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Article

Analysis of Heat and Mass Transfer Effects on an Isothermal Vertical Oscillating Plate

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Abstract

The objective of this paper is to investigate the heat and mass transfer effects on an isothermal vertical oscillating plate. The dimensionless governing equations are solved using a finite element method. The velocity, temperature and concentration fields for different physical parameters like thermal Grashof number, mass Grashof number, prandtl number, Schmidt number, phase angle and time are shown graphically.

Keywords: Heat and mass transfer, oscillating plate, FEM

Introduction

Heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines, and various propulsion devices for aircraft, missiles, satellites, and space vehicles. If the temperature of the surrounding fluid is rather high, radiation effects play an important role and this situation does exist in space technology. The effect of free convection flow of a viscous incompressible fluid past an infinite vertical plate has many important technological applications in the astrophysical, geophysical and engineering problems. Siegel [1] was the first to study the transient free convective flow past a semi-infinite vertical plate by an integral method. The same problem was studied by Gebhart [2] by an approximate method.

Soundalgekar [3] presented convection effects on the Stokes problem for an infinite vertical plate. The flow of a viscous, incompressible fluid past an infinite isothermal vertical plate, oscillating in its own plane, was solved by Soundalgekar [4]. The effect on the flow past a vertical oscillating plate due to a combination of concentration and temperature differences was studied extensively by Soundalgekar and Akolkar [5].

It is proposed to study heat and mass transfer effects on an isothermal vertical oscillating plate. The dimensionless governing equations are solved using the finite element method. The effects of various governing parameters on the velocity, temperature, concentration are shown in figures and discussed in detail.

Formulation of the problem

Heat and mass transfer effects on unsteady flow of a viscous incompressible fluid past an infinite isothermal vertical oscillating plate with variable mass diffusion is studied. Here the unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate of temperature, T_∞ and concentration, C'_∞. Here, the x'-axis is taken along the plate in the vertical upward direction and the y'-axis is taken normal to the plate. **Figure 1** is the physical model of the problem and coordinate system. Initially, it is assumed that the plate and the fluid are of the same temperature and concentration. At time $t' > 0$, the plate starts oscillating in its own plane with frequency w' and the temperature of the plate is raised to T_W and the concentration level near the plate is raised linearly with respect to time. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. Then by the usual Boussinesq's approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta\left(T - T_\infty\right) + g\beta^*\left(C' - C'_\infty\right) \quad (1)$$

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y'^2} \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} \quad (3)$$

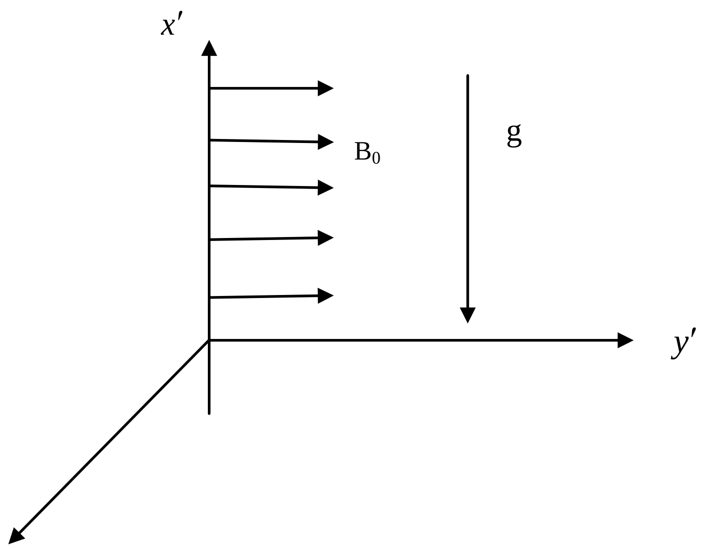

Figure 1 The physical model of the problem and coordinate system.

