WALAILAK JOURNAL
http://wjst.wu.ac.th Mathematics

On Bifuzzy SU-Subalgebras

Prakasam MURALIKRISHNA[1,*] and Mahadevan CHANDRAMOULEESWARAN[2]

[1]*School of Advanced Sciences, VIT University, Tamilnadu, Vellore 632014, India*
[2]*Department of Mathematics, SBK College, Tamilnadu, Aruppukottai 626101, India*

([*]Corresponding author's e-mail: pmkrishna@rocketmail.com)

Received: 16 July 2012, Revised: 23 September 2012, Accepted: 5 April 2013

Abstract

The study of SU-Algebra was initiated by Supawadee Keawrahun and Utsanee Leerawat. This paper introduces the notion of Bifuzzy SU-subalgebra and deals with some of their basic but interesting properties related to the Cartesian product and Homomorphism by applying the idea of a Bifuzzy subset.

Keywords: SU-algebra, subalgebra, Bifuzzy subset, sup-inf property, Bifuzzy SU-subalgebra

Introduction

The concept of a fuzzy set was introduced by Zadeh [1], and it is now a rigorous area of research with manifold applications ranging from engineering and computer science to medical diagnosis and social behavior studies. In 1986, Atanassov [2] introduced the notion of intuitionistic fuzzy sets as a generalization of fuzzy sets. In 1995, Gerstenkorn and Manko [3] re-named the intuitionistic fuzzy sets as bifuzzy sets. The elements of the bifuzzy sets are featured by an additional degree which is called the degree of uncertainty. Bifuzzy sets have also been defined by Takeuti and Titanti [4]. The bifuzzy sets have drawn the attention of many researchers in the last decades. This is mainly due to the fact that bifuzzy sets are consistent with human behavior, by reflecting and modeling the hesitancy present in real-life situations. These kind of fuzzy sets have gained a wide recognition as a useful tool in the modeling of some uncertain phenomena.

In 1966, Imai and Iseki [5] introduced two classes of abstract algebras; BCK-algebras and BCI-algebra. It is known that the BCK-algebras are a proper sub class of the class of BCI-algebras. Neggers and Kim [6] introduced the notion of B-algebras. With these ideas, Keawrahun and Leerawat [7] introduced new structured algebra: SU-Algebra 2011. Using the fuzzy concept, during 2009 fuzzy *BF*_subalgebras were developed by Saeid and Rezvani [8]. Motivated by this we have introduced Intuitionistic L-fuzzy BF-algebras [9] in 2010. Recently Intuitionistic (T,S)-fuzzy CI-algebras were developed by Saeid and Rezaei [10]. In this paper we investigate the Bifuzzy SU-subalgebra of a SU-algebra and establish some of their basic properties.

Preliminaries

In this section the basic definitions of a SU-algebra, Fuzzy subset and Bifuzzy subset are recalled. We start with;

Definition 2.1 [7] A SU-algebra is a non-empty set X with a consonant 0 and a single binary operation $*$ satisfying the following axioms for any $x, y, z \in X$

a) $((x*y)*(x*z))*(y*z)=0$;
b) $x*0=x$;
c) If $x*y=0 \;\Rightarrow\; x=y$.

Example 2.2 Let X = {0,1,2,3} be a set with the following table.

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Then $(X,*,0)$ is SU-algebra.

Definition 2.3 A binary relation "$\leq$" on X can be defined as $x \leq y$ if and only if $x * y = 0$.

Definition 2.4 A non-empty subset S of a SU-algebra X is said to be a subalgebra if $x * y \in S \quad \forall x, y \in S$.

Definition 2.5 A function $f : X \to Y$ of SU-algebras X and Y is called homomorphism if $f(x * y) = f(x) * f(y) \quad \forall\, x, y \in X$. and $f : X \to Y$ is called anti-homomorphism if $f(x * y) = f(y) * f(x) \quad \forall\, x, y \in X$.

Remark 2.6 If $f : X \to Y$ is a homomorphism on SU-algebras then $f(0_X) = 0_Y$.

Definition 2.7 A fuzzy subset μ in a non-empty set X is a function $\mu : X \to [0,1]$.

