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Irreversibility Analysis of Magnetohydrodynamic Flow over an Impulsively Started Plate

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Abstract

The aim of the present paper is to analyze the irreversibility effects in magnetohydrodynamic flow over an impulsively started plate. The transformed dimensionless governing equations for the flow and heat equations are solved using the Laplace transform technique. The velocity and the temperature profiles are obtained for the parameters involved in the problem. The expression for the entropy generation number and the Bejan number are computed using the velocity and the temperature expressions. The graphs are plotted for velocity, temperature, entropy generation number and Bejan number and are discussed.

Keywords: Magnetohydrodynamic, Laplace transform, entropy generation

Introduction

Irreversibility analysis of flow and heat transfer systems is an important tool used to predict energy losses during the processes. This analysis can be useful to design systems with more utilization of available resources and less energy losses. A standard metric used to study the irreversibility effects is entropy generation rate which is a measure of the destruction of available work of the system. Different sources are responsible for entropy production such as heat transfer along a temperature gradient, fluid friction, magnetic field effects etc. This method was introduced by Bejan [1,2]. Since then, many researchers have studied the entropy generation in flow and heat transfer processes over static and moving surfaces. Al-Odat *et al*. [3] studied the influence of magnetic field on entropy generation due to laminar forced flow past a horizontal flat plate. Saouli and Saouli [4] studied the entropy generation in laminar falling liquid film along an inclined permeable heated plate with upper surface of the liquid film free. Esfahani and Jafarian [5] made entropy analysis of boundary layer flow over a flat plate through various solution methods. Saouli *et al*. [6] examined the entropy effects in gravity driven liquid film along an inclined plate in the presence of hydromagnetic and viscous dissipation effects. Irreversibility analysis for a gravity driven non-Newtonian liquid film along an inclined isothermal plate was made by Makinde [7]. Makinde [8] analyzed the entropy effects in magnetohydrodynamic (MHD) flow and heat transfer over a flat plate with convective boundary conditions. Makinde [9] studied the entropy generations effects for boundary layer flow over a flat plate having variable viscosity in the presence of thermal radiation and Newtonian heating. Butt *et al*. [10] studied the entropy generation effects in Blasius flow under the influence of thermal radiation. Recently, Butt *et al*. [11] investigated the hydrodynamic slip effects on entropy generation over a vertical plate with a convective boundary.

In recent years, numerous investigations have been conducted on the MHD flows because of its important applications in the metallurgical industry. Recently, Ramzan *et al*. [12] studied the axisymmetric flow and heat transfer effects over a nonlinear radially stretching sheet and obtained exact solutions.

The objective of the present work is to study the entropy generation in MHD flow over an impulsively started plate. The solutions are obtained using the Laplace transform method and the results are discussed and analyzed through graphs.

Mathematical formulation

Consider the flow of an electrically conducting incompressible, viscous fluid over an impulsively started plate. The coordinate system is chosen such that the x-axis is taken along the plate and the y-axis is normal to the plate. A magnetic field of strength B_0 is assumed to be applied perpendicular to the plate. The induced magnetic field is neglected by assuming that the magnetic Reynolds number is very small. Initially for time $t^* \leq 0$, the plate and the fluid are at the same temperature T_∞^*. At $t^* > 0$, the plate is assumed to be moving impulsive with a constant velocity U_o and the temperature of the plate is raised to T_w^*. Then the flow and heat transfer phenomenon are governed by the equations;

$$\frac{\partial u^*}{\partial t^*} = \nu \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma B_0^2 u^*}{\rho}, \tag{1}$$

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{\rho c_p} \frac{\partial^2 T^*}{\partial y^{*2}}, \tag{2}$$

The initial and the boundary conditions are;

$$u^* = 0,\ T^* = T_\infty^*,\ \ \text{for all }\ y^*, t^* \leq 0, \tag{3}$$

$$\left.\begin{array}{r} u^* = U_o,\ T^* = T_w^* \ \text{ at } \ y^* = 0, \\ u^* = 0,\ T^* = T_\infty^* \quad \text{ at } \ y^* \to \infty \end{array}\right\} \ t^* > 0. \tag{4}$$

