}'' + Cf_{k+1}' + Df_{k+1} = E, \tag{25}$$

where

$$A = 1 - \lambda\left(f_k\right)^2, \tag{26}$$

$$B = f_k\left(1 + 2\lambda f_k'\right), \tag{27}$$

$$C = 2\left(\lambda f_k f_k'' - f_k'\right), \tag{28}$$

$$D = \left\{1 - \lambda\left(f_k\right)^2\right\}^{-1}\left[f_k'' + 2\lambda f_k' f_k'' + 2\lambda^2\left(f_k\right)^2 f_k' f_k'' - 2\lambda f_k\left(f_k'\right)^2 + \lambda\left(f_k\right)^2 f_k''\right], \tag{29}$$

$$E = \left\{1 - \lambda\left(f_k\right)^2\right\}^{-1}\left[f_k f_k'' - \left(f_k'\right)^2 + 4\lambda f_k f_k' f_k'' - \lambda\left(f_k f_k'\right)^2 + \lambda\left(f_k\right)^3\left(f_k''\right)^2\right], \tag{30}$$

and the boundary conditions becomes;

$$f_{k+1}(0)=0,\quad f_{k+1}^{'}(0)=1-\lambda\left[\left\{f_{k+1}^{'}(0)\right\}^{2}-f_{k+1}^{'}(0)\right]+\gamma f_{k+1}^{''}(0),\qquad f_{k+1}^{'}(\infty)=0, \tag{31}$$

where the subscript k and $k+1$ respectively represent the k^{th} and $(k+1)^{\text{th}}$ approximations to the solution. By the method of superposition we define the following expression;

$$f_{k+1}=u_1+\mu_1 u_2, \tag{32}$$

where

$$f_{k+1}''(0)=\mu_1, \tag{33}$$

in which μ_1 is a constant which has to be determined. Using Eq. (32) the original boundary value problem is transformed into two initial value problems in the following form;

$$Au_1^{'''}+Bu_1^{''}+Cu_1^{'}+Du_1=E, \tag{34}$$

$$Au_2^{'''}+Bu_2^{''}+Cu_2^{'}+Du_2=0, \tag{35}$$

with the initial conditions;

$$u_1(0)=0,\quad u_2(0)=0, \tag{36}$$

$$u_1^{'}(0)=1,\quad u_2^{'}(0)=\frac{\gamma}{1+\lambda}, \tag{37}$$

$$u_1^{''}(0)=0,\quad u_2^{''}(0)=1, \tag{38}$$

and the condition at infinity yield;

$$\mu_1=-\frac{u_1'(\infty)}{u_2'(\infty)}. \tag{39}$$

To start the iteration procedure we have used the Crane's closed form solution given by;

$$f_0(\eta)=1-e^{-\eta}. \tag{40}$$

For the solution of Eqs. (34) and (35) subject to initial conditions (36) - (38) for different values of the parameters λ and γ using the Runge-Kutta method we choose a numerical value of ∞ as η_∞ and for our calculations the solution up to an accuracy of 10^{-10} is obtained for $\eta_\infty=8$ and $h=0.001$ for 9 to 10 iteration steps. First we integrate above initial value problems and obtain $u_1^{'}(\eta_\infty)$ and $u_2^{'}(\eta_\infty)$. Once these values are known μ_1 can be obtained through Eq. (39) numerically, finally a complete solution is given by Eq. (32).