Definition 2.8 An Intuitionistic Fuzzy Subset (IFS) A in a non-empty set X is defined as an object of the form $A = \{ < x, \mu_A(x), \nu_A(x) > / \ x \in X \}$ where $\mu_A : X \to [0,1]$ is the degree membership and $\nu_A : X \to [0,1]$ is the degree non-membership of the element $x \in X$ satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Note 2.9 In 1995, Gerstenkorn and Manko [3] re-named the intuitionistic fuzzy sets as bifuzzy sets. Here after we also referred to an Intuitionistic Fuzzy Subset (IFS) as a Bifuzzy set (BFS).

Definition 2.10 Let A and B be two Bifuzzy Subsets of a non-empty set X. Then;

a) $A \cap B = \{ \langle x, \min[\mu_A(x), \mu_B(x)], \max[\nu_A(x), \nu_B(x)] \rangle ; \forall\, x \in X \}$;

b) $\Box A = \{ < x, \mu_A(x), \overline{\mu}_A(x) > \ \forall\, x \in X \}$;

c) $\Diamond A = \{ < x, \overline{\nu}_A(x), \nu_A(x) > \ \forall\, x \in X \}$.

Definition 2.11 A Bifuzzy Subset A in a SU-algebra X with the degree membership $\mu_A : X \to [0,1]$ and the degree non-membership $\nu_A : X \to [0,1]$ is said to have ***Sup-Inf property*** [11], if for any subset $T \subseteq X$ there exists $x_0 \in T$ such that $\mu_A(x_0) = \underset{t \in T}{Sup}\, \mu_A(t)$ and $\nu_A(x_0) = \underset{t \in T}{Inf}\, \nu_A(t)$.

Definition 2.12 Let $f : X \to Y$ be a function and A and B be the Bifuzzy subsets of X and Y where $A = \{ < x, \mu_A(x), \nu_A(x) > / \ x \in X \}$ and $B = \{ < x, \mu_B(x), \nu_B(x) > / \ x \in Y \}$.

Then the image of A under f is defined as $f(A) = \{ < y, \mu_{f(A)}(y), \nu_{f(A)}(y) > / \ y \in Y \}$ such that

$$\mu_{f(A)}(y) = \begin{cases} \sup\limits_{z \in f^{-1}(y)} \mu_A(z) & if \quad f^{-1}(y) = \{x : f(x) = y\} \neq \phi \\ 0 & otherwise \end{cases}$$

and

$$\nu_{f(A)}(y) = \begin{cases} \inf\limits_{z \in f^{-1}(y)} \nu_A(z) & if \quad f^{-1}(y) = \{x : f(x) = y\} \neq \phi \\ 0 & otherwise \end{cases}.$$

Definition 2.13 Let $f : X \to Y$ be a function and A and B be the Bifuzzy subsets of X and Y where $A = \{ < x, \mu_A(x), \nu_A(x) > / \ x \in X \}$ and $B = \{ < x, \mu_B(x), \nu_B(x) > / \ x \in Y \}$.

Then the inverse image of *B* under *f* is defined as $f^{-1}(B) = \{ < x, \mu_{f^{-1}(B)}(x), \nu_{f^{-1}(B)}(x) > / \ x \in X \}$

such that $\mu_{f^{-1}(B)}(x) = \mu_{(B)}(f(x)) \quad and \quad \nu_{f^{-1}(B)}(x) = \nu_{(B)}(f(x)) \quad \forall\, x \in X$.

Bifuzzy SU-subalgebra

Here we introduce the notions of Bifuzzy SU-subalgebra in a SU-algebra X. Here after unless otherwise specified X denotes a SU-algebra.

Definition 3.1 A Fuzzy Subset μ in a SU-algebra X is said to be a Fuzzy SU-subalgebra of X if $\mu(x*y) \geq \min\{\mu(x), \mu(y)\}$ $\forall\ x, y \in X$.

Example 3.2 Consider the SU-algebra $X = \{0, 1, 2, 3\}$ in Example 2.2. and μ is the fuzzy Subset of X defined by $\mu(x) = \begin{cases} 0.1 & ; \quad x = 1, 2 \\ 0.3 & ; \quad x = 0, 3 \end{cases}$. Then μ is fuzzy SU-subalgebra of X.

