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Article

Some New Results in Linear Programming Problems with Fuzzy Cost Coefficients

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Abstract

The fuzzy primal simplex method proposed by Mahdavi-Amiri *et al.* and the fuzzy dual simplex method proposed by SH Nasseri and A Ebrahimnejad are two current procedures for solving linear programming problems with fuzzy cost coefficients known as reduced fuzzy numbers linear programming (RFNLP) problems. In this paper, we prove that in the absence of degeneracy these fuzzy methods stop in a finite numbers of iterations. We also prove the fundamental theorem of linear programming in a crisp environment to a fuzzy one. Finally, we illustrate our proof by use of a numerical example.

Keywords: Fuzzy numbers linear programming, fuzzy primal simplex algorithm, fuzzy dual simplex algorithm, trapezoidal fuzzy number

Introduction

The fuzzy set theory is being applied extensively to many fields these days. One of these is linear programming problems. Therefore, a number of researchers have shown an interest in the area of fuzzy linear programming (FLP) problems [1-11]. Since the fuzziness may appear in a linear programming problem in many ways, the definition of fuzzy linear programming problem is not unique. In this paper we try to study a special kind of fuzzy linear programming problem in which only the cost coefficients are represented by trapezoidal fuzzy numbers. We name this special kind of FLP problems as reduced fuzzy numbers linear programming (RFNLP) problem. Mahdavi-Amiri *et al.* [12] proposed a fuzzy primal dual simplex method for solving RFNLP problems using the concept of comparison of fuzzy numbers based on linear ranking functions. It is important to note that the primal simplex method for solving the RFNLP problem is efficient when an initial basis, which is feasible, is at hand. In certain instances, it is difficult to find such basis. In these same instances, it is often possible to find an initial basis (not necessarily feasible) that is dual feasibility. Thus, Nassei and Ebrahimnejad [13] proposed a fuzzy dual simplex method to overcome this shortcoming. Ebrahimnejad [14] proposed a new method, called the fuzzy primal-dual algorithm, similar to the fuzzy dual simplex method, which begins with dual feasibility and proceeds to obtain primal feasibility while maintaining complementary slackness. An important difference between the dual simplex method and the primal-dual simplex method is that it does not require a dual feasible solution to be basic. In this paper we show that in the absence of primal and dual degeneracy, these fuzzy methods stop in a finite number of iterations. In addition, the key of these fuzzy algorithms is that the optimal solution is obtained at a basic solution. Thus we prove that if the RFNLP problem has an optimal solution, then it also has a basic optimal solution.

This paper is organized as follows: in Section 2, we give some necessary concepts and the background of fuzzy arithmetic. In Section 3, we review the fuzzy simplex methods for solving

RFNLP problems. The main results are given in Section 4. Conclusions are discussed in Section 5.

Preliminaries

In this section we review some of the basic terminologies of fuzzy set theory and the main concepts needed in the rest of the paper.

Definition 2.1 The characteristic function χ_A of a crisp set A assigns a value of either one or zero to each individual in the universal set X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the values assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{A}}: X \to [0,1]$. The assigned value indicates the membership grade of the element in the set $\mu_{\tilde{A}}$. Larger values denote the higher degrees of set membership.

The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \left\{ \left(x, \mu_{\tilde{A}}(x)\right) \mid x \in X \right\}$ defined by $\mu_{\tilde{A}}$ for each $x \in X$ is called a fuzzy set.

Definition 2.2 A fuzzy set $\tilde{A}$, defined on universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:

i) $\tilde{A}$ is convex, i.e. $\forall\, x, y \in R, \forall\, \lambda \in [0,1], \mu_{\tilde{A}}\left(\lambda x + (1-\lambda) y\right) \geq \min\left\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\right\},$

ii) $\tilde{A}$ is normal, i.e., $\exists\, \bar{x} \in R;\ \mu_{\tilde{A}}(\bar{x}) = 1,$

iii) $\mu_{\tilde{A}}$ is piecewise continues.