The initial and boundary conditions are:

$$\begin{aligned} & t' \le 0: \; u' = 0, T = T_\infty, C' = C'_\infty \;\; for\ all\ \ y' \\ & t' > 0: \; u' = u_0 \cos w't', \;\; T = T_w, \\ & \quad C = C'_w + \left(C'_w - C'_\infty\right)At', \;\; at \;\; y' = 0 \\ & \quad u' = 0, \;\; T \to T_\infty, C' \to C'_\infty \;\; as\ y' \to \infty \end{aligned} \quad (4)$$

It is assumed that the temperature differences within the flow are sufficiently small such that they may be expressed as a linear function of the temperature. This is accomplished by expanding a Taylor series about and neglecting higher-order terms, thus:

The dimensional quantities are defined as:

$$u = \frac{u'}{u_0}, \quad y = \frac{u_0 y'}{\nu}, \quad t = \frac{u_0^2 t'}{\nu},$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}$$

$$\Pr = \frac{\mu C_p}{k}, \quad Sc = \frac{\nu}{D}, \quad Gr = \frac{g\beta\nu\left(T_w - T_\infty\right)}{u_0^3},$$

$$Gc = \frac{g\beta^*\nu\left(C'_w - C'_\infty\right)}{u_0^3},$$

$$M = \frac{\sigma B_0^2 \nu}{\rho u_0^3}, w = \frac{w' \nu}{u_0^2}, \; A = \frac{u_0^2}{\nu} \tag{5}$$

In view of Eq. (5) and Eq. (1) - (3) this reduces to the following dimensionless form.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + GcC \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \tag{8}$$

where Gr, Gc, Pr, Sc are the thermal Grashof number, mass Grashof number, Prandtl number, Schmidt number respectively.

The initial and boundary conditions in non-dimensional form are:

$$\left.\begin{array}{ll} t \le 0: & u = 0, \quad \theta = 0, \\ & C = 0, \quad for\ all\ \ y \\ t > 0: & u = \cos wt, \quad \theta = 1, \\ & C = t, \quad at \quad y = 0 \\ & u = 0,\ \theta \to 0,\ C \to 0,\ as\, y \to \infty \end{array}\right\} \tag{9}$$

Solution of the problem

The linear functional for Eq. (6) over a typical line segment element (e), ($y_j \le y \le y_k$) is

$$J^{(e)}(u) = \frac{1}{2}\int_{y_j}^{y_k} \left\{ \begin{array}{l} \left(\dfrac{\partial u^{(e)}}{\partial y}\right)^2 + 2u^{(e)}\dfrac{\partial u^{(e)}}{\partial t} \\ -2u^{(e)}\left(Gr\theta + GcC\right) \end{array} \right\} dy = \text{minimum.}$$

Let $u^{(e)} = N_j u_j + N_k u_k$ be the linear piecewise approximation solution over the element (e), ($y_j \le y \le y_k$), where u_j, u_k are the values of the function u at the ends of the element (e) and $N_j = \dfrac{y_k - y}{y_k - y_j}$, $N_k = \dfrac{y - y_j}{y_k - y_j}$ are the basis functions. One obtains:

$$\frac{1}{l^{(e)}}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix} + \frac{l^{(e)}}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} \dot{u}_j \\ \dot{u}_k \end{bmatrix} - \left(Gr\theta + GcC\right)\frac{l^{(e)}}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0$$

where the dot denotes the differentiation with respect to t and $l^{(e)} = y_k - y_j$. Assembling the element equations for two consecutive elements ($y_{i-1} \le y \le y_i$) and ($y_i \le y \le y_{i+1}$), the following is obtained:

$$\frac{1}{l^{(e)}}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} + \frac{l^{(e)}}{6}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} \dot{u}_{i-1} \\ \dot{u}_i \\ \dot{u}_{i+1} \end{bmatrix} = \left(Gr\theta + GcC\right)\frac{l^{(e)}}{2}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

Now put row corresponding to the node i to zero, the difference schemes with $l^{(e)} = h$ is

$$\left(\dot{u}_{i-1} + 4\dot{u}_i + \dot{u}_{i+1}\right) = \frac{6}{h^2}\left(u_{i-1} + u_{i+1}\right) + \frac{12}{h^2}u_i + 6\left(Gr\theta + GcC\right)$$