Definition 3.3 A Bifuzzy Subset A in a SU-algebra X is said to be a Bifuzzy SU-subalgebra of X if for any $x, y \in X$.

a) $\mu_A(x*y) \geq \min\{\mu_A(x), \mu_A(y)\}$;

b) $\nu_A(x*y) \leq \max\{\nu_A(x), \nu_A(y)\}$.

Example 3.4 Consider the SU-algebra $X = \{0, 1, 2, 3\}$ in Example 2.2 and $A = \{ < x, \mu_A(x), \nu_A(x) > / \ x \in X\}$ is the Bifuzzy Subset of X defined by $\mu_A(x) = \begin{cases} 0.2 & ; \quad x = 1, 2 \\ 0.4 & ; \quad x = 0, 3 \end{cases}$ *and* $\nu_A(x) = \begin{cases} 0.5 & ; \quad x = 1, 2 \\ 0.3 & ; \quad x = 0, 3 \end{cases}$.

Then $A = \{ < x, \mu_A(x), \nu_A(x) > / \ x \in X\}$ is Bifuzzy SU-subalgebra of X.

Lemma 3.5 In a Bifuzzy SU-subalgebra A of X we have for any $x \in X$;

a) $\mu_A(0) \geq \mu_A(x)$;
b) $\nu_A(0) \leq \nu_A(x)$.

Proof. For $\mu_A(0) = \mu_A(x*x) \geq \min\{\mu_A(x), \mu_A(x)\} = \mu_A(x)$
and $\nu_A(0) = \nu_A(x*x) \leq \max\{\nu_A(x), \nu_A(x)\} = \nu_A(x)$.

Theorem 3.6 Intersection of any two Bifuzzy SU-subalgebras of X is also a Bifuzzy SU-subalgebra of X.

Proof. Let A and B be any two Bifuzzy SU-subalgebras of X.

Let $A = \{ < x, \mu_A(x), \nu_A(x) > / \ x \in X\}$ and $B = \{ < x, \mu_B(x), \nu_B(x) > / \ x \in X\}$.

Take $C = A \cap B = \{ < x, \mu_C(x), \nu_C(x) > / \ x \in X\}$.

where $\mu_C(x) \geq \min\{\mu_A(x), \mu_B(x)\}$ and $\nu_C(x) \leq \max\{\nu_A(x), \nu_B(x)\}$.

Let $x, y \in X$.

We have to prove;

$\mu_C(x*y) \geq \min\{\mu_C(x), \mu_C(y)\}$ and $\nu_C(x*y) \leq \max\{\nu_C(x), \nu_C(y)\}$ $\forall\ x, y \in X$.

For $\mu_C(x*y) = \min\{\mu_A(x*y), \mu_B(x*y)\}$

$$\geq \min\{\min(\mu_A(x), \mu_A(y)), \min(\mu_B(x), \mu_B(y))\}$$

$$= \min\{\min(\mu_A(x), \mu_B(x)), \min(\mu_A(y), \mu_B(y))\}$$
$$= \min\{\mu_C(x), \mu_C(y)\} \qquad \forall\ x, y \in X.$$

Similarly we can prove $\nu_C(x*y) \le \max\{\nu_C(x), \nu_C(y)\}\ \ \forall\ x, y \in X$.

This proves that the intersection of any two Bifuzzy SU-subalgebras of X is also a Bifuzzy SU-subalgebra of X.

The above theorem can be generalized as follows.

Theorem 3.7 The intersection of a family of Bifuzzy SU-subalgebras of X is also a Bifuzzy SU-subalgebra of X.

Theorem 3.8 A BFS $A = \{ < x, \mu_A(x), \nu_A(x) > \ /\ x \in X\}$ is a Bifuzzy SU-subalgebra of X if and only if the fuzzy subsets μ_A and $\overline{\nu}_A$ are fuzzy SU-subalgebras of X.

Proof. Let $A = \{ < x, \mu_A(x), \nu_A(x) > \ /\ x \in X\}$ be a Bifuzzy SU-subalgebra of X.