Definition 2.3 A fuzzy number $\tilde{A} = (m, n, \alpha, \beta)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-(m-\alpha)}{\alpha} & m-\alpha \leq x \leq m \\ 1 & m \leq x \leq n \\ \dfrac{(n+\beta)-x}{\beta} & n \leq x \leq n+\beta \\ 0 & else \end{cases}$$

Now we define the arithmetic operations of trapezoidal fuzzy numbers.

Definition 2.4 Let $\tilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)$ and $\tilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)$ be two trapezoidal fuzzy numbers. Then the arithmetic operations on $\tilde{A}_1$ and $\tilde{A}_2$ are given by:

i) $\tilde{A}_1 + \tilde{A}_2 = (m_1 + m_2, n_1 + n_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2)$

ii) $\tilde{A}_1 - \tilde{A}_2 = (m_1 - n_2, n_1 - m_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2)$

iii) $k > 0, k \in R,\ k\tilde{A}_1 = (k\, m_1, k\, n_1, k\alpha_1, k\beta_1)$

iv) $k < 0, k \in R,\ k\tilde{A}_1 = (k\, n_1, k\, m_1, -k\beta_1, -k\alpha_1)$

Ranking procedures are useful in various applications and one of them is in the study of fuzzy mathematical programming in later sections. There are numerous methods proposed in the literature for the ranking of fuzzy numbers. Here, we describe only a simple method for the ordering of fuzzy numbers.

An efficient approach to order fuzzy numbers is based on the concept of comparison of fuzzy numbers by the use of ranking functions, in which a ranking function $\Re : F(R) \to R$ that maps each fuzzy number into the real line is defined for ranking the elements of $F(R)$. Thus, using the natural order of the real numbers we can compare fuzzy numbers easily as follows:

$$\tilde{A}_1 \tilde{\geq} \tilde{A}_2 \iff \Re(\tilde{A}_1) \geq \Re(\tilde{A}_2)$$
$$\tilde{A}_1 \tilde{\leq} \tilde{A}_2 \iff \Re(\tilde{A}_1) \leq \Re(\tilde{A}_2)$$
$$\tilde{A}_1 \cong \tilde{A}_2 \iff \Re(\tilde{A}_1) = \Re(\tilde{A}_2)$$

Several ranking functions have been proposed by researchers to suit their requirements of the problems under consideration. We restrict our attention to linear ranking functions, that is, a ranking function $\Re$ such that $\Re(k\tilde{A}_1 + \tilde{A}_2) = k\Re(\tilde{A}_1) + \Re(\tilde{A}_2)$ for any $\tilde{A}_1$ and $\tilde{A}_2$ belonging to $F(R)$ and any $k \in R$. For a trapezoidal fuzzy number $\tilde{A} = (m, n, \alpha, \beta)$, one of the most linear ranking functions introduced by Yager [15] is as follows:

$$Y_2(\tilde{A}) = \frac{1}{2}\left[m + n + \frac{\beta - \alpha}{2}\right] \tag{1}$$

Fuzzy numbers linear programming problems

The aim of linear programming problems in the crisp environment is to maximize or minimize a linear objective function under linear constraints. But in many practical situations, the decision maker may not be in a position to specify the parameters precisely but rather can specify them in a fuzzy sense. In such situations, it is desirable to use some fuzzy linear programming type of modeling so as to provide more flexibility to the decision maker. Therefore, a number of researchers have shown interest in the area of fuzzy linear programming (FLP) problems. A special kind of FLP problem in which all decision parameters except of decision variables are represented by trapezoidal fuzzy numbers is called fuzzy numbers linear programming (FNLP) problem. In this paper, we focus on a type of FNLP problems in which only the cost coefficients are represented by trapezoidal fuzzy numbers. We name this type of FNLP problem as a reduced fuzzy numbers linear programming (RFNLP) problem.

