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Analysis of Volume Fraction, Diffusion and Electrification of Suspended Particulate Matter in a Two-Phase Couette Flow with Heat Transfer in an Inertial Frame of Reference

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Abstract

The paper deals with the effects of volume fraction, diffusion and electrification of suspended particulate matter on a two-phase flow and heat transfer between 2 infinite parallel plates due to the impulsive start of the upper plate in an inertial frame of reference. The Crank-Nicholson finite difference implicit technique is employed to find the solutions of the governing equations of flow field. An illustration of dependence of physical variables on diffusion parameter, finite volume fraction & magnetic parameter is depicted through figures and tables. The electrification of particles has the effects of decreasing the velocities and increasing the temperature of carrier fluid and suspended particulate matter (SPM). This type of study helps in understanding large-scale phenomena in atmospheric and oceanic circulations.

Keywords: Particulate suspension, volume fraction, diffusion, electrification, equilibrium flow

Nomenclature

p	Pressure of fluid phase
h	Distance between two parallel plates
t	Time
$\bar{t}$	Dimensionless time
(x, y, z)	Space coordinates
(u, v, w)	Velocity components of fluid phase
(u_p, v_p, w_p)	Velocity components of particle phase
$(\bar{u}, \bar{v}, \bar{w})$	Dimensionless velocity components of fluid phase
$(\bar{u}_p, \bar{v}_p, \bar{w}_p)$	Dimensionless velocity components of particle phase
(F_x, F_y, F_z)	Components of the force due to electrification of particles
$\vec{V}, \vec{V}_p$	Velocity vectors of fluid & particle phases respectively
T	Temperature of fluid phase
T_p	Temperature of particle phase
T_0, T_h	Temperatures of the lower & upper plates respectively
C_{f_0}, C_{f_1}	Skin friction coefficients at the lower and upper plates respectively

C_p, C_s	Specific heats of fluid and SPM respectively
K	Thermal conductivity
E_r	Rotation parameter
R_e	Fluid phase Reynolds number
E_c	Eckert number
P_r	Prandtl number
N_u	Nusselt number
B	Electromagnetic field intensity
H	Magnetic field strength
E	Electric field intensity
J	Current density
B_0	Constant field intensity (= $\mu_e H_0$)
H_0	Constant field strength
$\vec{F}_e$	Force due to electrification of charged SPM
M	Magnetic number
D_p	Binary diffusion coefficient
U	Free stream velocity
σ	Electrical conductivity of the medium
J^2/σ	Energy source due to magnetic field
η	Dimensionless co-ordinate perpendicular to the plate
μ_e	Magnetic permeability
(μ, μ_p)	Coefficients of viscosities of fluid and particle phases
(υ,υ_p)	Kinematic coefficient of viscosities of fluid and particle phases respectively
(ρ,ρ_p)	Densities of fluid and particle phases respectively
α	Concentration parameter (ρ_p/ρ)
ε	Diffusion parameter (υ_p/υ)
ρ_{p_∞}	Density of the particles in the free stream
ρ_s	Density of the material of the particle
ϕ	Volume fraction of the suspended particles
τ_p	The velocity equilibration time
τ_T	The thermal equilibration time
τ_w	Skin friction
(θ, θ_p)	Non-dimensional temperature of fluid and particle phases respectively
γ	Ratio of specific heats (C_s/C_p)
λ	Velocity equilibration length
λ_T	Thermal equilibration length
Λ	Non-dimensional velocity relaxation length
Ω	Angular velocity of the rotating frame of reference

Introduction

There have been several investigations of the flow of fluids with suspended particulate matter (SPM) between 2 parallel plates in a rotating frame of reference. The Couette flow and heat transfer in a rotating system with closed form solutions to study heat transfer characteristics was considered in [1]. The unsteady flow of a dusty gas due to non-torsional oscillations of a plate in a rotating frame was discussed in [2]. The Couette flow of a dusty gas between 2 parallel infinite plates for impulsive start as well as for uniformly accelerated start of one of the plates was studied in [3] by obtaining the solution with the help of the Laplace transform technique. The unsteady hydro magnetic laminar flow of a conducting dusty fluid between 2 parallel plates, started impulsively from rest was studied in [4]. The problem on the motion of a dusty gas under a rotating system of coordinates was analysed by obtaining the closed form solutions in [5]. Unsteady Couette flow and heat transfer of a dusty fluid filling the gap between 2 infinite parallel plates kept at arbitrary temperature was studied in [6] by finding the solution to be valid for any time with the help of the Laplace transform followed by its numerical inversion. The Couette flow of a dusty fluid in a rotating frame of reference for impulsive start as well as for uniformly accelerated start of one of the plates was studied by using the Laplace transform technique followed by numerical inversion in [7]. The transient features of the Couette flow of an electrically conducting fluid subject to rotation and magnetic field by setting one of the plates into uniformly accelerated motion was analysed by obtaining the exact solution in [8]. The unsteady Couette flow and heat transfer of a dusty fluid with the inclusion of volume fraction and Brownian diffusion of SPM in the mathematical formulation was studied in [9] by obtaining the solution with the help of the Crank-Nicholson finite implicit scheme. An initial value investigation was made on the motion of an incompressible visco-elastic (Rivlin-Ericksen) fluid with small particles between 2 infinite moving parallel plates in the presence of a transverse magnetic field by obtaining the solution with the help of the Laplace transform technique in [10]. The unsteady laminar flow of an electrically conducting viscous incompressible fluid with embedded non-conducting identical spherical particles bounded by 2 infinite flat plates under the influence of a uniform magnetic field was studied by obtaining the solution with the help of the Laplace transform technique in [11]. The problem of flow of a viscous incompressible embedded fluid with dust particles between 2 oscillating parallel plates using differential geometry techniques was discussed in [12]. The laminar flow of an unsteady viscous fluid with uniform distribution of dust particles through a rectangular channel under the influence of pulsatile pressure gradient was considered by solving the partial differential equations using the variable separable method and the Laplace transform technique in [13]. The unsteady flow and heat transfer of a dusty fluid between 2 parallel plates with variable viscosity, electric conductivity and Navier slip boundary condition was studied by solving the governing equations numerically with the help of a semi-implicit finite difference scheme in [14]. The unsteady laminar free convection flow of an incompressible, viscous, electrically conducting dusty fluid through a porous medium was studied by applying the technique of the state space approach as well as the inversion of the Laplace transformation in [15].The unsteady flow and heat transfer of a dusty fluid between 2 parallel plates was considered in [16]. The effects of the temperature dependent viscosity and electric conductivity, Reynolds number and particle concentration on the unsteady MHD flow and heat transfer of a dusty, electrically conducting fluid between parallel plates were studied in the presence of an external uniform magnetic field using the network simulation method in [17].

However, most of the theoretical and experimental studies were based on negligible volume fractions, and the diffusion of particles through carrier fluid has not been considered. Also, the effect of charged SPM on the two-phase flow field has been neglected.

The consideration of finite volume fraction in flow analysis at high fluid density and at high particle mass fraction was well justified in [18]. The errors that would result from neglecting the particle volume range from insignificant to large. The random motion of the SPM must be taken into account for all practical purposes. Practical contact and separation between the suspended particles and a wall of different materials or similar material with different surface conditions cause the electrification of suspended particles, by virtue of which they are charged. The investigators [19,20] suggested that a very slight charge on the suspended particles in the two-phase flow field has tremendous influence on the dynamics of particulate system.