Then clearly μ_A is a fuzzy SU-subalgebra of X.

Now $\forall\ x, y \in X$, $\overline{\nu}_A(x*y) = 1 - \nu_A(x*y) \ge 1 - \max[\nu_A(x), \nu_A(y)]$
$$= \min\{(1-\nu_A(x)), (1-\nu_A(y))\} = \min\{\overline{\nu}_A(x), \overline{\nu}_A(y)\}.$$

$\therefore\ \overline{\nu}_A$ is a fuzzy SU-subalgebra of X.

Conversely, assume μ_A and $\overline{\nu}_A$ are fuzzy SU-subalgebras of X.

So we have $\mu_A(x*y) \ge \min\{\mu_A(x), \mu_A(y)\}$ and $\overline{\nu}_A(x*y) \ge \min\{\overline{\nu}_A(x), \overline{\nu}_A(y)\}\ \ \forall\ x, y \in X$.

Hence to prove $A = \{ < x, \mu_A(x), \nu_A(x) > \ /\ x \in X\}$ is a Bifuzzy SU-subalgebra of X it is enough to prove $\nu_A(x*y) \le \max\{\nu_A(x), \nu_A(y)\}\ \forall\ x, y \in X$.

For $1 - \nu_A(x*y) = \overline{\nu}_A(x*y) \ge \min\{\overline{\nu}_A(x), \overline{\nu}_A(y)\} = \min\{(1-\nu_A(x)), (1-\nu_A(y))\}$
$$= 1 - \max[\nu_A(x), \nu_A(y)].$$

i.e, $\nu_A(x*y) \le \max\{\nu_A(x), \nu_A(y)\}\ \ \forall\ x, y \in X$.

This completes the proof.

Using this theorem and by definition 2.10 we have the following.

Theorem 3.9 A BFS $A = \{ < x, \mu_A(x), \nu_A(x) > \ /\ x \in X\}$ is a Bifuzzy SU-subalgebra of X if and only if $\Box A$ and $\Diamond A$ are also Bifuzzy SU-subalgebras of X.

Theorem 3.10 Let f be a homomorphism from SU-algebras X onto Y and A be an Bifuzzy SU-subalgebra of X with Sup-Inf property. Then the image of A, $f(A) = \{ < y, \mu_{f(A)}(y), \nu_{f(A)}(y) > \ /\ y \in Y\}$ is a Bifuzzy SU-subalgebra of Y.

Proof. Let $a, b \in Y$ with $x_0 \in f^{-1}(a)$ and $y_0 \in f^{-1}(b)$ such that;

$\mu_A(x_0) = \sup\limits_{t \in f^{-1}(a)} \mu_A(t)$; $\mu_A(y_0) = \sup\limits_{t \in f^{-1}(b)} \mu_A(t)$ and

$\nu_A(x_0) = \inf_{t\in f^{-1}(a)} \nu_A(t) \quad ; \cdot \nu_A(y_0) = \inf_{t\in f^{-1}(b)} \nu_A(t)$.

Now by definitions 2.11 and 2.12, $\mu_{f(A)}(a*b) = \sup_{t\in f^{-1}(a*b)} \mu_A(t) \geq \mu_A(x_0 * y_0)$

$$\geq \min\{\mu_A(x_0), \mu_A(y_0)\}$$

$$= \min\left\{ \sup_{t\in f^{-1}(a)} \mu_A(t) \ , \ \sup_{t\in f^{-1}(b)} \mu_A(t) \right\}$$

$$= \min\{\mu_{f(A)}(a) \ , \ \mu_{f(A)}(b)\}.$$

Also, $\nu_{f(A)}(a*b) = \inf_{t\in f^{-1}(a*b)} \nu_A(t) \leq \nu_A(x_0 * y_0)$

$$\leq \max\{\nu_A(x_0), \nu_A(y_0)\}$$

$$= \max\left\{ \inf_{t\in f^{-1}(a)} \nu_A(t) \ , \ \inf_{t\in f^{-1}(b)} \nu_A(t) \right\}$$

$$= \max\{\nu_{f(A)}(a) \ , \ \nu_{f(A)}(b)\}.$$

Hence the image $f(A) = \left\{ < y, \mu_{f(A)}(y), \nu_{f(A)}(y) > \ / \ y \in Y \right\}$ is a Bifuzzy SU-subalgebra of Y.