Applying the trapezoidal rule, the following system of equations using the Crank-Nicholson method is obtained:

$$\begin{aligned} &(1-3r)u_{i-1}^{j+1} + (4+6r)u_i^{j+1} + (1-3r)u_{i+1}^{j+1} = \\ &(1+3r)u_{i-1}^{j} + (4-6r)u_i^{j} + (1+3r)u_{i+1}^{j} \\ &+ 6k\left(Gr\theta_i^j + GcC_i^j\right) \end{aligned} \tag{10}$$

Now from Eq. (7) and Eq. (8) the following equations are obtained:

$$\left(\Pr-3r\right)\theta_{i-1}^{j+1}+\left(4\Pr+6r\right)\theta_{i}^{j+1}+\left(\Pr-3r\right)\theta_{i+1}^{j+1} = \left(\Pr+3r\right)\theta_{i-1}^{j}+\left(4\Pr-6r\right)\theta_{i}^{j}+\left(\Pr+3r\right)\theta_{i+1}^{j} \quad (11)$$

$$\left(2Sc-6r\right)C_{i-1}^{j+1}+\left(4Sc+6r\right)C_{i}^{j+1}+\left(2Sc-6r\right)C_{i+1}^{j+1}= (2Sc+6r)C_{i-1}^{j}+\left(4Sc-6r\right)C_{i}^{j}+\left(2Sc+6r\right)C_{i+1}^{j} \quad (12)$$

Here $r=\frac{k}{h^2}$ and h,k are mesh sizes along *y*-direction and time *t*-direction respectively. Index *i* refers to the space and *t* refers to the time. In the above Eq. (10) - (12) taking i = 1(1)*n* and using initial and boundary conditions (9), the following tri-diagonal system of equations are obtained:

$$Au = B \quad (13)$$

$$D\theta = E \quad (14)$$

$$FC = G \quad (15)$$

where *A*, *D* and *F* are tri-diagonal matrices of order – *n* and whose elements are given by

$$a_{i,i}=4+6r,\ d_{i,i}=4\Pr+6r,\quad f_{i,i}=4Sc+6r$$
$$,\ i=1(1)\ n$$
$$a_{i-1,j}=a_{i,j-1}=1-3r,\ \ d_{i-1,j}=d_{i.j-1}=\Pr-3r$$
$$f_{i-1,j}=f_{i,j-1}=2Sc-6r,\qquad i=2(1)n$$

Here *u*, θ, *C* and *B*, *E*, *G* are column matrices having the *n*-components $u_i^{j+1}, \theta_i^{j+1}, C_i^{j+1}$ and $u_i^{j}, \theta_i^{j}, C_i^{j}$ respectively. The solutions of the above system of equations are obtained by using the Thomas algorithm for velocity, temperature and concentration. Also, the numerical solutions for these equations are obtained by the *C*-program. In order to prove the convergence and stability of the Ritz finite element method, the computations are carried out for slightly changed values of *h* and *k* by running the same *C*-program. No significant change was observed in the values of *u*, θ, and *C*. Hence, the finite element method is convergent and stable.

Results and discussion

In order to get a physical insight into the problem numerical calculations carried out for the different physical parameters viz., thermal Grashof number, mass Grashof number, Prandtl number, Schmidt number, on the physical flow field, computations are carried out for velocity, temperature and concentration and they are presented in the figures below. In the present study we adopted the following default parameter values of finite element computations: *Gr* = 2.0, *Gc* = 2.0, Pr = 0.71, *Sc* = 0.6, *wt* = π/4, *t* = 0.4. All graphs therefore correspond to these values unless specifically indicated on the appropriate figures.

The influence of the thermal Grashof number *Gr* on the velocity is shown in **Figure 2**. The thermal Grashof number signifies the relative effect of the thermal buoyancy force on the viscous hydrodynamic force. The flow is accelerated due to the enhancement in buoyancy force corresponding to an increase in the thermal Grashof number i.e., free convection effects. The positive values of *Gr* correspond to cooling of the plate by natural convection. Heat is therefore conducted away from the vertical plate into the fluid which increases the temperature and thereby enhances the buoyancy force. In addition, it is seen that the peak values of the velocity increase rapidly near the plate as the thermal Grashof number increases and then decays smoothly to the free stream velocity.