Results and discussions

We compute the velocity field by solving the nonlinear ordinary differential Eq. (23) with nonlinear boundary conditions (24) numerically first by linearizing the governing equation using quasilinearization. Then the developed linear boundary value problem is converted to an initial value problem using method of superposition. In order to see the influences of the non-dimensional velocity component in the *x*-direction f' and the velocity component in *y*-direction *f* for different values of the dimensionless relaxation time and slip parameters, we plotted in **Figures 1 - 5**. **Figure 1** is made to see the effects of the velocity slip parameter γ on the velocity f when λ is fixed. It is evident from this figure that the fluid velocity decreases and the boundary layer thickness increases with an increase in velocity slip parameter γ. The effects of the velocity slip parameter on the velocity component f' are presented in **Figure 2**. This figure illustrates that the fluid velocity decreases with increasing values of the velocity slip parameter. For the no-slip case when $\lambda \to 0$ the value of the fluid velocity f' approaches asymptotically equal to 1. **Figure 3** shows the effects of dimensionless relaxation time λ on the velocity component f for a fixed value of the slip parameter. This figure depicts that no variation in f exists inside the boundary layer however, outside the boundary layer the velocity decreases by an increase in the relaxation time parameter λ. The influence of the relaxation time parameter λ on the velocity component f' is displayed in **Figure 4**. This figure elucidates that the fluid velocity oscillates near the boundary. It is also consistent with the results presented in [14]. **Figure 5** is devoted to see the effects of the relaxation time parameter λ and the velocity slip parameter γ on the skin friction coefficient $-f''(0)$. It is found that the solution exists up to critical values of the parameter λ and beyond which no solution is possible. The critical values for the present numerical solution are denoted by λ_c and are found to be equal to 1 hence the solutions are possible only for $0 \le \lambda \le 1$. Therefore, the graphical results are given for the solution within the range of admissible values of λ. However, a solution exists for all values of the slip parameter γ from $\gamma = 0$ (no-slip case) to $\gamma \to \infty$ (full slip case). The behavior of skin friction $-f''(0)$ is plotted against the parameter λ for different values of the slip parameter. It is observed from this figure that $-f''(0)$ is an increasing function of parameter λ and decreasing function of parameter γ. It is also important to mention that this kind of slip flow for a rate type non-Newtonian fluid has never been considered before.

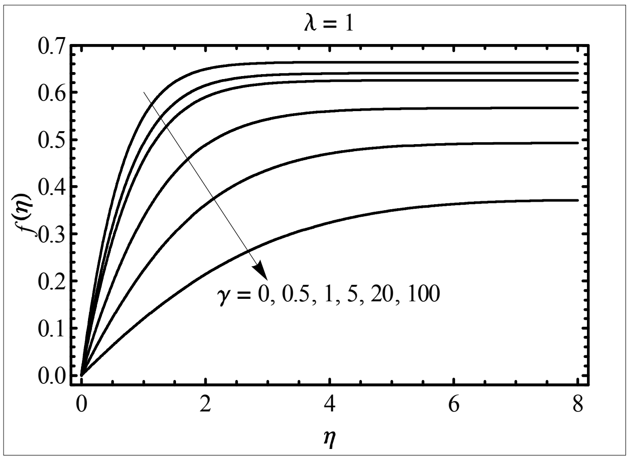

Figure 1 Influence of the slip parameter on the velocity component *f* when $\lambda = 1$.

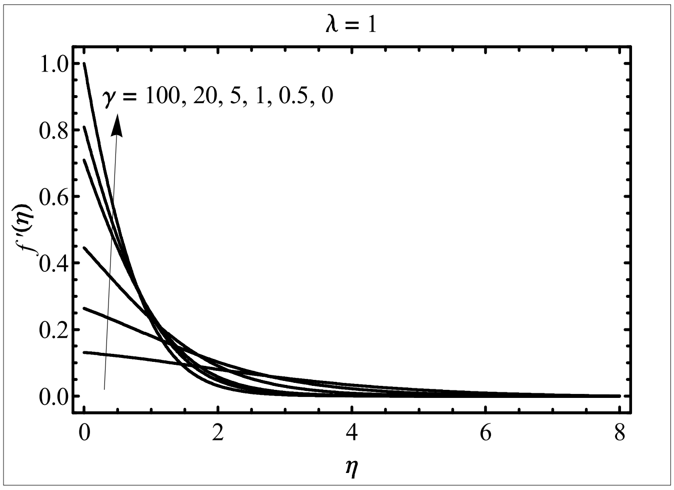

Figure 2 Influence of the slip parameter on the velocity component f' when $\lambda = 1$.