In the present study, the finite volume fraction, Brownian diffusion and electrification of SPM are considered in a non-conducting fluid with charged SPM to show their effects on the unsteady flow with heat transfer between 2 parallel plates in a rotating system.

Mathematical formulation

Consider the motion of a non-ionized fluid with uniformly distributed electrified dust particles filling the gap between 2 non-conducting infinite parallel plates as shown in **Figure 1**. A cartesian coordinate system of axes is chosen with *X*-axis along the lower plate, *Y*-axis lying on the lower plate and *Z*-axis perpendicular to it. At time $t < 0$, both the fluid with dust particles and the plates are assumed to be at rest and at a uniform temperature T_0. At time $t = 0$, the lower plate, coincident with *X*-*Y* plane, remains fixed, and the upper plate at a distance *h* apart begins to move impulsively in its own plane with a velocity U in a rotating frame of reference having a uniform angular velocity $\vec{\Omega}$ about *Z*-axis. The temperatures of the lower and upper plates at $t > 0$ are maintained at T_0 and T_h respectively. A uniform magnetic field H_0 fixed relative to the pseudo fluid acts along *Z*-axis and the magnetic permeability μ_e is constant throughout the flow field.

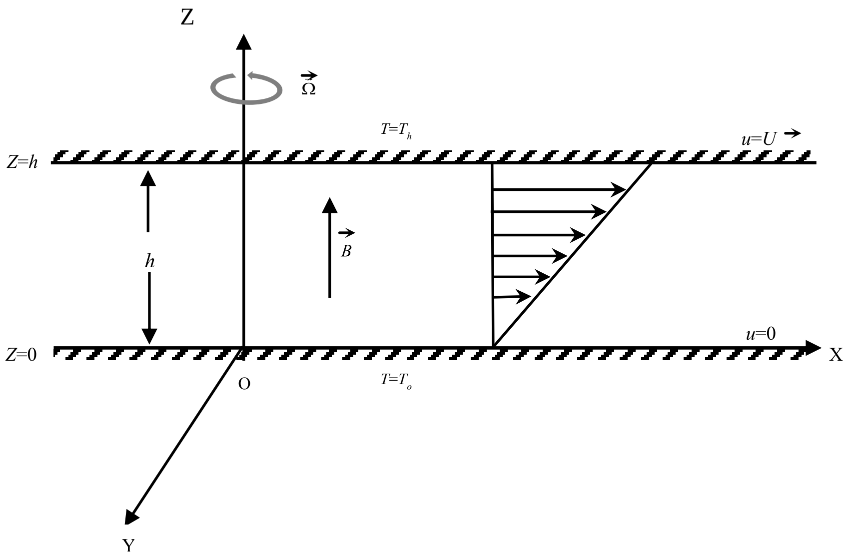

Figure 1 Geometry of the problem.

Assuming that the induced magnetic field produced by the motion of the electrically charged suspended particles is negligible, $\vec{B} = (0,0,B_0)$, where $B_0 = \mu_e H_0$. As no applied and polarization voltage exists, it is assumed that the electric field $\vec{E} = 0$. Since the plates are infinite, all the physical quantities $\vec{V}, \vec{V}_p, T, T_p, p$ in this problem are functions of z and t only. Again, no-slip condition at the plate $z = 0$ implies that $w = 0 = w_p$ throughout the flow field.

In the present problem the components of $\vec{V}, \vec{V}_p, \vec{B}, \vec{E}$ are given by;

$$\vec{V} = (u, v, 0)\ ;\ \vec{V}_p = (u_p, v_p, 0);\ \vec{B} = (0, 0, B_0);\ \vec{E} = (0, 0, 0) \tag{1}$$

The Lorentz force $\vec{F}_e$ due to electrification SPM has the components;

$$\vec{F}_e = \left(-\sigma B_0^2\ u_p, -\sigma B_0^2\ v_p,\ 0\right) \tag{2}$$

and the energy source due to magnetic field is;

$$\frac{J^2}{\sigma} = \sigma B_0^2\left(u_p^2 + v_p^2\right). \tag{3}$$

Introducing the non-dimensional variables;

$$\eta = \frac{z}{h},\ \bar{u} = \frac{u}{U}, \bar{u}_p = \frac{u_p}{U},\ \bar{\rho}_p = \frac{\rho_p}{\rho_{p_\infty}},\ \bar{t} = \frac{t\upsilon}{h^2},\ \theta = \frac{T - T_0}{T_h - T_0},\ \theta_p = \frac{T_p - T_0}{T_h - T_0}, \bar{v} = \frac{v}{U},\ \bar{v}_p = \frac{v_p}{U}, \tag{4}$$

the governing equations of flow field for unsteady non-ionized incompressible fluid with charged SPM in inertial frame of reference as discussed in the references [1,3,6,7,9], after dropping bars are given by;

$$\frac{\partial \rho_p}{\partial t} = \varepsilon\ \frac{\partial^2 \rho_p}{\partial \eta^2} \tag{5}$$

$$(1-\phi)\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial \eta^2} + \frac{\alpha \rho_p R_e}{\Lambda}\left(u_p - u\right) + 2E_r v \tag{6}$$

$$(1-\phi)\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial \eta^2} + \frac{\alpha \rho_p R_e}{\Lambda}\left(v_p - v\right) - 2E_r u \tag{7}$$

$$\phi\frac{\partial u_p}{\partial t} = \frac{R_e}{\Lambda}\left(u - u_p\right) - \varepsilon\ \frac{\partial^2 u_p}{\partial \eta^2} + 2E_r v_p - M\ \frac{u_p}{\rho_p} \tag{8}$$

$$\phi\frac{\partial v_p}{\partial t} = \frac{R_e}{\Lambda}\left(v - v_p\right) - \varepsilon\ \frac{\partial^2 v_p}{\partial \eta^2} - 2E_r u_p - M\ \frac{v_p}{\rho_p} \tag{9}$$

$$(1-\phi)\frac{\partial \theta}{\partial t} = \frac{1}{P_r}\left(\frac{\partial^2 \theta}{\partial \eta^2}\right) + \frac{2}{3}\frac{\alpha\, \rho_p\, R_e}{P_r\, \Lambda}\left(\theta_p - \theta\right) + E_c\left[\left(\frac{\partial v}{\partial \eta}\right)^2 + \left(\frac{\partial u}{\partial \eta}\right)^2\right]$$
$$+ \frac{\rho_p\ \alpha E_c R_e}{\Lambda}\left\{\left(u_p - u\right)^2 + \left(v_p - v\right)^2\right\} \tag{10}$$

$$\phi\frac{\partial\theta_p}{\partial t}=\frac{2}{3}\frac{R_e}{\Lambda P_r\gamma}\left(\theta-\theta_p\right)-\frac{\varepsilon E_c}{\gamma}\frac{\partial}{\partial\eta}\left(u_p\frac{\partial u_p}{\partial\eta}\right)$$

$$-\frac{\varepsilon E_c}{\gamma}\frac{\partial}{\partial\eta}\left(v_p\frac{\partial v_p}{\partial\eta}\right)+\frac{M\,E_c}{\gamma}\frac{\left(u_p^2+v_p^2\right)}{\rho_p} \tag{11}$$

From the geometry of the problem, the initial and boundary conditions are given by;