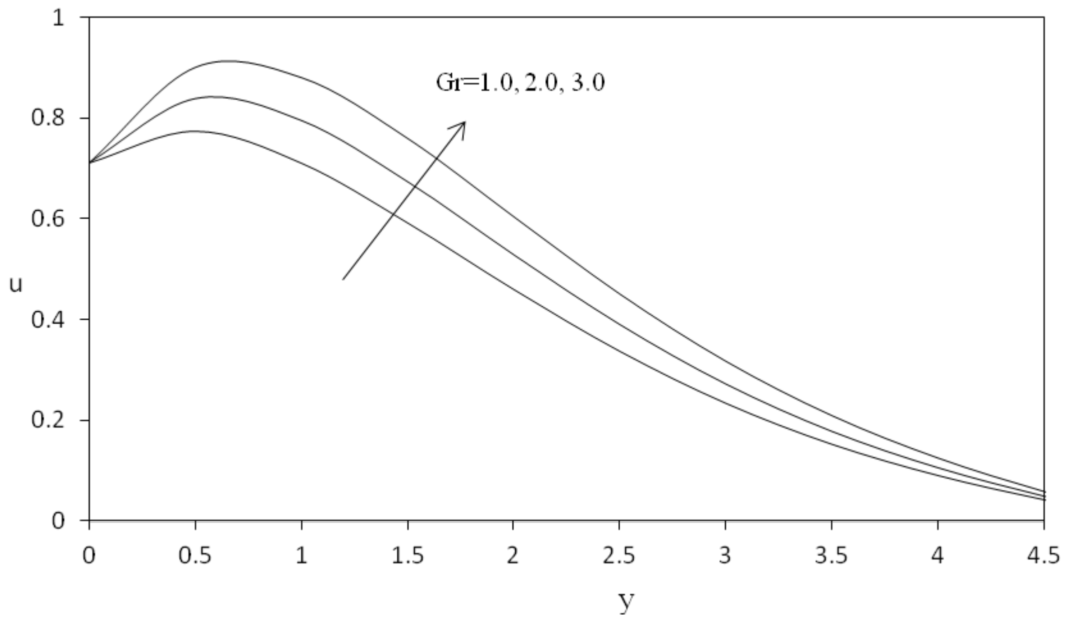

Figure 2 Velocity profile for different values of *Gr*.

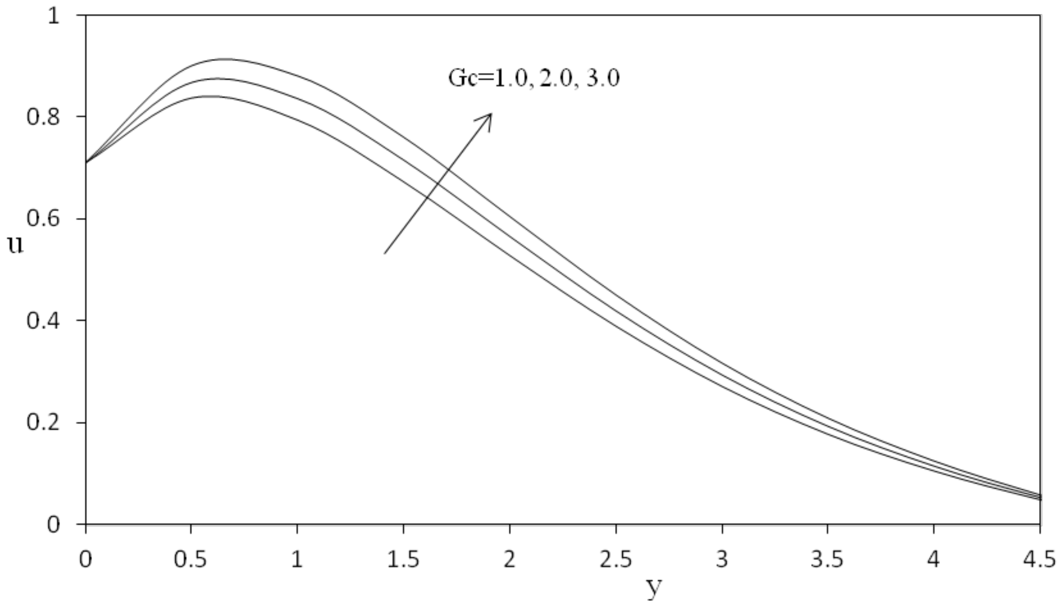

Figure 3 Velocity profile for different values of *Gc*.