An RFNLP is defined as follows:

$$\begin{aligned} \min\ & \tilde{z} \approx \tilde{c}\,x \\ s.t.\ & Ax \geq b \\ & x \geq 0 \end{aligned} \tag{2}$$

In problem (2) $b \in R^m$, $\tilde{c}^T \in F(R)^n$, $A \in R^{m \times n}$ are given and $x \in R^n$ is to be determined.

Definition 3.1 Any vector $x \in R^n$ which satisfies the constraints and nonnegative restrictions of (2) is said to be a feasible solution.

Definition 3.2 Let S be the set of all feasible solutions of (2). Any fuzzy vector $x_* \in S$ is said to be a fuzzy optimum solution to (2) if $\tilde{c}x_* \tilde{\leq} \tilde{c}x$ for all $x \in S$, where $\tilde{c} = (\tilde{c}_1, \tilde{c}_2, \ldots, \tilde{c}_n)$ and $\tilde{c}x = \tilde{c}_1 x_1 + \tilde{c}_2 x_2 + \cdots + \tilde{c}_n x_n$.

Definition 3.3 (Basic solution) Suppose $\bar{x} = (\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n)$ solves $Ax = b$. If all $\bar{x}_j = 0$, then $\bar{x}$ is said to be a basic solution. Otherwise, $\bar{x}$ has some non-zero components, say $\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_k, 1 \leq k \leq m$. Then $Ax = b$ can be written as:

$$a_1 \bar{x}_1 + a_2 \bar{x}_2, \ldots + a_k \bar{x}_k + a_{k+1} x_{k+1} + \cdots + a_n x_n = b$$

If the columns $a_1, a_2, \ldots, a_k$ corresponding to non-zero components $\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_k$ are linear independent, then $\bar{x}$ is said to be a fuzzy basic solution.

Remark 3.1 Consider the system of constraints (2) where A is a matrix of the order $(m \times n)$ and $rank(A) = m$. Any $(m \times m)$ matrix B formed by m linearly independent columns of A

is known as a basis for this system. The column vectors of A and the variables in the problem, can be partitioned into the basic and the nonbasic part with respect to this basis B. Each column vector of A, which is in the basis B, is known as a basic column vector. All the remaining column vectors of A are called the nonbasic column vectors.

Remark 3.2 Let x_B be the vector of the variables associated with the basic column vectors. The variables in x_B are known as the basic variables with respect to basis B, and x_B is the basic vector. Also, let x_N and N be the vector and the matrix of the remaining variables and columns, which are called the nonbasic variables and nonbasic matrix, respectively. In this case, $x=(x_B, x_N)=(B^{-1}b, 0)$ is a basic solution too.

Definition 3.4 Suppose $\bar{x}$ is a basic feasible solution of fuzzy system $Ax=b, x\geq 0$. If the number of positive variables $\bar{x}$ is exactly *m*, then it is called a non-degenerate basic feasible solution, i.e. $\bar{x}=(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_m)\succ(0,0,\cdots,0)$. If the number of positive $\bar{x}$ is less than of m, then $\bar{x}$ is called a degenerate basic feasible solution.

Suppose $\bar{x}$ is a basic feasible solution of (2). Let y_k and w be the solutions to $By_k=a_k$ and $wB=\tilde{c}_B$, respectively and define $z_j=\tilde{w}a_j$. Mahdavi-Amiri *et al.* [10] proved some important theorems of RFNLP problems concerning improving a fuzzy feasible solution, unbounded criteria and the optimality conditions and then proposed a new algorithm for solving RFNLP problems. Here, we give a summary of their method in tableau format.

Algorithm 3.1 A fuzzy primal simplex for FVLP problem

Initialization step

Suppose a basic feasible solution with basic B is at hand. Form the initial tableau as **Table 1**.

Table 1 The initial FVLP simplex tableau.