I. C. $$u=u_p=v=v_p=0\ ,\ T=T_p=T_0\ \ in\ 0\le z\le h\ \ for\ t\le 0 \tag{12}$$

B. C. $$\left.\begin{array}{l} u=0, v=0,\ T=T_0\ at\ \ z=0 \\ u=U, v=0, T=T_h\ at\ \ z=h \end{array}\right\}\ for\ t>0, \tag{13}$$

with the help of the non-dimensional variables (4), the initial and boundary conditions (12) and (13) are reduced to (after dropping bars);

I. C. $$u=u_p=v=v_p=0\ ,\ \theta=\theta_p=0\ \ in\ \ 0\le\eta\le 1\ \ for\ t\le\ 0 \tag{14}$$

B. C. $$\left.\begin{array}{l} u=0,\ v=0,\ \theta=0\ \ at\ \eta=0 \\ u=1,\ v=0,\ \theta=1\ \ at\ \eta=1 \end{array}\right\}\ for\ t>0 \tag{15}$$

Method of solution

By employing Crank-Nicholson finite difference methods Eq. (5) is written as;

$$\frac{\rho_{p_{i,n+1}}-\rho_{p_{i,n}}}{\Delta t}=\varepsilon.\frac{1}{2}\left[\frac{\rho_{p_{i+1,n}}-2\rho_{p_{i,n}}+\rho_{p_{i-1,n}}}{\Delta\eta^2}+\frac{\rho_{p_{i+1,n+1}}-2\rho_{p_{i,n+1}}+\rho_{p_{i-1,n+1}}}{\Delta\eta^2}\right] \tag{16}$$

$$\Rightarrow\quad -\rho_{p_{i-1,n+1}}+\left(2+\frac{2\Delta\eta^2}{\varepsilon\,\Delta t}\right)\rho_{p_{i,n+1}}-\rho_{p_{i+1,n+1}}=\rho_{p_{i+1,n}}+\left(-2+\frac{2\Delta\eta^2}{\varepsilon\,\Delta t}\right)\rho_{p_{i,n}}+\rho_{p_{i-1,n}} \tag{17}$$

$$\Rightarrow\quad -A_i\,\rho_{p_{i-1,n+1}}+B_i\,\rho_{p_{i,n+1}}-C_i\rho_{p_{i+1,n+1}}=D_i \tag{18}$$

where $$A_i=1,\ C_i=1,\ \ \lambda=\frac{\Delta\eta^2}{\Delta t},B_i=\left(2+\frac{2\lambda}{\varepsilon}\right)\ \ \&\ \ D_i=\rho_{p_{i+1,n}}+\left(-2+\frac{2\lambda}{\varepsilon}\right)\rho_{p_{i,n}}+\rho_{p_{i-1,n}} \tag{19}$$

Eq. (6) is written as;

$$\left(1-\phi\right)\frac{u_{i,n+1}-u_{i,n}}{\Delta t}=\frac{1}{2}\left[\frac{u_{i+1,n}-2u_{i,n}+u_{i-1,n}}{\Delta\eta^2}+\frac{u_{i+1,n+1}-2u_{i,n+1}+u_{i-1,n+1}}{\Delta\eta^2}\right] \tag{20}$$

$$+\frac{\alpha\,\rho_{p_{i,n+1}}R_e}{\Lambda}\left[u_{p_{i,n}}-\frac{u_{i,n+1}+u_{i,n}}{2}\right]+2E_r v_{i,n}$$

$$\Rightarrow \quad -u_{i-1,n+1}+\left(2+\frac{\alpha R_e \Delta\eta^2 \rho_{p_{i,n+1}}}{\Lambda}+2(1-\phi)\frac{\Delta\eta^2}{\Delta t}\right)u_{i,n+1}-u_{i+1,n+1} \tag{21}$$

$$=u_{i-1,n}+\left(2(1-\phi)\frac{\Delta\eta^2}{\Delta t}-2-\frac{\alpha R_e \Delta\eta^2 \rho_{p_{i,n+1}}}{\Lambda}\right)u_{i,n}+u_{i+1,n}+2\Delta\eta^2\left(\frac{\alpha R_e \rho_{p_{i,,n+1}}}{\Lambda}u_{p_{i,n}}+2E_r v_{i,n}\right)$$

$$\Rightarrow \; -A_i u_{i-1,n+1}+E_i u_{i,n+1}-C_i u_{i+1,n+1}=F_i \tag{22}$$

where $A_i=1,\ C_i=1,\ \lambda=\dfrac{\Delta\eta^2}{\Delta t},\ E_i=2+\dfrac{\alpha R_e \Delta\eta^2 \rho_{p_{i,n+1}}}{\Lambda}+2(1-\phi)\lambda$ &

$$F_i=u_{i-1,n}+\left(2(1-\phi)\lambda-2-\frac{\alpha R_e \Delta\eta^2 \rho_{p_{i,n+1}}}{\Lambda}\right)u_{i,n}+u_{i+1,n}+2\Delta\eta^2\left(\frac{\alpha R_e \rho_{p_{i,,n+1}}}{\Lambda}u_{p_{i,n}}+2E_r v_{i,n}\right) \tag{23}$$

Similarly to the above, Eqs. (7) - (11) are expressed in the forms of the Eqs. (24) - (28) respectively as follows;

$$-A_i v_{i-1,n+1}+R_i v_{i,n+1}-C_i v_{i+1,n+1}=G_i \tag{24}$$

$$-A_i u_{p_{i-1,n+1}}+E_{p_i} u_{p_{i,n+1}}-C_i u_{p_{i+1,n+1}}=F_{p_i} \tag{25}$$

$$-A_i v_{p_{i-1,n+1}}+R_{p_i} v_{p_{i,n+1}}-C_i v_{p_{i+1,n+1}}=G_{p_i} \tag{26}$$

$$-A_i \theta_{i-1,n+1}+Q_i \theta_{i,n+1}-C_i \theta_{i+1,n+1}=H_i \tag{27}$$

$$\theta_{p_{i,n+1}}=\frac{BB\ \theta_{p_{i,n}}+R\ D_i}{DD}\ ; \quad i=0,\ N \tag{28}$$

where $A_i=1,\ C_i=1,\ \lambda=\dfrac{\Delta\eta^2}{\Delta t},\ R_i=2+\dfrac{\alpha R_e \Delta\eta^2 \rho_{p_{i,n+1}}}{\Lambda}+2(1-\phi)\lambda,$ (29)

$$E_{p_i}=2-\frac{R_e \Delta\eta^2}{\Lambda\varepsilon}-\frac{2\phi\lambda}{\varepsilon}-\frac{M\cdot\Delta\eta^2}{\varepsilon\rho_{p_{i,n+1}}},\quad R_{p_i}=2-\frac{R_e\Delta\eta^2}{\Lambda\varepsilon}-\frac{2\phi\lambda}{\varepsilon}-\frac{M\cdot\Delta\eta^2}{\varepsilon\rho_{p_{i,n+1}}}, \tag{30}$$

$$Q_i=2+\frac{2\alpha R_e \Delta\eta^2 \rho_{p_{i,n+1}}}{3\Lambda}+2P_r\lambda(1-\phi),\ BB=\phi\lambda-\frac{R_e\ \Delta\eta^2}{3\ \gamma P_r\ \Lambda},\ DD=\phi\lambda+\frac{R_e\ \Delta\eta^2}{3\ \gamma P_r\ \Lambda}, \tag{31}$$