Figure 3 presents typical velocity profiles in the boundary layer for various values of the mass Grashof number, *Gc*. The mass Grashof number, *Gc* defines the ratio of the species buoyancy force to the viscous hydrodynamic force. It is noticed that the velocity increases with increasing values of the mass Grashof number.

Figures 4 and **5** illustrate the velocity and temperature profiles for different values of the Prandtl number, Pr. The numerical results show that the effect of increasing the Prandtl number results in a decreasing velocity. It is observed that an increase in the Prandtl number results in a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities, and therefore heat is able to diffuse away from the heated surface more rapidly than for higher values of Pr. Hence in the case of smaller Prandtl numbers the boundary layer is thicker and the rate of heat transfer is reduced.

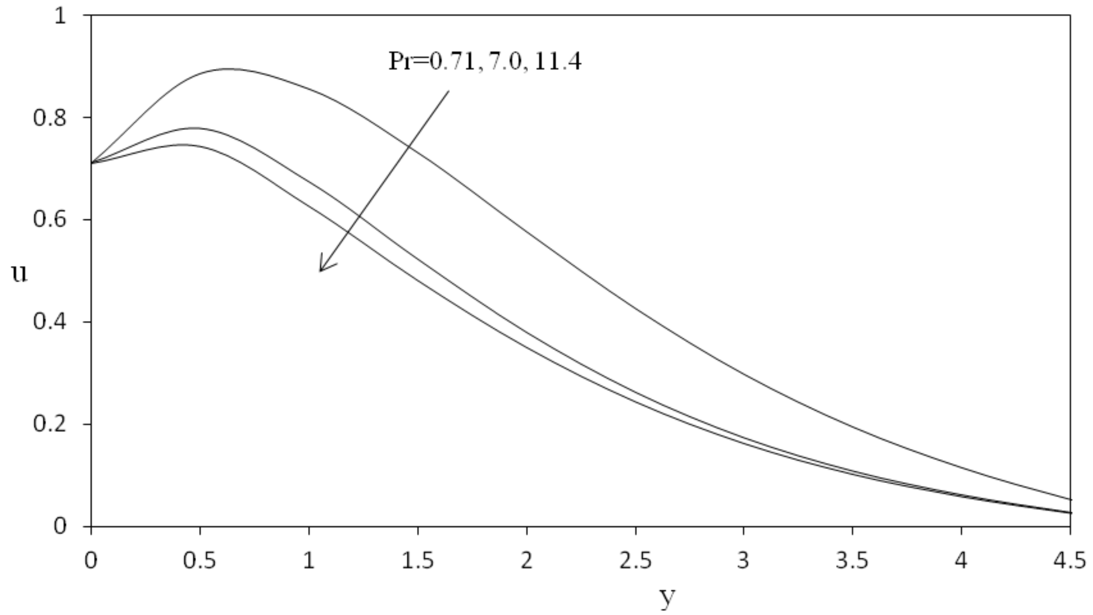

Figure 4 Velocity profile for different values of Pr.