Theorem 3.11 Let f be a homomorphism from SU-algebras X onto Y and B be a Bifuzzy SU-subalgebra of Y. Then the inverse image of B, $f^{-1}(B) = \left\{ < x, \mu_{f^{-1}(B)}(x), \nu_{f^{-1}(B)}(x) > \ / \ x \in X \right\}$ is a Bifuzzy SU-subalgebra of X.

Proof. Let $x, y \in X$.

Then $\mu_{f^{-1}(B)}(x*y) = \mu_B(f(x*y)) = \mu_B(f(x)*f(y)) \geq \min\{\mu_B(f(x)), \mu_B(f(y))\}$

$$= \min\{\mu_{f^{-1}(B)}(x) \ , \ \mu_{f^{-1}(B)}(y)\}.$$

Also $\nu_{f^{-1}(B)}(x*y) = \nu_B(f(x*y)) = \nu_B(f(x)*f(y)) \leq \max\{\nu_B(f(x)), \nu_B(f(y))\}$

$$= \max\{\nu_{f^{-1}(B)}(x), \nu_{f^{-1}(B)}(y)\}.$$

Then the inverse image of $f^{-1}(B) = \left\{ < x, \mu_{f^{-1}(B)}(x), \nu_{f^{-1}(B)}(x) > \ / \ x \in X \right\}$ is a Bifuzzy SU-subalgebra of X.

Remark 3.12 One can verify the theorem 3.10 and 3.11 for an anti-homomorphism on SU-algebras.

Product on Bifuzzy SU-subalgebras

In this section we introduce the notions of Cartesian product of Bifuzzy SU-subalgebras in a SU-algebra and discuss some properties.

Definition 4.1 Let A and B be any two BFS of X. The Cartesian product of A and B is defined as $A\times B = (X\times Y \ , \ \mu_A \times \mu_B \ , \nu_A \times \nu_B)$ with $(\mu_A \times \mu_B)(x, y) = \min\{\mu_A(x) \ , \ \mu_B(y)\}$ and $(\nu_A \times \nu_B)(x, y) = \max\{\nu_A(x) \ , \nu_B(y)\}$ where $\mu_A \times \mu_B : X \times Y \to [0,1]$ and $\nu_A \times \nu_B : X \times Y \to [0,1]$ $\forall \ x \ , \in X \ and \ y \in Y$.

Theorem 4.2 Let A and B be any two Bifuzzy SU-subalgebras of X and Y respectively. Then $A \times B$ is a Bifuzzy SU-subalgebra of $X \times Y$.

Proof. Take (x_1 , y_1) *and* $(x_2 , y_2) \in X \times Y$.

Then $(\mu_A \times \mu_B)[(x_1, y_1) * (x_2, y_2)]$

$$= (\mu_A \times \mu_B)[(x_1 * x_2), (y_1 * y_2)]$$
$$= \min\{\mu_A(x_1 * x_2), \mu_B(y_1 * y_2)\}$$
$$\geq \min[\min(\mu_A(x_1), \mu_A(x_2)), \min(\mu_B(y_1), \mu_B(y_2))]$$
$$\geq \min\{\min(\mu_A(x_1), \mu_B(y_1)), \min(\mu_A(x_2), \mu_B(y_2))\}$$
$$= \min\{(\mu_A \times \mu_B)(x_1, y_1), (\mu_A \times \mu_B)(x_2, y_2)\}.$$

And $(\nu_A \times \nu_B)[(x_1, y_1) * (x_2, y_2)]$

$$= (\nu_A \times \nu_B)[(x_1 * x_2), (y_1 * y_2)]$$
$$= \max\{\nu_A(x_1 * x_2), \nu_B(y_1 * y_2)\}$$
$$\leq \max[\max(\nu_A(x_1), \nu_A(x_2)), \max(\nu_B(y_1), \nu_B(y_2))]$$
$$\leq \max\{\max(\nu_A(x_1), \nu_B(y_1)), \max(\nu_A(x_2), \nu_B(y_2))\}$$
$$= \max\{(\nu_A \times \nu_B)(x_1, y_1), (\nu_A \times \nu_B)(x_2, y_2)\}.$$

This completes the proof.