Here u^* is the velocity components in the *x*-direction, t^* is the time, ν is the kinematic viscosity, σ is the electric conductivity, B_0 is the uniform magnetic field strength, k is the thermal conductivity, c_p is the specific heat of the fluid at a constant pressure, ρ is the density, T^* is the temperature of the fluid, T_w^* is the temperature of the plate and T_∞^* is the temperature of fluid far away from the plate. In order to reduce Eqs. (1) - (4) into a non-dimensional form, the following dimensionless variables and parameters are introduced;

$$u = \frac{u^*}{U_0},\ t = \frac{U_0^2\, t^*}{\nu},\ y = \frac{U_0\, y^*}{\nu},\ M = \frac{\sigma B_0^2 \nu}{\rho U_0^2},\ \Pr = \frac{\mu c_p}{k},\ \theta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}, \tag{5}$$

where M is the magnetic field parameter and $\Pr$ is the Prandtl number. With the help of (5), the governing Eqs. (1) - (2) reduce to;

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - M\, u, \tag{6}$$

$$\Pr\frac{\partial\theta}{\partial t}=\frac{\partial^2\theta}{\partial y^2}. \tag{7}$$

The corresponding initial and boundary conditions in dimensionless form are;

$$u=0,\ \theta=0,\quad \text{for all}\ \ y,t\le 0, \tag{8}$$

$$\left.\begin{aligned} u=1,\ \theta=1 \quad &\text{at}\ \ y=0,\\ u=0,\ \ \theta=0\ \ &\text{as}\ \ y\to\infty \end{aligned}\right\}\ t>0. \tag{9}$$

Entropy generation

The local volumetric rate of entropy generation for a viscous fluid in the presence of magnetic field is defined by;

$$S_G=\frac{k}{T_\infty^{*2}}\left[\left(\frac{\partial T^*}{\partial y^*}\right)^2\right]+\frac{\mu}{T_\infty^*}\left[\left(\frac{\partial u^*}{\partial y^*}\right)^2\right]+\frac{\sigma B_0^2}{T_\infty^*}u^{*2}. \tag{10}$$

It is quite evident from Eq. (10) that 3 sources are responsible for entropy generation in the considered problem. The first term on the right hand side of Eq. (10) is the local entropy generation due to heat transfer, the second is the local entropy generation due to fluid friction and the third term is the entropy generation due to the magnetic field. In terms of dimensionless variables, the entropy generation has the form;

$$Ns=\frac{S_G}{S_{Go}}=\left(\frac{\partial\theta}{\partial y}\right)^2+\frac{Br}{\Omega}\left(\frac{\partial u}{\partial y}\right)^2+\frac{Br}{\Omega}M\,u^2. \tag{11}$$

where $S_G=\frac{k\,(T_w^{*2}-T_\infty^{*2})^2\,U_o^2}{T_\infty^{*2}\,\nu^2}$ is the characteristic entropy generation rate, $\Omega=\frac{T_\infty^*}{T_w^*-T_\infty^*}$ is the dimensionless temperature difference and $Br=\frac{\mu U_o^2}{k(T_w^*-T_\infty^*)}$ is the Brinkman number.

An alternative irreversibility distribution parameter is the Bejan number, which gives an idea whether the fluid friction and magnetic field irreversibility dominates or the heat transfer irreversibility. It is simply the ratio of entropy generation due to heat transfer to the total entropy generation.

$$Be=\frac{Entropy\ generation\ due\ to\ heat\ transfer}{Total\ entropy\ generation}. \tag{12}$$

Clearly, the Bejan number ranges from 0 to 1. When the value of Be is greater than 0.5, the irreversibility due to heat transfer dominates, where as $Be<0.5$ refers to irreversibility due to fluid friction and magnetic field. When $Be=0.5$, the contribution of the heat transfer and fluid friction and magnetic field entropy generation are equal.

Solution of the problem

The solution of Eqs. (6) - (7) with conditions (8) - (9) are obtained using the Laplace transform technique defined as;

$$\overline{F}(y,p)=\int_0^\infty F(y,t)e^{-pt}dt, \tag{13}$$

where p is the Laplace transformation parameter.