Basic	x_B	x_N	***R.H.S***
$\Re(\tilde{z})$	0	$\Re(\tilde{z}_N-\tilde{c}_N)=\Re(\tilde{c}_BY_N-\tilde{c}_N)$	$\Re(\tilde{\bar{z}})=\Re(\tilde{c}_BB^{-1}b)$
$\tilde{z}$	$\tilde{0}$	$\tilde{z}_N-\tilde{c}_N=\tilde{c}_BY_N-\tilde{c}_N$	$\tilde{\bar{z}}=\tilde{c}_BB^{-1}b$
x_B	I	Y_N	$\bar{b}=B^{-1}b$

Main step

1) Calculate $\Re(\tilde{z}_j-\tilde{c}_j)$ for all nonbasic variables. Let $\Re(\tilde{z}_k-\tilde{c}_k)=\max\{\Re(\tilde{z}_j-\tilde{c}_j), j\in T\}$ in which T is the index set of the current nonbasic variables.

If $\Re(\tilde{z}_k-\tilde{c}_k)\leq 0$, then stop; the current solution is fuzzy optimal. Otherwise go to step (2).

2) Let $y_k=B^{-1}a_k$. If $y_k\leq 0$, then stop; the problem is unbounded. Otherwise, determine the index of variable x_{B_r} leaving the basic as follows:

3) $$\frac{\bar{b}_r}{y_{rk}}=\min_{1\leq i\leq m}\left\{\frac{\bar{b}_i}{y_{ik}} \mid y_{ik}>0\right\} \tag{3}$$

4) Update the tableau by pivoting at y_{rk}, Update the basic and nonbasic variable where x_k enters the basic and x_{B_r} leaves the basic and go to step (1).

Remark 3.3 In step 3 of the above mentioned method, new basic B_{new}, new basic solution and new objectives are obtained as follows, respectively:

$$B_{new} = \{a_{B_1}, a_{B_2}, \ldots, a_{B_{r-1}}, a_k, a_{B_{r+1}}, \ldots, a_{B_m}\} \tag{4}$$

$$\hat{x}_{B_i} = \left(\bar{x}_{B_i} - \frac{\bar{x}_{B_r}}{y_{rk}} y_{rj}\right),\ i \neq r,\ \hat{x}_{B_k} = \bar{x}_{B_r} = \frac{\bar{x}_{B_r}}{y_{rk}} \tag{5}$$

$$\hat{z} = \tilde{\bar{z}} - \frac{\bar{x}_{B_r}}{y_{rk}}(\tilde{z}_k - \tilde{c}_k) \tag{6}$$

In contrast to classical linear programming, the concept of duality of the RFNLP problem is not uniquely defined. Mahdavi-Amiri and Nasseri [16] defined the duality in the RFNLP problem by using linear ranking functions leading to a standard primal-dual linear programming pair.

The dual of the RFNLP problem (2) is defined as follows:

$$\begin{aligned} \max\ & \tilde{y} \approx \tilde{w} b \\ s.t.\ & \tilde{w} A \tilde{\le} \tilde{c} \\ & \tilde{w} \ge \tilde{0} \end{aligned} \tag{7}$$

In addition, Mahdavi-Amiri and Nasseri [16] proved some important results concern to the FNLP problem and its dual problem. Based on these results, Nasseri and Ebrahimnejad [13] proposed a new algorithm for solving RFNLP problems. Here, we give a summary of their method in tableau format leading to the new algorithm as a fuzzy dual simplex method.

Algorithm 3.2 A fuzzy dual simplex for the RFNLP problem

Initialization step

Suppose that basic B be the dual feasible for the RFNLP problem (2) in its standard form, i.e. $\tilde{z}_j - \tilde{c}_j \tilde{\le} \tilde{0},\ j = 1, 2, \cdots, n$. Form **Table 1** as an initial dual simplex tableau.

Main step

(1) Suppose $\bar{b} = B^{-1} b$. If $\bar{b} \ge 0$, then stop; the current solution is optimal.