$$G_i = v_{i-1,n} - \left(2 + \frac{\alpha R_e \Delta\eta^2 \rho_{p_{i,n+1}}}{\Lambda} - 2(1-\phi)\lambda \right) v_{i,n} + v_{i+1,n} + 2\Delta\eta^2 \left(\frac{\alpha R_e \rho_{p_{i,n+1}}}{\Lambda} v_{p_{i,n}} - 2E_r u_{i,n+1} \right), \quad (32)$$

$$F_{pi} = u_{p_{i-1,n}} + \left(-2 + \frac{R_e \Delta\eta^2}{\Lambda\varepsilon} - \frac{2\phi\lambda}{\varepsilon} + \frac{M \cdot \Delta\eta^2}{\varepsilon\rho_{p_{i,n+1}}} \right) u_{p_{i,n}} + u_{p_{i+1,n}} - 2\frac{\Delta\eta^2}{\varepsilon} \left(\frac{R_e}{\Lambda} u_{i,n+1} + 2E_r v_{p_{i,n}} \right), \quad (33)$$

$$G_{p_i} = v_{p_{i-1,n}} + \left(-2 + \frac{R_e \Delta\eta^2}{\varepsilon\Lambda} - \frac{2\phi\lambda}{\varepsilon} + \frac{M \cdot \Delta\eta^2}{\varepsilon\rho_{p_{i,n+1}}} \right) v_{p_{i,n}} + v_{p_{i+1,n}} - \frac{2\Delta\eta^2}{\varepsilon} \left(\frac{R_e}{\Lambda} v_{i,n+1} - 2E_r u_{p_{i,n+1}} \right), \quad (34)$$

$$\begin{aligned} RD_i = {} & \frac{2R_e \Delta\eta^2}{3\gamma P_r \Lambda} \theta_{i,n+1} - \frac{\varepsilon E_c}{\gamma} \left[u_{p_{i,n+1}} \left(u_{p_{i+1,n+1}} - 2u_{p_{i,n+1}} + u_{p_{i-1,n+1}} \right) + \left(u_{p_{i+1,n+1}} - u_{p_{i,n+1}} \right)^2 \right] \\ & - \frac{\varepsilon E_c}{\gamma} \left[v_{p_{i,n+1}} \left(v_{p_{i+1,n+1}} - 2v_{p_{i,n+1}} + v_{p_{i-1,n+1}} \right) + \left(v_{p_{i+1,n+1}} - v_{p_{i,n+1}} \right)^2 \right] + \frac{M.E_c \Delta\eta^2}{\gamma} \frac{\left(u^2_{p_{i,n+1}} + v^2_{p_{i,n+1}} \right)}{\rho_{p_{i,n+1}}} \quad ; \; i = 0, N \end{aligned} \quad (35)$$

$$\begin{aligned} H_i = {} & \theta_{i-1,n} - \left(2 + \frac{2\alpha R_e \Delta\eta^2 \rho_{p_{i,n+1}}}{3\Lambda} - 2P_r \lambda (1-\phi) \right) \theta_{i,n} + \theta_{i+1,n} \\ & + \frac{4\alpha R_e \Delta\eta^2 \rho_{p_{i,n+1}}}{3\Lambda} \theta_{p_{i,n}} + 2E_c P_r \left[\left(u_{i+1,n+1} - u_{i,n+1} \right)^2 + \left(v_{i+1,n+1} - v_{i,n+1} \right)^2 \right] \\ & + \frac{2\alpha P_r E_c R_e \Delta\eta^2 \rho_{p_{i,n+1}}}{\Lambda} \left\{ \left(u_{p_{i,n+1}} - u_{i,n+1} \right)^2 + \left(v_{p_{i,n+1}} - v_{i,n+1} \right)^2 \right\} \end{aligned} \quad (36)$$

As no slip condition is not satisfied by the particles, so the compatibility conditions at both lower and upper plates for ρ_p, u_p & v_p are considered.

Using the compatibility condition at lower and upper plates for ρ_p in Eq. (5);

$$\left. \frac{\partial \rho_p}{\partial t} \right|_{\eta=0,1} = \varepsilon \left. \frac{\partial^2 \rho_p}{\partial \eta^2} \right|_{\eta=0,1} \quad (37)$$

for $i = 0$, $\rho_{p_{0,n+1}} = 2\rho_{p_{0,n}} - \rho_{p_{0,n-1}}$ (38)

and for $i = N$, $\rho_{p_{N,n+1}} = 2\rho_{p_{N,n}} - \rho_{p_{N,n-1}}$ (39)

Again using the compatibility condition at the lower and upper plates for u_p in Eq. (8);

$$\phi \left. \frac{\partial u_p}{\partial t} \right|_{\eta=0,1} = \frac{R_e}{\Lambda} \left. \left(u - u_p \right) \right|_{\eta=0,1} - \varepsilon \left. \frac{\partial^2 u_p}{\partial \eta^2} + 2E_r v_p \right|_{\eta=0,1} - M \left. \frac{u_p}{\rho_p} \right|_{\eta=0,1} \quad (40)$$

for $i = 0$:

$$u_{p_{0,n+1}} = \left[2 + \frac{M\,\Delta t}{\phi}\left\{\frac{1}{\rho_{p_{0,n}}} - \frac{1}{\rho_{p_{0,n+1}}}\right\}\right]u_{p_{0,n}} - u_{p_{0,n-1}} + \frac{R_e\,\Delta t}{\phi\Lambda}\left[u_{0,n+1} - u_{0,n}\right] + \frac{2E_r\,\Delta t}{\phi}\left[v_{p_{0,n}} - v_{p_{0,n-1}}\right] \tag{41}$$

and for $i = N$:

$$u_{p_{N,n+1}} = \left[2 + \frac{M\,\Delta t}{\phi}\left\{\frac{1}{\rho_{p_{N,n}}} - \frac{1}{\rho_{p_{N,n+1}}}\right\}\right]u_{p_{N,n}} - u_{p_{N,n-1}} + \frac{R_e\,\Delta t}{\phi\Lambda}\left[u_{N,n+1} - u_{N,n}\right] + \frac{2E_r\Delta t}{\phi}\left[v_{p_{N,n}} - v_{p_{N,n-1}}\right] \tag{42}$$

Similarly, using the compatibility condition at the lower and upper plates for v_p in Eq. (9);

$$\phi\frac{\partial v_p}{\partial t}\bigg|_{\eta=0,1} = \frac{R_e}{\Lambda}\left(v - v_p\right)\bigg|_{\eta=0,1} - \varepsilon\,\frac{\partial^2 v_p}{\partial\eta^2} + 2E_r u_p\bigg|_{\eta=0,1} - M\,\frac{v_p}{\rho_p}\bigg|_{\eta=0,1} \tag{43}$$

for $i = 0$:

$$v_{p_{0,n+1}} = \left[2 + \frac{M\,\Delta t}{\phi}\left\{\frac{1}{\rho_{p_{0,n}}} - \frac{1}{\rho_{p_{0,n+1}}}\right\}\right]v_{p_{0,n}} - v_{p_{0,n-1}} + \frac{R_e\,\Delta t}{\phi\Lambda}\left[v_{0,n+1} - v_{0,n}\right] + \frac{2E_r\Delta t}{\phi}\left[u_{p_{0,n}} - u_{p_{0,n-1}}\right] \tag{44}$$

and for $i = N$:

$$v_{p_{N,n+1}} = \left[2 + \frac{M\,\Delta t}{\phi}\left\{\frac{1}{\rho_{p_{N,n}}} - \frac{1}{\rho_{p_{N,n+1}}}\right\}\right]v_{p_{N,n}} - v_{p_{N,n-1}} + \frac{R_e\,\Delta t}{\phi\Lambda}\left[v_{N,n+1} - v_{N,n}\right] + \frac{2E_r\Delta t}{\phi}\left[u_{p_{N,n}} - u_{p_{N,n-1}}\right] \tag{45}$$