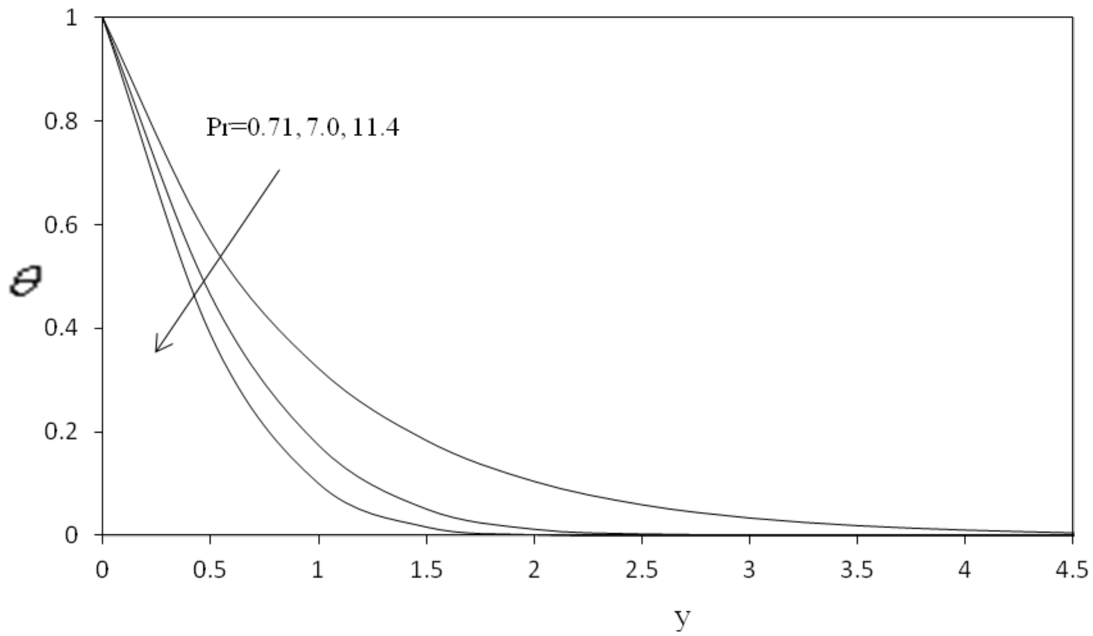

Figure 5 Temperature profile for different values Pr.

For different values of the Schmidt number *Sc*, the velocity and concentration profiles are plotted in **Figures 6** and **7** respectively. The Schmidt number *Sc* embodies the ratio of the momentum diffusivity to the mass (species) diffusivity. It physically relates the relative thickness of the hydrodynamic boundary layer and mass-transfer (concentration) boundary layer. As the Schmidt number increases the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The reductions in the velocity and concentration profiles are accompanied by simultaneous reductions in the velocity and concentration boundary layers, which is evident from **Figures 6** and **7**.