Theorem 4.3 If $A \times B$ is a Bifuzzy SU-subalgebra of $X \times Y$, then either A is a Bifuzzy subalgebra of X or B is a Bifuzzy subalgebra of Y.

Proof.

Suppose we assume that $A \times B$ is a Bifuzzy SU-subalgebra of $X \times Y$. Then

$$(\mu_A \times \mu_B)[(x_1, y_1) * (x_2, y_2)] \geq \min\{(\mu_A \times \mu_B)(x_1, y_1), (\mu_A \times \mu_B)(x_2, y_2)\} \quad (1)$$

Putting $x_1 = x_2 = 0.$ in (1) we get,

$(\mu_A \times \mu_B)[(0, y_1) * (0, y_2)] \geq \min\{(\mu_A \times \mu_B)(0, y_1), (\mu_A \times \mu_B)(0, y_2)\}$ and

$$(\mu_A \times \mu_B)[(0, y_1 * y_2)] \geq \min\{(\mu_A \times \mu_B)(0, y_1), (\mu_A \times \mu_B)(0, y_2)\} \quad (2)$$

Thus we have, $\mu_B(y_1 * y_2) \geq \min\{\mu_B(y_1), \mu_B(y_2)\}$.

In a similar way we can prove $\nu_B(y_1 * y_2) \leq \max\{\nu_B(y_1), \nu_B(y_2)\}$.

This proves B is a Bifuzzy SU-subalgebra of Y.

This completes the proof.

Theorem 4.4 For any Bifuzzy SU-subalgebra A and B of X and Y respectively, $A \times B$ is a Bifuzzy SU-subalgebra of $X \times Y$ if and only if $(\mu_A \times \mu_B)(x, y)$ and $(\overline{\nu_A \times \nu_B})(x, y)$ are fuzzy SU-subalgebras of $X \times Y$.

Proof. Let $A \times B$ be a Bifuzzy SU-subalgebra of $X \times Y$.

Clearly $(\mu_A \times \mu_B)(x, y) = \min\{\mu_A(x), \mu_B(y)\}$ is a fuzzy SU-subalgebra of $X \times Y$.

We have $(\nu_A \times \nu_B)(x, y) = \max\{\nu_A(x), \nu_B(y)\}$

$$\Rightarrow 1 - (\overline{\nu_A} \times \overline{\nu_B})(x, y) = \max\{(1 - \overline{\nu_A(x)}), (1 - \overline{\nu_B(y)})\}$$

$$\Rightarrow\ 1-\max\left\{\left(1-\overline{\nu_A(x)}\ \right),\left(1-\overline{\nu_B(y)}\right)\right\}=\left(\overline{\nu_A}\times\overline{\nu_B}\right)(x,y)$$

$$\Rightarrow\ \left(\overline{\nu_A}\times\overline{\nu_B}\right)(x,y)=\min\left\{\overline{\nu_A(x)}\ ,\overline{\nu_B(y)}\right\}.$$

Thus $\left(\overline{\nu_A}\times\overline{\nu_B}\right)(x,y)=\min\left\{\overline{\nu_A(x)}\ ,\overline{\nu_B(y)}\right\}$ is a fuzzy BF- subalgebra of $X\times Y$.

Conversely, $(\mu_A\times\mu_B)(x,y)$ and $\left(\overline{\nu_A\times\nu_B}\right)(x,y)$ are fuzzy SU- subalgebras of $X\times Y$.

Now $A\times B=\left(X\times X\ ,\ \mu_A\times\mu_B\ ,\nu_A\times\nu_B\right)$.

Since $\left(\overline{\nu_A}\times\overline{\nu_B}\right)(x,y)=\min\ \left\{\overline{\nu_A(x)}\ ,\overline{\nu_B(y)}\right\}\Rightarrow(\nu_A\times\nu_B)(x,y)=\max\{\nu_A(x)\ ,\nu_B(y)\}$,

we can easily observe that $A\times B$ is a Bifuzzy SU-subalgebra of $X\times Y$.