Heat transfer

The heat transfer characteristic is expressed in terms of the Nusselt number, defined as;

$$N_u = \frac{h\left(q_{w_1} - q_{w_0}\right)}{K\left(T_h - T_0\right)} \tag{46}$$

where q_{w0} and q_{w1} represent the rates of heat transfer per unit area at the plates $z = 0$ and $z = h$ respectively, and are given by;

$$\left.\begin{aligned} q_{w_0} &= -K\left.\frac{\partial T}{\partial z}\right|_{z=0} = -K\left.\frac{\partial T}{\partial \eta}\cdot\frac{\partial \eta}{\partial z}\right|_{\eta=0} \\ q_{w1} &= -K\left.\frac{\partial T}{\partial z}\right|_{z=h} = -K\left.\frac{\partial T}{\partial \eta}\cdot\frac{\partial \eta}{\partial z}\right|_{\eta=1} \end{aligned}\right\} \tag{47}$$

Skin friction

Skin friction coefficients at the lower and upper plates are respectively given by;

$$C_{f_0} = \frac{2\tau_w}{\rho U^2} = \frac{2}{\rho U^2}\left.\mu\frac{\partial u}{\partial z}\right|_{z=0} = \frac{2\mu U}{\rho U^2\sqrt{\upsilon\tau_p}}\left.\frac{\partial \bar{u}}{\partial \eta}\right|_{\eta=0} = \frac{2}{U\sqrt{\frac{\tau_p}{\upsilon}}}\left.\frac{\partial \bar{u}}{\partial \eta}\right|_{\eta=0}$$

$$= \frac{2}{R_e}\left.\frac{\partial u}{\partial \eta}\right|_{\eta=0} \text{(after dropping bar)} = \frac{2}{R_e}\left(\frac{u_{1,n} - u_{0,n}}{\Delta\eta}\right) \tag{48}$$

$$C_{f_1} = \frac{2\tau_w}{\rho U^2} = \frac{2}{\rho U^2}\left.\mu\frac{\partial u}{\partial z}\right|_{z=h} = \frac{2\mu U}{\rho U^2\sqrt{\upsilon\tau_p}}\left.\frac{\partial \bar{u}}{\partial \eta}\right|_{\eta=1} = \frac{2}{U\sqrt{\frac{\tau_p}{\upsilon}}}\left.\frac{\partial \bar{u}}{\partial \eta}\right|_{\eta=1}$$

$$= \frac{2}{R_e}\left.\frac{\partial u}{\partial \eta}\right|_{\eta=1} \text{(after dropping bar)} = \frac{2}{R_e}\left(\frac{u_{N,n} - u_{N-1,n}}{\Delta\eta}\right) \tag{49}$$

where $R_e = U\sqrt{\frac{\tau_p}{\upsilon}}$. (50)

Results and discussion

From **Tables 1 - 4**, the following observations are made in the frozen flow & equilibrium flow regimes, which refer to small as well as large times. The Nusselt number increases, whereas the skin friction at the lower plate increases and decreases at the upper plate for all times with the increase in ε (**Table 1**). The Nusselt number and the skin friction at the lower plate increase for all times, whereas the skin friction increases for small times and decreases for large times at the upper plate with the increase in the magnetic number *M* (**Table 2**).

Table 1 Variations of N_u, C_{f_0}, C_{f_1} with ε.

t	ε	N_u	C_{f_0}	C_{f_1}
0.0225	0.01	21.076	0.0005265	-0.01820
0.0225	0.02	21.777	0.0005295	-0.01830
0.0225	0.03	26.680	0.0005315	-0.01840
0.4775	0.01	29.005	0.0030790	-0.02250
0.4775	0.02	30.635	0.0030840	-0.02255
0.4775	0.03	33.550	0.0030920	-0.02267

Table 2 Variations of N_u, C_{f_0}, C_{f_1} with M.

t	M	N_u	C_{f_0}	C_{f_1}
0.0225	1.0	20.839	0.0003162	-0.01860
0.0225	2.0	21.042	0.0004878	-0.01850
0.0225	3.0	21.076	0.0005265	-0.01820
0.4775	1.0	26.069	0.0030695	-0.02240
0.4775	2.0	26.080	0.0030732	-0.02245
0.4775	3.0	29.005	0.0030790	-0.02250

The Nusselt number decreases and the skin friction at the upper plate increases for all times, whereas the skin friction increases for small times and decreases for large times at the lower plate with the increase in ϕ (**Table 3**).There is no change in the Nusselt number in the frozen flow regime, but it decreases in equilibrium flow regime, whereas the skin friction at either of the plates increases in frozen flow regime and decreases in equilibrium flow regime with the increase in rotation parameter E_r (**Table 4**).

Table 3 Variations of N_u, C_{f_0}, C_{f_1} with ϕ .

t	ϕ	N_u	C_{f_0}	C_{f_1}
0.0225	0.01	21.076	0.0052650	-0.01830
0.0225	0.02	21.023	0.0055200	-0.01810
0.0225	0.03	20.971	0.0005745	-0.01780
0.4775	0.01	29.005	0.0030790	-0.02250
0.4775	0.02	26.114	0.0030785	-0.02240
0.4775	0.03	26.073	0.0030782	-0.02238

Table 4 Variations of N_u, C_{f_0}, C_{f_1} with E_r.

t	E_r	N_u	C_{f_0}	C_{f_1}
0.0225	0.4	21.076	0.0004950	-0.01815
0.0225	0.8	21.076	0.0005160	-0.01820
0.0225	1.0	21.076	0.0005265	-0.01830
0.4775	0.4	28.681	0.0031027	-0.02235
0.4775	0.8	28.425	0.0030905	-0.02245
0.4775	1.0	28.005	0.0030790	-0.02250

From **Tables 5 - 7**, it is observed that the Nusselt number remains positive for all values of $P_r\, E_c \geq\, < 2$ and for all times, i.e. for small as well as large times, which refer to frozen and equilibrium flow regimes respectively. It shows that heat is transferred from the fluid to the upper plate for all times. This result agrees with the result of Datta and Mishra [6] for small times. Again, the Nusselt number decreases in the frozen flow regime, whereas it increases in the equilibrium flow regime with the increase of time. The result in the frozen flow regime is in agreement with the result of Datta and Mishra [6].

Table 5 $P_r = 0.72$, $E_c = 0.05$ ($P_rE_c < 2$).

t	q_{w_0}	q_{w_1}	N_u
0.0075	2.016	-25.75	27.766
0.0175	2.023	-22.85	24.873
0.0275	2.032	-21.60	23.632
0.2525	3.125	-22.90	26.025
0.4325	4.428	-22.80	27.228
0.4775	6.255	-22.75	29.005

Table 6 $P_r = 0.72$, $E_c = 2.77$ ($P_rE_c \approx 2$).

t	q_{w_0}	q_{w_1}	N_u
0.0075	2.797	-28.23	31.027
0.0175	3.152	-27.17	30.322
0.0275	4.028	-25.30	29.328
0.2525	5.220	-23.82	29.040
0.4325	6.450	-23.58	30.030
0.4775	8.400	-23.37	31.770

Table 7 $P_r = 0.72$, $E_c = 4.16$ ($P_rE_c > 2$).

t	q_{w_0}	q_{w_1}	N_u
0.0075	3.196	-30.15	33.346
0.0175	3.730	-28.74	32.470
0.0275	4.372	-26.75	31.122
0.2525	6.270	-24.97	31.240
0.4325	7.930	-24.73	32.660
0.4775	9.820	-24.29	34.110

From the **Table 8** it is observed that the Nusselt number increases with the increase of the concentration parameter α . The result with respect to the concentration parameter α is in agreement with the result of Datta and Mishra [6].