Else select the pivot row r, with $\bar{b}_r < 0$.

(2) If $y_{rj} \ge 0$ for all j, then stop; the problem is infeasible. Else select the pivot column k by the following test:

$$\frac{\Re(\tilde{z}_k - \tilde{c}_k)}{y_{rk}} = \min_{1 \le j \le m}\left\{\frac{\Re(\tilde{z}_j - \tilde{c}_j)}{y_{rj}} \,\middle|\, y_{ik} < 0\right\} \tag{8}$$

(3) Update the tableau by pivoting at y_{rk}. Update the basic and nonbasic variable where x_k enters the basic and x_{B_r} leaves the basic and go to step (1).

Remark 3.4 In step (3) of the above mentioned method new fuzzy objective value is obtained as follows:

$$\tilde{\hat{z}} = \tilde{\bar{z}} - \frac{\bar{b}_r}{y_{rk}}(\tilde{z}_k - \tilde{c}_k) \tag{9}$$

Definition 3.5 A basis B for the RFNLP problem (2) is said to be dual degenerate if for at least one nonbasic variable, say x_j, we have $\Re(\tilde{z}_j - \tilde{c}_j) = 0$. Otherwise, it is said to be dual non-degenerate. The RFNLP problem (2) is said to be totally dual non-degenerate, if for all nonbasic variables in the dual simplex tableau with respect to any basis for (2), we have $\Re(\tilde{z}_j - \tilde{c}_j) \neq 0$.

New results

In this section, we extend some important results related to linear programming problems to reduced fuzzy linear programming problems. It needs to be pointed out that the fuzzy primal method for RFNLP problems, starting at a basic feasible solution moves to another basic solution with a better (at least not worse) objective value until it finds an optimal basic feasible solution after a finite number of steps. Here, we prove that in the absence of degeneracy, the primal method stops in a finite number of iterations.

Theorem 4.1 In the absence of primal degeneracy, the fuzzy primal simplex algorithm stops in a finite number of iterations, either with an optimal basic feasible solution or with the conclusion that the optimal value is unbounded.

Proof. In the absence of primal degeneracy, every basic feasible solution has exactly m positive components and has a unique associated basis. Also, one of the following three actions is executed in each iteration stage of the method. It may stop with an optimal basic solution if $\Re(\tilde{z}_k - \tilde{c}_k) \le 0$; it may stop with an unbounded solution if $\Re(\tilde{z}_k - \tilde{c}_k) > 0$ and $y_{rk} < 0$; or else it gives a new basic feasible solution if $\Re(\tilde{z}_k - \tilde{c}_k) < 0$ and $y_k \nleq 0$. In the absence of degeneracy, $\bar{b}_r > 0$, i.e. $x_{B_r} = \bar{b}_r > 0$ and hence $\frac{x_{B_r}}{y_{rk}} > 0$. By (6), the difference between the fuzzy objective values at the previous iteration and the current iteration is $\frac{\bar{x}_{B_r}}{y_{rk}}(\tilde{z}_k - \tilde{c}_k) \succ \tilde{0}$. Thus, the fuzzy objective value decreases strictly in each step. Hence a basis that appears once in the course of method can never reappear. Also the total number of bases for (2) is less than or equal to $\begin{pmatrix} n \\ m \end{pmatrix}$. Hence, the method would stop in a finite number of steps with a finite optimal basic solution or with an unbounded optimal solution.

We note that the fuzzy dual simplex algorithm for RFNLP problems starts with a dual basic feasible solution, but primal basic infeasible solution and walks to an optimal solution by moving among adjacent dual basic feasible solutions. Now, we show that in the absence of dual degeneracy, this fuzzy dual method stops in a finite number of iterations.

Theorem 4.2 In the absence of dual degeneracy, the dual primal simplex algorithm stops in a finite number of iterations, either with an optimal basic feasible solution or with the conclusion that the problem is infeasible.