Theorem 4.5 For any BFS A and B of X and Y respectively, A and B are Bifuzzy SU-subalgebras of X and Y respectively if and only if

(a) $\square\ (A\times B)=\left(X\times X\ ,\ \mu_A\times\mu_B\ ,\ \overline{\mu_A}\times\overline{\mu_B}\right)$ and

(b) $\Diamond\ (A\times B)=\left(X\times X\ ,\overline{\nu_A}\times\overline{\nu_B}\ ,\nu_A\times\nu_B\right)$ are Bifuzzy SU-subalgebras of $X\times Y$.

Proof.

Since $(\mu_A\times\mu_B)(x,y)=\min\{\mu_A(x)\ ,\mu_B(y)\}\Rightarrow\left(\overline{\mu_A}\times\overline{\mu_B}\right)(x,y)=\max\left\{\overline{\mu_A(x)}\ ,\overline{\mu_B(y)}\right\}$ and $(\nu_A\times\nu_B)(x,y)=\max\{\nu_A(x)\ ,\nu_B(y)\}\Rightarrow\left(\overline{\nu_A}\times\overline{\nu_B}\right)(x,y)=\min\left\{\overline{\nu_A(x)}\ ,\overline{\nu_B(y)}\right\}$, the proof is clear.

Conclusions

In this article, we have directly extended the notions of Bifuzzy SU-subalgebras of SU-algebras and verified some of their basic properties and the characteristics of their homomorpic(anti-homomorphic) image(pre image). We have also introduced the idea of product on Bifuzzy SU-subalgebras. This paper itself gives the notions of fuzzy SU-subalgebras, which can be obtained by taking the membership alone from the Bifuzzy extensions. The surprising point is that Supawadee Keawrahun and Utsanee Leerawat [7] says that the structure SU-algebra becomes a TM-algebra, QS-algebra and a BF-algebra. Also Andrzej Walendziak [12] proves that a BF-algebra is BG-algebra. Hence we conclude that whatever result we have proved for SU-algebras can directly be carried over to TM-algebras, QS-algebras, BF-algebras and BG-algebras. These concepts can further be generalized and we feel that there is further scope of study on SU-algebras that may find some applications in the real field.

References

[1] LA Zadeh. Fuzzy sets. *Inform. Control*. 1965; **8**, 338-53.

[2] KT Atanassov. Intuitionistic fuzzy sets. *Fuzzy Set. Syst.* 1986; **20**, 87-96.

[3] T Gerstenkorn and J Manko. Bifuzzy probabilistic sets. *Fuzzy Set. Syst.* 1995; **71**, 207-14.

[4] G Takeuti and S Titants. Intuitionistic fuzzy logic and intuitionistic fuzzy set theory. *J. Symbolic Logic* 1984; **49**, 851-66.

[5] Y Imai and K Iseki. On axiom systems of propositional calculi 15. *Proc. Jap. Acad.* 1966; **42**, 19-22.

[6] J Neggers and HS Kim. On B_algebras. *Math. Vensik.* 2002; **54**, 21-9.

[7] S Keawrahun and U Leerawat. On a classification of a structure algebra: SU-Algebra. *Scientia Magna* 2011; **7**, 69-76.

[8] AS Borumand and MA Rezvani. On fuzzy BF-algebras. *Int. Math. Forum* 2009, **4**, 13-25.

[9] M Chandramouleeswaran and P Muralikrishna. The intuitionistic L-Fuzzy

*BF*_subalgebras. *Global J. Pure Appl. Math.* 2010; **6**, 1-6.

[10] AS Borumand and A Rezaei. Intuitionistic (T,S)-fuzzy CI-algebras. *Comput. Math. Appl.* 2012; **63**, 158-66.

[11] P Muralikrishna and M Chandramouleeswaran. Homomorphism on intuitionistic L-fuzzy BF/BG-subalgebras. *Adv. Fuzzy Math.* 2010; **5**, 311-6.

[12] A Walendziak. On BF-algebras. *Mathematica Slovaca* 2007; **57**, 119-28.