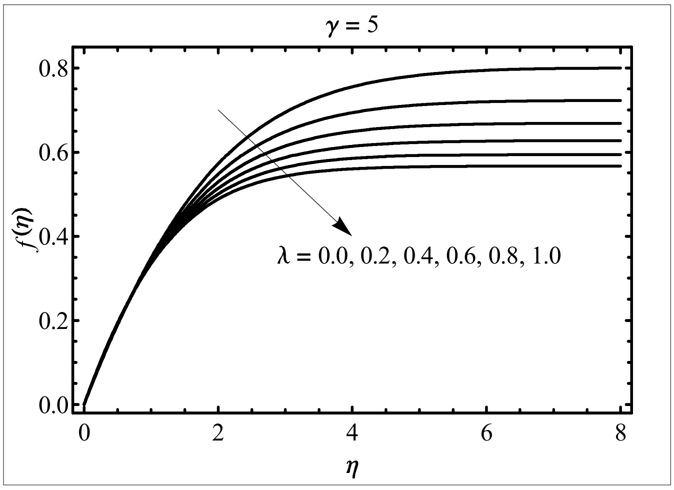

Figure 3 Influence of the relaxation time parameter on the velocity component f when $\gamma = 5$.

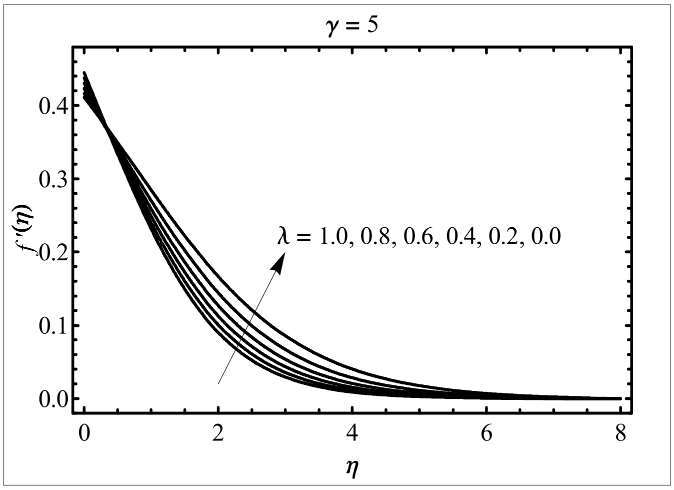

Figure 4 Influence of the relaxation time parameter on the velocity component f' when $\gamma = 5$.

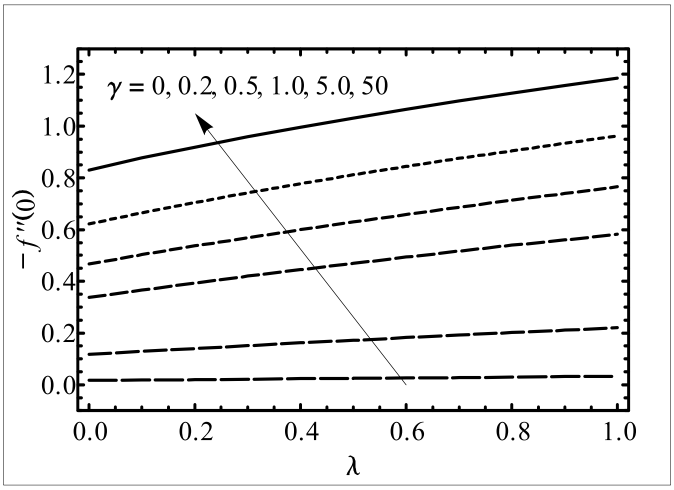

Figure 5 Variation of the skin friction $-f''(0)$ for different values of λ and γ.

Concluding remarks

In this paper the slip boundary condition for the Maxwell fluid is derived for the flow past a stretching sheet. A similar solution is found numerically using the method of superposition together with a Runge-Kutta algorithm. It is found that numerical solutions exist only up to a critical value of the dimensionless relaxation time and for the present case this critical value is found to be one. The results obtained are presented graphically and discussed under the influence of the parameters of interest. The results are valid for all the values of the slip parameter ranging from zero (no-slip) to infinity (full slip). It is the first time that the slip flow of a Maxwell fluid is available in the literature.

Acknowledgement

One of the authors M. Sajid acknowledges the support from Abdus Salam International Centre for Theoretical Physics, Trieste, Italy.

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