Proof. We know that the fuzzy dual simplex method moves among dual feasible bases. Also, in each iteration stage of the method, one of the following three actions is executed. It may stop with an optimal basic solution if $\bar{b}_r \ge 0$; it may stop with the conclusion that the problem is infeasible, if $\bar{b}_r < 0$ and $y_{rj} \ge 0, j = 1,2,...,n$; or else it gives a new basic feasible solution if $\bar{b}_r < 0$ and $\exists j; y_{rj} < 0$. In addition, the difference in the dual fuzzy objective values between two successive iterations is. Note that $\bar{b}_r < 0$ and $y_{rk} < 0$. Moreover, by Remark 3.5 in the absence of dual degeneracy we have $\Re(\tilde{z}_k - \tilde{c}_k) < 0$. Thus, the fuzzy objective value increases strictly in each iteration stage and hence no basis can be repeated and algorithm must converge in a finite number of steps.

It needs to note that the key of the fuzzy simplex methods is that the optimal solution is obtained at a basic solution. Thus we prove that if a RFNLP problem has an optimal solution, then it also has a basic optimal solution.

Theorem 4.3 (Generalized fundamental theorem of linear programming). If RNFLP (2) in the standard form has an optimum feasible solution, then it has a basic feasible solution that is optimal.

Proof. Suppose $\bar{x}$ be an optimum feasible solution for the RFNLP problem (2). Let $\{a_j \mid \bar{x}_j > 0\} = \{a_1, a_2, \ldots, a_k\}$. So, we have

$$a_1 \bar{x}_1 + a_2 \bar{x}_2 + \ldots + a_k \bar{x}_k = \bar{b} \tag{10}$$

If $\{a_1, a_2, \cdots, a_k\}$ is linear independent, then $\bar{x}$ is a fuzzy basic feasible solution of (2) and we are done. Suppose this set is linearly dependent. So, there exists $y = (y_1, y_2, \cdots, y_k) \neq (0, 0, \cdots, 0)$ such that

$$a_1 y_1 + a_2 y_2 + \cdots + a_k y_k = 0 \tag{11}$$

We now show that the assumption $\bar{x}$ is fuzzy optimal implies that any $(y_1, y_2, \cdots, y_k)$ satisfying (11) must also satisfy

$$\tilde{c}_1 y_1 + \tilde{c}_2 y_2 + \cdots + \tilde{c}_k y_k \cong \tilde{0} \tag{12}$$

Suppose not. Using Eq. (10) and (11), we get the following relation in which θ is a real number.

$$a_1(\bar{x}_1 + \theta y_1) + a_2(\bar{x}_2 + \theta y_2) + \ldots + a_k(\bar{x}_k + \theta y_k) = \bar{b} \tag{13}$$

Define the fuzzy vector $\bar{x}(\theta) = (\bar{x}_1(\theta), \bar{x}_2(\theta), \ldots, \bar{x}_n(\theta)))$, where

$$\bar{x}_j(\theta) = \begin{cases} \bar{x}_j + \theta y_j & j = 1, 2, \cdots, k \\ 0 & j = k+1, \cdots, n \end{cases} \tag{14}$$

Clearly, $\bar{x}(\theta)$ satisfies $Ax = b$. Define

$$\theta_1 = \max_{1 \le j \le k} \left\{ -\frac{\bar{x}_j}{y_j} \,\middle|\, y_j > 0 \right\} \tag{15}$$

$$\theta_2 = \min_{1 \le j \le k} \left\{ -\frac{\bar{x}_j}{y_j} \,\middle|\, y_j < 0 \right\} \tag{16}$$