The Laplace transform of the Eqs. (6) - (7) and the boundary conditions (9) are given by;

$$\frac{d^2\overline{u}}{dy^2}-(M+p)\overline{u}=0, \tag{14}$$

$$\frac{d^2\overline{\theta}}{dy^2}-p\Pr\overline{\theta}=0, \tag{15}$$

$$\begin{aligned}&\overline{u}=\frac{1}{p},\ \overline{\theta}=\frac{1}{p} \quad \text{at} \quad y=0, t>0,\\ &\overline{u}=0,\ \overline{\theta}=0 \quad \text{as} \quad y\to\infty, t>0.\end{aligned} \tag{16}$$

Solving Eqs. (14) - (16), we get;

$$\overline{u}(y,p)=\frac{1}{p}\exp(-\sqrt{M+p}\ y), \tag{17}$$

$$\overline{\theta}(y,p)=\frac{1}{p}\exp(-\sqrt{\Pr p}\ y). \tag{18}$$

Inverting Eqs. (17) and (18) we get the solution for the temperature $\theta(y,t)$ and velocity $u(y,t)$ for $t>0$ in non-dimensional form as;

$$\theta(y,t)=\operatorname{erf}c\left(\frac{\sqrt{\Pr}\ y}{2\sqrt{t}}\right), \tag{19}$$

$$u(y,t)=\frac{1}{2}\left[\exp(\sqrt{M}\ y)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}+\sqrt{M\,t}\right)+\exp(-\sqrt{M}\ y)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}-\sqrt{M\,t}\right)\right]. \tag{20}$$

Here $\operatorname{erf}c(x)$ is the complementary error function defined as;

$$\operatorname{erf}c(x)=1-\operatorname{erf}(x), \qquad \operatorname{erf}(x)=\frac{2}{\sqrt{\pi}}\int_0^x \exp(-\varsigma^2)d\varsigma. \tag{21}$$

Results and discussion

In this section, the effects of the parameters such as magnetic field parameter *M*, Prandtl number *Pr* and the dimensionless time *t* on the velocity and temperature profiles are discussed. Morover, the effects of these parameters as well as the group parameter Br/Ω on local entropy generation number and Bejan number are observed.

In **Figure 1**, it is observed that there is a decrease in the velocity profile with an increase in the magnetic field parameter M. This is due to the increasing opposing force which ultimately slows down the fluid. **Figure 2** shows that for fixed values of the parameters, an increase in velocity is noticed with an increase in time *t*.

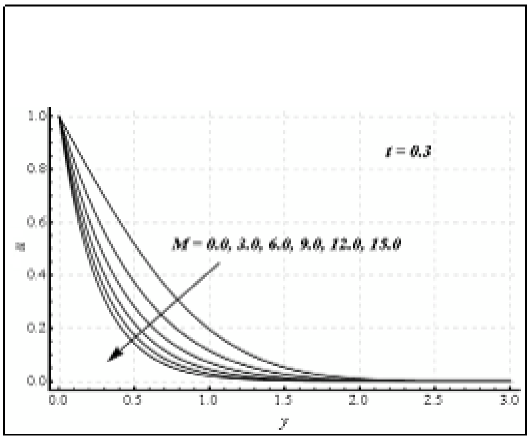

Figure 1 Effects of *M* on *u(y,t)*.

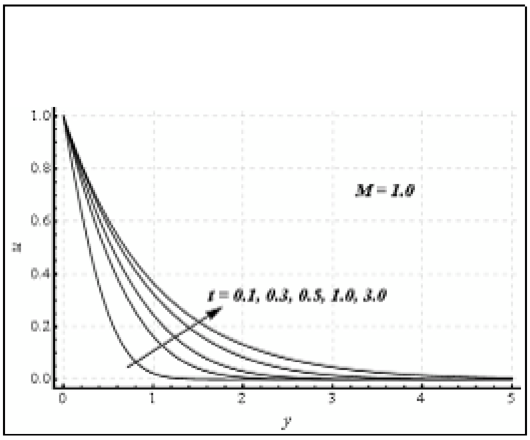

Figure 2 Effects of *t* on *u(y,t)*.

Figure 3 shows the influence of the Prandtl number *Pr* on the temperature profile. It is noteworthy that a decrease in temperature is observed with an increase in *Pr*. Thus for high values of the Prandtl number, the temperature is small and this increases the gradient of temperature. **Figure 4** depicts that the temperature profile increases with non-dimensional time *t*.