Table 8 Variations of N_u with α.

α	Nu, obtained in Datta and Mishra [6]	Nu, obtained in the present work
0.1	0.51358270	0.52079683
0.2	0.53162026	0.53883739
0.3	0.54992568	0.55713981

The increase in magnetic number M decreases the magnitude of fluid, as well as the particle velocities u, u_p & v_p (**Figures 2 - 4**), whereas the fluid and particle temperature θ and θ_p increase (**Figures 5** and **6**) throughout the flow regime. The increase in rotation parameter E_r has the effect of increasing the magnitudes of the fluid and particle velocities u & u_p along the direction of motion of the plate (**Figures 7** and **8**), and of decreasing the magnitudes of the fluid and particle velocities v & v_p in the direction normal to the direction of motion of the plate (**Figures 9** and **10**). The result with respect to the rotation parameter E_r agrees with the result of Mitra [5] except in the case of u_p as it decreases with the increase of the rotation parameter. The carrier fluid moves faster (**Figure 11**) and the particles move slower (**Figure 12**) as ϕ increases but the particle velocity v_p in the transverse direction (**Figure 13**) increases with the increase in ϕ.The temperature of the carrier fluid and particles (**Figures 14** and **15**) decrease near the lower plate and increase towards the upper plate as ϕ increases. The temperature of the carrier fluid increases with the increase in ε (**Figure 16**). There is rarefaction of the particles as ε increases (**Figure 17**).