Since $y = (y_1, y_2, \cdots, y_k) \neq (0, 0, \cdots, 0)$, at least one of θ_1 or θ_2 must be finite. It is clear that $\theta_1 < 0$, $\theta_2 > 0$. Let $0 < \varepsilon \le \min\{|\theta_1|, \theta_2\}$. Therefore, $\bar{x}(\theta)$ is a feasible solution for all θ satisfying $-\varepsilon \le \theta \le \varepsilon$. In addition, $\tilde{z}(\bar{x}(\theta)) = \tilde{z}(\bar{x}) + \theta(\tilde{c}_1 y_1 + \tilde{c}_2 y_2 + \cdots + \tilde{c}_k y_k)$. If $\tilde{c}_1 y_1 + \tilde{c}_2 y_2 + \cdots + \tilde{c}_k y_k \succ \tilde{0}$, let $\pi = -\varepsilon$ and if $\tilde{c}_1 y_1 + \tilde{c}_2 y_2 + \cdots + \tilde{c}_k \tilde{y}_k \prec \tilde{0}$, let $\pi = \varepsilon$. Then $\bar{x}(\pi)$ is a feasible solution of (2) and $\tilde{z}(\tilde{\bar{x}}(\pi)) \prec \tilde{z}(\tilde{\bar{x}})$, which contradicts the assumption that $\bar{x}$ is an optimum solution of (2). Hence, (12) must hold. Using (11) we can obtain another feasible solution $\hat{x}$ in which the number of positive variables is at least one less than the number of positive variables of $\bar{x}$. By (12), any such fuzzy feasible $\hat{x}$ that we obtain must also satisfy $\tilde{z}(\bar{x}) \cong \tilde{z}(\hat{x})$, and thus $\hat{x}$ is an another optimum feasible solution. Hence, when this procedure is applied repeatedly, an optimal fuzzy basic feasible solution of (2) will be obtained after at most $(k-1)$ applications of the procedure.

Here, for an illustration of the above theorem we consider the following fuzzy variable linear programming problem.

Example 4.1 Consider the following RFNLP problem:

$$\begin{aligned} \max\ & \tilde{z} \cong (1,3,1,1)\,x_1 + (1,3,1,1)\,x_2 \\ s.t.\ & x_1 + x_2 + x_3 \quad = 10 \\ & \quad x_2 + \quad x_4 = 8 \\ & x_1,\, x_2,\, x_3,\, x_4 \geq 0 \end{aligned}$$

It should be note that $\bar{x} = (6,4,0,4)$ is an optimal solution for the above problem with optimal objective value $\tilde{z}^* = (10,30,10,10)$ and $\Re(\tilde{z}^*) = 20$. It is clear that $\{a_j \,|\, \bar{x}_j > 0\} = \left\{ a_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, a_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, a_4 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ are linearly dependent. So, there exists $\mathrm{y} = (\mathrm{y}_1, \mathrm{y}_2,\, \mathrm{y}_4) \neq (0,0,,0)$ such that $y_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + y_4 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

Let $y = (y_1, y_2,\, y_4) \neq (1,-1,,1)$. From the definition of θ_1 and θ_2, we have $\theta_1 = -4$ and $\theta_2 = 4$. Thus we get $\varepsilon = 4$. In this case if we suppose $\pi = -\varepsilon = -4$ we obtain a new feasible solution as $\hat{x} = \bar{x} + \theta\, y = (2,8,0,0)$. For this new solution we have $\tilde{z}_{new}{}^* = (10,30,10,10)$ with $\Re(\tilde{z}_{new}{}^*) = 20$. This shows that the new solution is optimal too. In addition, for this solution we have $\{a_j \,|\, \hat{x}_j > 0\} = \left\{ a_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, a_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$ which are linearly independent. Thus, the new optimal solution is an optimal feasible basis solution.

Conclusions

The field of fuzzy linear programming has recently attracted significant interest. The key of the fuzzy primal and fuzzy dual simplex method is that the optimal solution is obtained at a basic solution. Thus, in this paper we proved that if an RFNLP problem has an optimal solution, then it also has a basic optimal solution. We also showed that in the absence of primal and dual degeneracy, these fuzzy methods stop in a finite number of iterations.

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