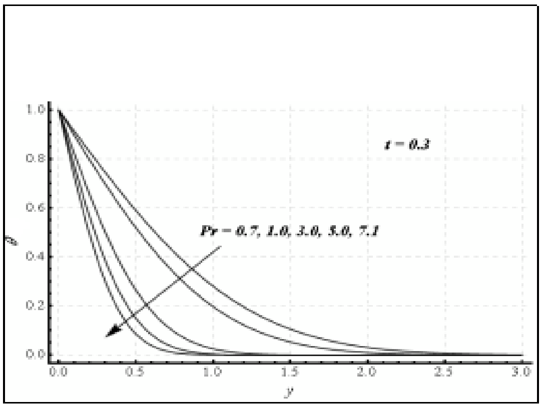

Figure 3 Effects of *Pr* on $\theta(y,t)$.

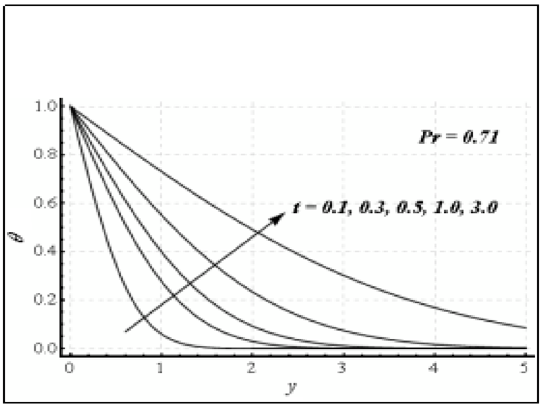

Figure 4 Effects of *t* on $\theta(y,t)$.

Figures 5 - 6 show the influence of physical parameters on the local entropy generation number Ns plotted against dimensionless distance y . The influence of the magnetic field parameter M on the local entropy generation number is presented in **Figure 5**. In the absence of the magnetic field, the entropy generation rate is low. However, the presence of the magnetic field causes more entropy generation in the fluid. Also it is noticed that for a fixed value of M , the entropy generation is maximum near the surface of the plate and decreases with *y*. The effects of the group parameter Br/Ω on Ns are increasing as presented in **Figure 6**. **Figure 7** gives the 3 dimensional plot of the local entropy generation number plotted against non-dimensional distance y and time t . It is observed that the entropy generation rate is high at the surface of the plate and decreases as the distance from the surface increases. Moreover, it is noticed that entropy production is high initially and decreases as the time passes by.

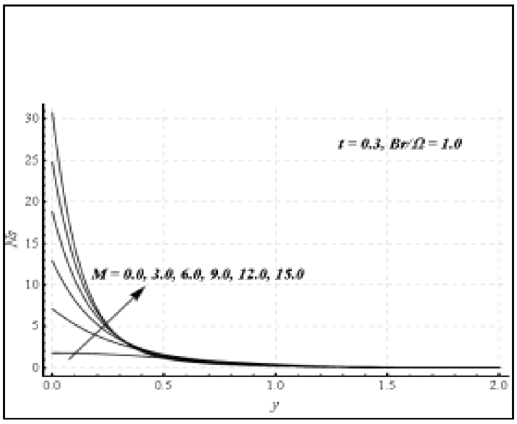

Figure 5 Effects of *M* on *Ns*.

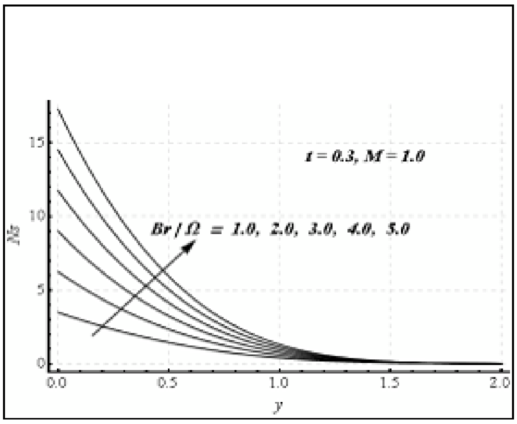

Figure 6 Effects of *Br*/*Ω* on *Ns*.

