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Short Communication

Solitary Wave Solutions for Zoomeron Equation

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Abstract

Tanh-Coth Method is applied to find solitary wave solutions of the Zoomeron equation which is of extreme importance in mathematical physics. The proposed scheme is fully compatible with the complexity of the problem and is highly efficient. Moreover, suggested combination is capable to handle nonlinear problems of versatile physical nature.

Keywords: Tanh-Coth method, Zoomeron equation, nonlinear equations, solitary wave solutions

Introduction

Most physical phenomenon are modeled by nonlinear differential equations [1-65]. A wide range of analytical and numerical techniques including Perturbation, Modified Adomian's Decomposition (MADM), Variational Iteration (VIM), Homotopy Perturbation (HPM), exp-function, Spline, Backlund transformation, Homotopy Analysis (HAM), have been developed to solve such equations, see [1-65] and the references therein. The basic motivation of this paper is the extension of a relatively new scheme called the Tanh-Coth Method [18-22] to obtain solitary wave solutions of the Zoomeron equation which is of utmost importance in mathematical physics. It is observed that the proposed scheme is fully compatible with the complexity of such problems. Moreover, suggested combination is highly capable to handle nonlinear problems of versatile physical nature. Numerical results are very encouraging and reveal the efficiency of the proposed scheme.

Tanh-Coth method [18-22]

Consider the following nonlinear partial differential equation for u(x, t) to be in the form

$$P(u,\ u_x, u_t,\ u_{xx}, u_{xt}, u_{tt}, \dots) = 0 \qquad (1)$$

where P is a polynomial in its arguments. The essence of the Tanh-Coth method expansion method can be presented in the following steps:

Step 1 Seek traveling wave solutions of Eq. (1) by taking $u(x,t) = u(\xi),\ \xi = x - ct,$ and transform Eq. (1) into an ordinary differential equation

$$Q(u, u', u'', \dots) = 0, \qquad (2)$$

where prime denotes the derivative with respect to ξ.

Step 2 If possible, integrate Eq. (2) term by term one or more times. This yields constant(s) of integration. For simplicity, the integral constant(s) may be zero.

Step 3 Introduce a new independent variable

$$\mathrm{Y} = \tanh(\mu\, \xi)\,,\ \xi = \mathrm{x} - \mathrm{ct}, \qquad (3)$$

that leads to a change of derivatives:

$$\frac{d}{d\xi} = (1-Y^2)\frac{d}{dY},$$
$$\frac{d^2}{d\xi^2} = -2\mu^2(1-Y^2)\frac{d}{dY} + \mu^2(1-Y^2)^2\frac{d^2}{dY^2},$$
$$\frac{d^2}{d\xi^2} = -2\mu^3(1-Y^2)(3Y^2-1)\frac{d}{dY} - 6\mu^3(1-Y^2)^2\frac{d^2}{dY^2} + \mu^3(1-Y^2)^3\frac{d^3}{dY^3}. \quad (4)$$

Other derivatives can be derived in a similar manner.

Step 4 We then propose the following finite series expansion

$$u(\mu\xi) = S(Y) = \sum_{k=0}^{m} a_k Y^k + \sum_{k=1}^{m} b_k Y^{-k}, \quad (5)$$

in which in most cases *m* is a positive integer. To determine the parameter *m*, we usually balance the linear terms of highest order in the equation (2) with the highest order nonlinear terms. Substituting (3), (4) and (5) into the ODE yields an equation in powers of Y.

Step 5 With m determined, we collect all coefficients of powers of *Y* in the resulting equation where these coefficients have to vanish. This will give a system of algebraic equations involving the parameters a_k, b_k, μ, c. Having determined these parameters and using (5) we obtain an analytic solution $u = u(x,t)$, in a closed form.