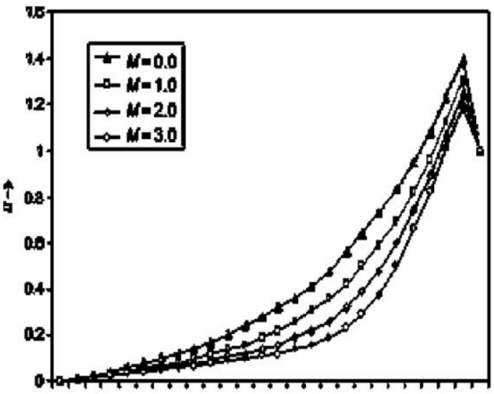

Figure 2 Variation of u with M

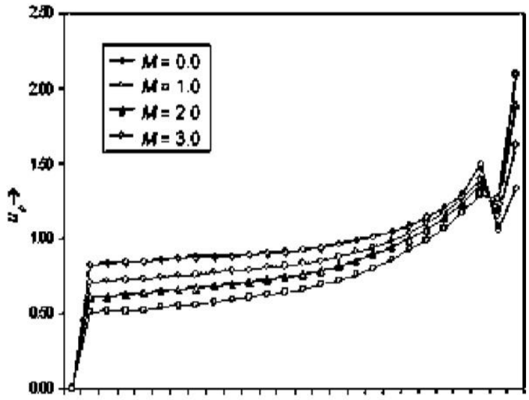

Figure 3 Variation of u_p with M

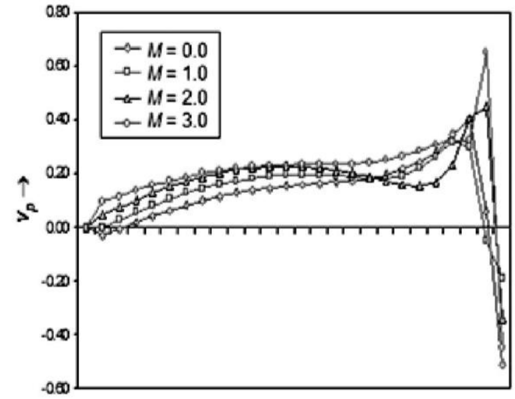

Figure 4 Variation of v_p with M

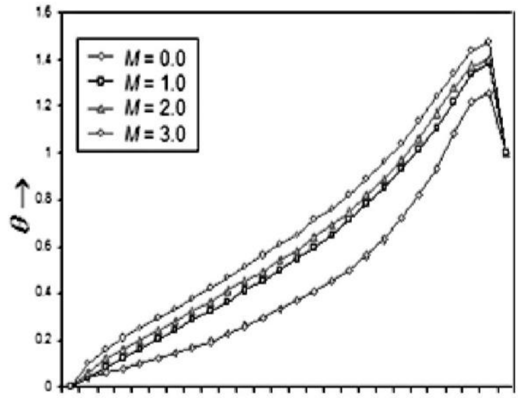

Figure 5 Variation of θ with M

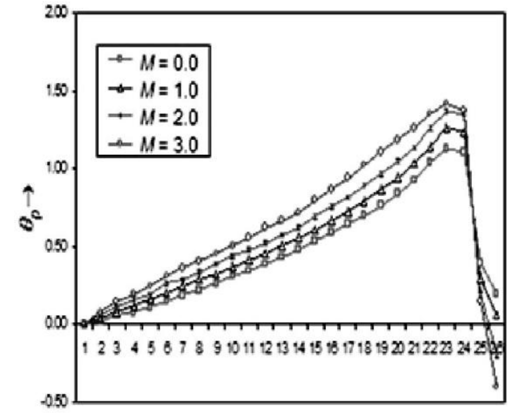

Figure 6 Variation of θ_p with M

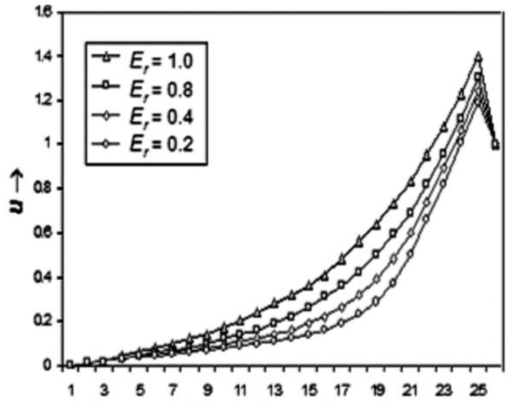

Figure 7 Variation of u with E_r

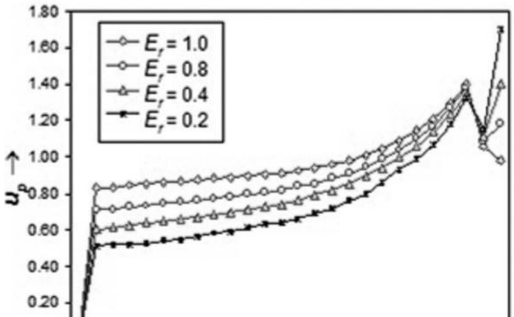

Figure 8 Variation of u_p with E_r

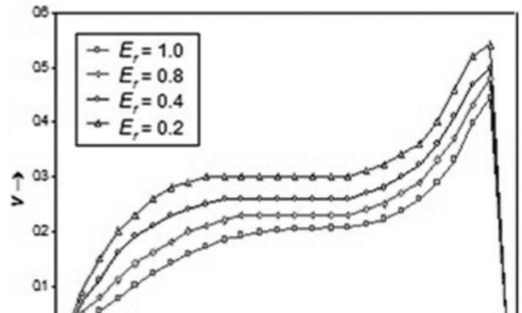

Figure 9 Variation of v with E_r

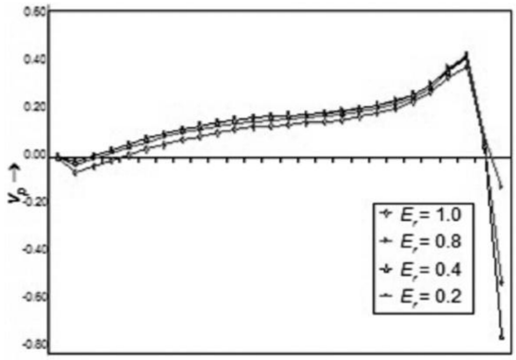

Figure 10 Variation of v_p with E_r

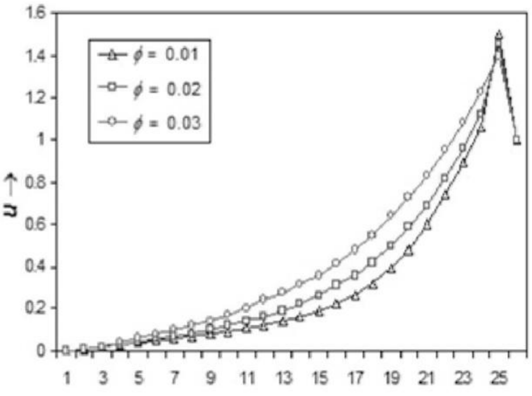

Figure 11 Variation of u with ϕ

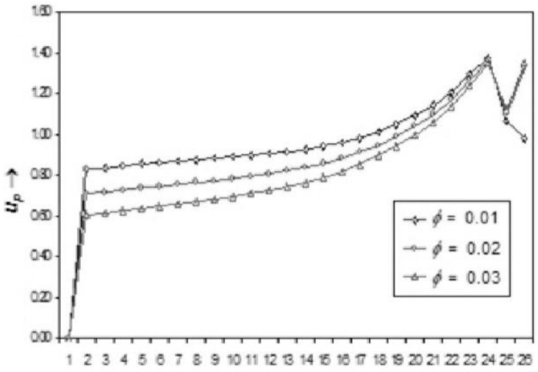

Figure 12 Variation of u_p with ϕ

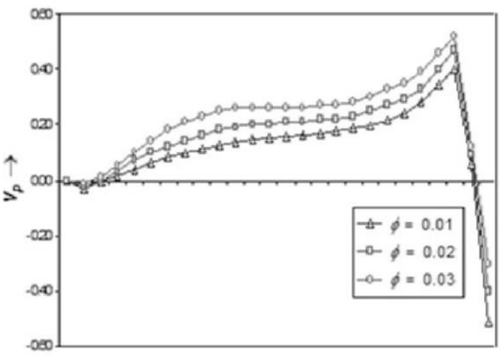

Figure 13 Variation of v_p with ϕ

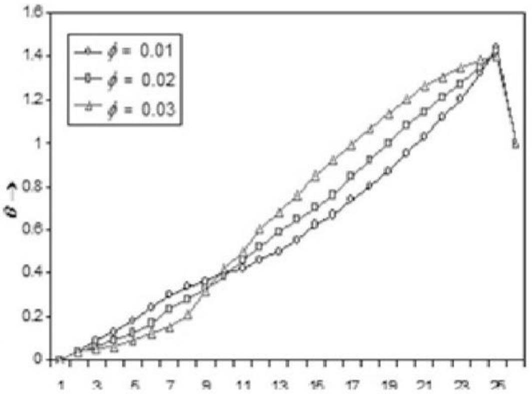

Figure 14 Variation of θ with ϕ

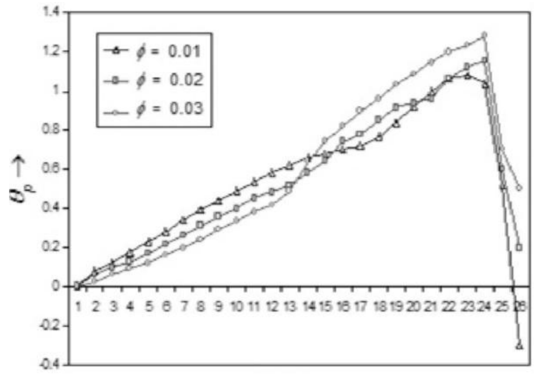

Figure 15 Variation of θ_p with ϕ

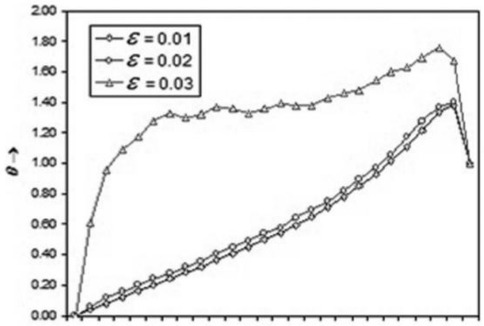

Figure 16 Variation of θ with ε

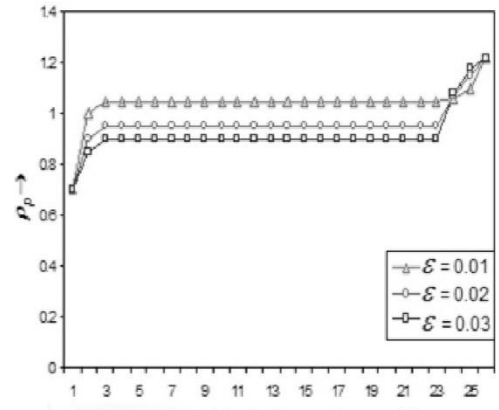

Figure 17 Variation of ρ_p with ε

Figures 18 - 22 illustrate the dependence of u, u_p, v_p, θ, θ_p on the concentration parameter α. The carrier fluid slows down near the lower plate, accelerates towards the upper plate, and becomes slower at the vicinity of the upper plate with the increase of α (**Figure 18**). The particles move slower (**Figure 19**), whereas the velocity of the particle phase in the transverse direction (**Figure 20**) increases with the increase of the parameter α. The temperatures of both the carrier fluid and the particles (**Figures 21** and **22**) increase with an increase of the parameter α.