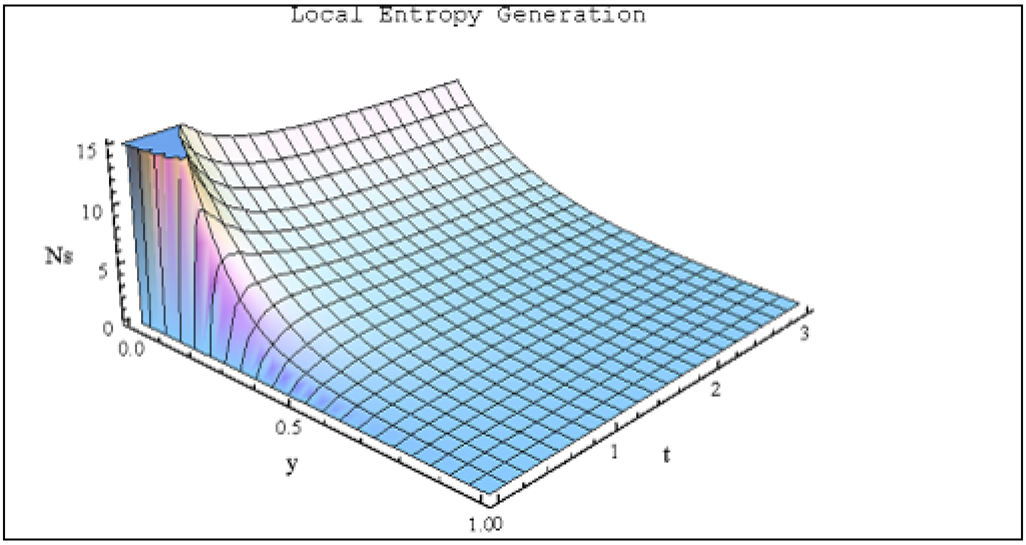

Figure 7 3 dimensional plot of *Ns* against *y* and *t*.

Figure 8 illustrates that with an increase in the magnetic field parameter M, the irreversibility effects due to fluid friction and magnetic field become dominant near the plate surface. Away from the surface, the situation in reversed and the heat transfer irreversibility dominates fully. For the case of the group parameter Br/Ω as presented in **Figure 9**, the fluid friction and magnetic field irreversibility strengthens near the surface with an increase in the group parameter and the irreversibility due to heat transfer is dominant in the far away region from the surface.

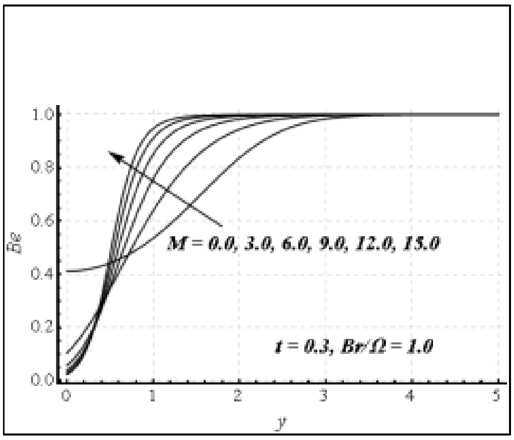

Figure 8 Effects of *M* on *Be*.

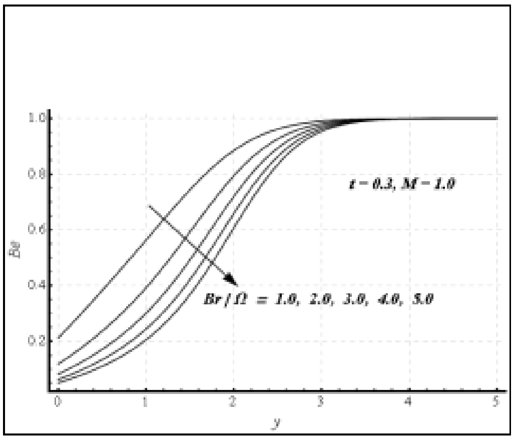

Figure 9 Effects of Br/Ω on *Be*.

Conclusions

This article analyzes the irreversibility effects in MHD flow over a impulsively started plate. The solutions are computed through the Laplace transform technique and the results are interpreted for different parameters. It is observed that the velocity profile decreases with an increase in the magnetic field parameter M and increases with the non-dimensional time t. The effects of Pr on temperature profile are decreasing. On the other hand, temperature increases as the time t passes by. The local entropy generation number increases with the magnetic field parameter M and the group parameter Br/Ω. The fluid friction and magnetic field irreversibility dominates near the surface and the heat transfer irreversibility is dominant in the region far away from the plate.

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