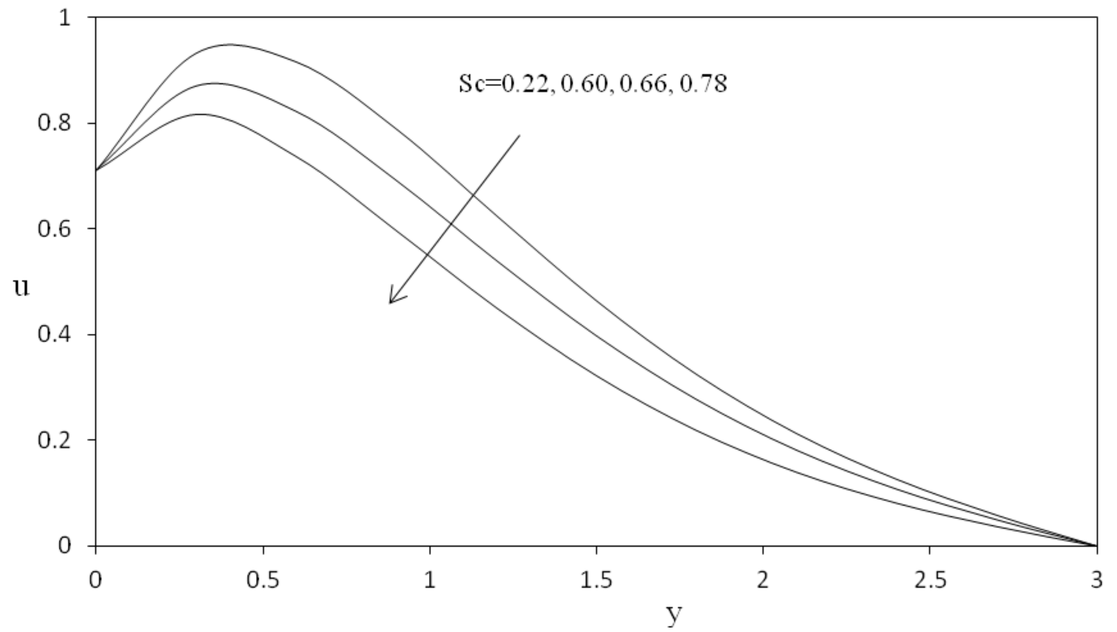

Figure 6 Velocity profile for different values of *Sc*.

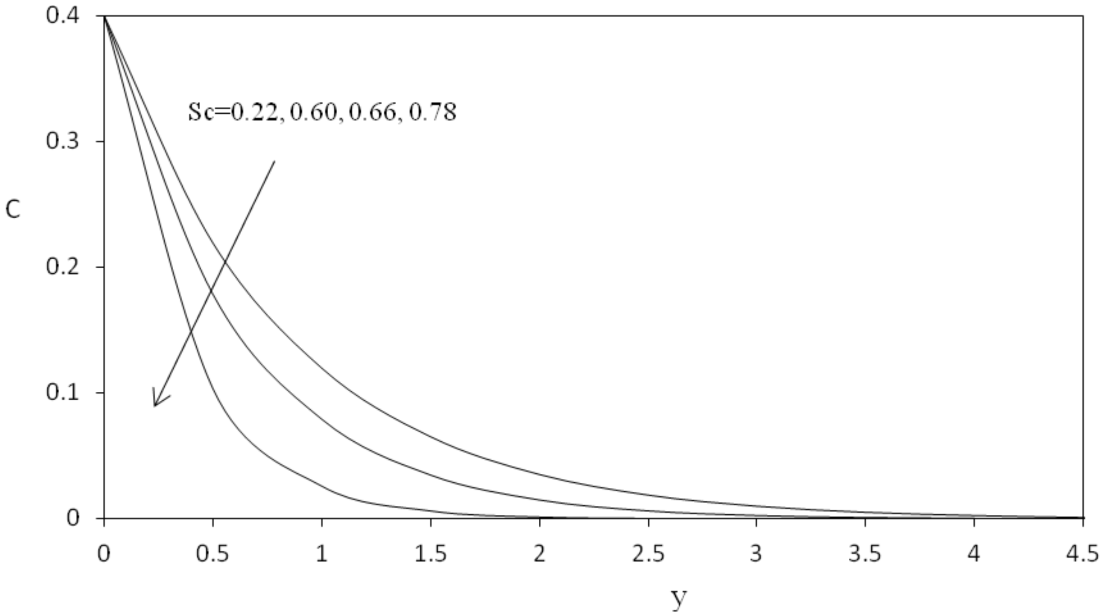

Figure 7 Concentration profile for different values of *Sc*.

Conclusions

An exact analysis is performed to study heat and mass transfer effects on an isothermal vertical oscillating plate. The governing equations are solved using the finite element method. The conclusions of the study are that the velocity increases with an increase in the thermal Grashof number and mass Grashof number and that the velocity and concentration decrease with increasing the Schmidt number. Finally, the velocity and temperature decrease when the Schmidt number increases.

References

[1] R Seigel. Transient free convection from a vertical flat plate. *J. Heat Tran.* 1958; **80**, 347-59.

[2] B Gebhart. Transient natural convection form vertical elements. *J. Heat Tran.* 1961; **83**, 61-70.

[3] VM Soundalgekar. Convection effects on the Stokes problem for infinite vertical plate, ASME. *J. Heat Trans.* 1977; **99**, 499-501.

[4] VM Soundalgekar. Free convection effects on the flow past a vertical oscillating plate. *Astrophys. Space Sci.* 1979; **64**, 165-72.

[5] VM Soundalgekar and SP Akolkar. Effects of free convection currents and mass transfer on the flow past a vertical oscillating plate. *Astrophys. Space Sci.* 1983; **89**, 241-54.