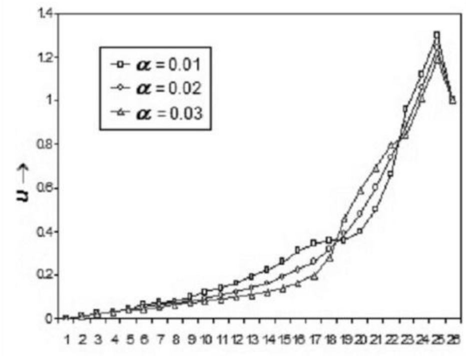

Figure 18 Variation of u with α

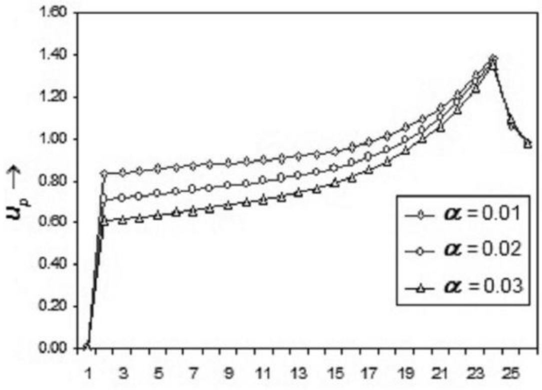

Figure 19 Variation of u_p with α

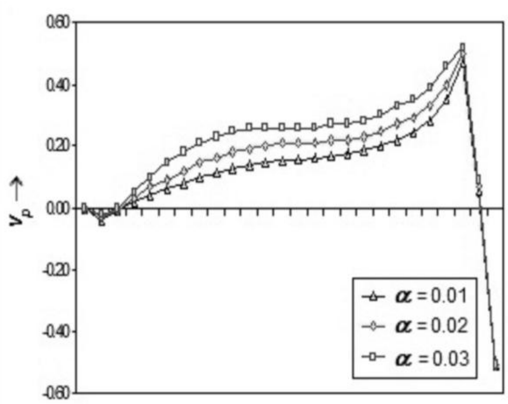

Figure 20 Variation of v_p with α

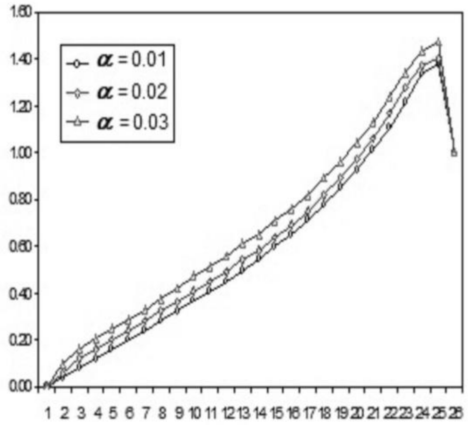

Figure 21 Variation of θ with α

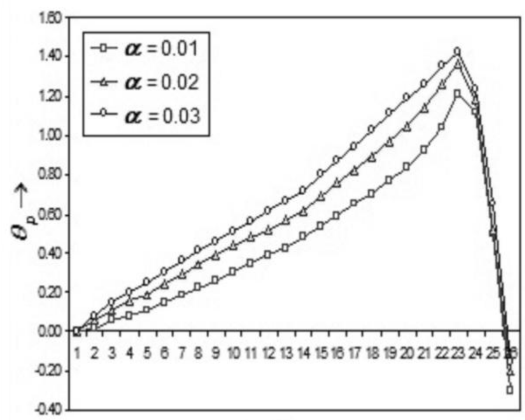

Figure 22 Variation of θ_p with α

Conclusions

The main results of the investigation may be briefly summarized as follows;

1) Higher magnetic numbers M induce lower velocities u, u_p & v_p and higher temperatures θ and θ_p.

2) A higher value of the rotation parameter E_r increases the fluid and particle velocities u, u_p along the direction of motion of the plate and reduces the fluid and particle velocities v, v_p in the transverse direction.

3) A higher diffusion parameter ε reduces the particle dencity ρ_p and increases the temperature θ throughout the flow regime.

4) A higher volume fraction ϕ reduces the particle velocity u_p and increases the fluid velocity u as well as the particle velocity v_p in the transverse direction.

5) Increasing the volume fraction ϕ decreases the temperatures θ and θ_p near the lower plate and increases towards the upper plate.

6) Increasing the concentration parameter α decreases the fluid velocity u near the lower plate, accelerates towards the upper plate and decreases at the vicinity of the upper plate, whereas it reduces the particle velocity u_p and increases the particle velocity v_p in the transverse direction.

7) Higher concentration parameters α induce higher temperatures θ and θ_p.

References

[1] RN Jana and N Datta. Couette flow and heat transfer in a rotating system. *Acta Mech*. 1977; **26**, 301-6.
[2] RN Jana and N Datta. Unsteady flow of a dusty gas due to non-torsional oscillations of a plate in a rotating frame. *Bull. Math. de la soc. De la R.S. de. Roumanic*. 1977; **21**, 309-16.
[3] SK Nag, RN Jana and N Datta. Couette flow of a dusty gas. *Acta Mech.* 1979; **33**, 179-87.
[4] P Mitra and P Bhattacharya. Unsteady hydro magnetic laminar flow of a conducting dusty fluid between two parallel plates started impulsively from rest. *Acta Mech.* 1981; **39**, 171-82.
[5] P Mitra. Flow of a dusty gas with suspended particles in a rotating frame of reference. *Defence Sci. J.* 1981; **31**, 23-30.
[6] N Datta and SK Mishra. Unsteady Couette flow and heat transfer in a dusty gas. *Int. Comm. Heat Mass Tran.* 1983; **10**, 153-62.
[7] N Datta and SK Mishra. Couette flow of a dusty fluid in a rotating frame of reference. *J. Math. Phys. Sci*. 1988; **22**, 421-30.
[8] AK Singh, NC Sacheti and P Chandran. Transient effects on magneto hydrodynamic Couette flow with rotation: Accelerated motion. *Int. J. Eng. Sci.* 1994; **32**, 133-9.
[9] TC Panda, SK Mishra and KC Panda. Volume fraction and diffusion analysis in *SPM* modeling in an inertial frame of reference. *Acta Ciencia Indica* 2001; **27**, 515-25.
[10] NC Ghosh, BC Ghosh and L Debnath. The hydromagnetic flow of a dusty visco-elastic fluid between two infinite parallel plates. *Comput. Math. Appl.* 2002; **39**, 103-16.
[11] BJ Gireesha, CS Bagewadi and P Venkatesh. Unsteady flow of a conducting dusty fluid between two parallel plates. *Appl. Sci.* 2007; **9**, 102-8.
[12] S Rashmi, V Kavitha, RSB Gurumurthy, BJ Gireesha and CS Bagewadi. Unsteady flow of a dusty fluid between two oscillating plates under varying constant pressure gradient. *J. Math. Novi Sad*. 2007; **37**, 25-34.
[13] BJ Gireesha, CS Bagewadi and BC Prasannakumara. Pulsatile flow of an unsteady dusty fluid through rectangular channel. *Comm. Nonlinear Sci. Numer. Simulat.* 2009; **14**, 2103-10.
[14] OD Makinde and T Chinyoka. MHD transient flows and heat transfer of dusty fluid in a channel with variable physical properties and Navier slip condition. *Comput. Math. Appl.* 2010; **60**, 660-9.
[15] MA Ezzat, AA El-Bary and MM Morsey. Space approach to the hydro-magnetic flow of a dusty fluid through a porous medium. *Comput. Math. Appl.* 2010; **59**, 2868-79.
[16] GS Roopa, BJ Gireesha and CS Bagewadi. Unsteady flow and heat transfer of a dusty fluid between two parallel plates. *Int. J. Comput. Sci. Math.* 2011; **3**, 421-33.
[17] P Eguía, J Zueco, E Granada and D Patiño. NSM solution for unsteady MHD Couette flow of a dusty conducting fluid with variable viscosity and electric conductivity. *Appl. Math. Model.* 2011; **35**, 303-16.
[18] G Rudinger. Some effects of finite particle volume on the dynamics of gas-particle mixtures. *AIAA J.* 1965; **3**, 1217-22.
[19] LB Loeb. *Static Electrification*. Springer-Verlag, Berlin, Germany, 1958.
[20] SL Soo. Effect of electrification on the dynamics of a particulate system. *Ind. Eng. Chem. Fundamen*. 1964; **3**, 75-80.