Solution procedure

Consider the following Zoomeron equation:

$$\left(\frac{u_{xy}}{u}\right)_{tt} - \left(\frac{u_{xy}}{u}\right)_{xx} + 2(u^2)_{xt} = 0, \quad (6)$$

assuming the solution in the following frame:

$$u = U(\xi),\ \xi = x - \omega y - ct, \quad (7)$$

where c, ω are constants. We substitute Eq. (7) into Eq. (6) and integrating twice with respect to ξ, by setting the second integration constant equal to zero, we obtain the following nonlinear ordinary differential equation

$$\omega(1-c^2)U'' - 2cU^3 - RU = 0, \quad (8)$$

where *R* is integration constant. Balancing the nonlinear term U^3 with the highest order derivative U'' that gives

$$3M = M + 2, \quad (9)$$

so that

$$M = 1. \quad (10)$$

The Tanh-Coth method admits the use of the substitution

$$u(x,t) = S(Y) = a_0 + a_1 Y + b_1 Y^{-1}, \quad (11)$$

Substituting (11) into (8), collecting the coefficients of each power of Y, setting each coefficient to zero, and solving the resulting system of algebraic equations, we find the following sets of solutions:

(i) $$a_0 = 0,\ a_1 = \sqrt{-\frac{R}{2c}},\ b_1 = 0,\ \mu = \sqrt{\frac{R}{2\omega(c^2-1)}}\ ,\ \ c > 0 \tag{12}$$

(ii) $$a_0 = 0,\ a_1 = 0,\ b_1 = \sqrt{-\frac{R}{2c}},\ \mu = \sqrt{\frac{R}{2\omega(c^2-1)}},\ \ c > 0 \tag{13}$$

(iii) $$a_0 = 0,\ a_1 = \frac{1}{4}\sqrt{-\frac{2R}{c}},\ b_1 = -\frac{1}{2}\frac{R}{c\sqrt{-\frac{R}{c}}},\ \mu = \sqrt{\frac{R}{8\omega(c^2-1)}},\ \ c > 0 \tag{14}$$

This in turn gives the front wave (kink) solution

$$u_1(x,t) = \frac{1}{2}\sqrt{-\frac{R}{2c}}\tanh\left[\sqrt{\frac{R}{2\omega(c^2-1)}}\ (x + \omega y - ct)\right], \tag{15}$$

and the travelling wave solutions

$$u_2(x,t) = \frac{1}{2}\sqrt{-\frac{R}{2c}}\tanh\left[\sqrt{\frac{R}{2\omega(c^2-1)}}\ (x + \omega y - ct)\right], \tag{16}$$

For $c > 0$, we obtain the solution

$$u_3(x,t) = \frac{1}{4}\left(\sqrt{\frac{-2R}{c}}\tanh\left[\sqrt{\frac{R}{8\omega(c^2-1)}}\ (x + \omega y - ct)\right] - \frac{R\sqrt{2}}{c\sqrt{-\frac{R}{c}}}\coth\left[\sqrt{\frac{R}{8\omega(c^2-1)}}\ (x + \omega y - ct)\right]\right), \tag{17}$$

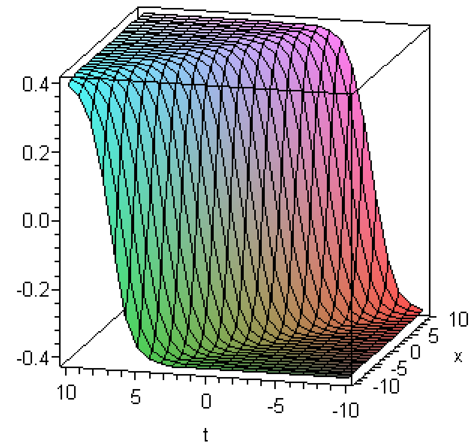

Figure 1 Kink solution of Eq. (15) when $R = 0.5,\ c = -1.5,\ \omega = 1.$

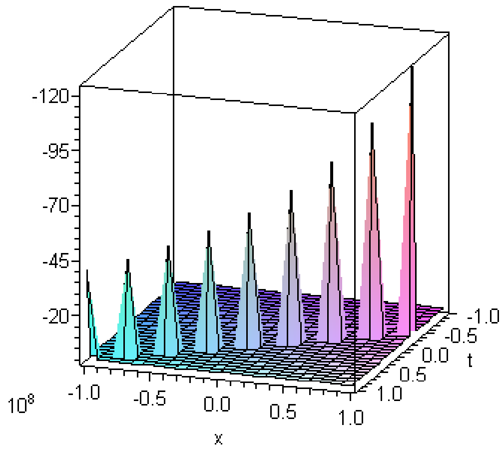

Figure 2 Travelling wave solution of Eq. (16) when $R = 0.5,\ c = -1.5,\ \omega = 1.$

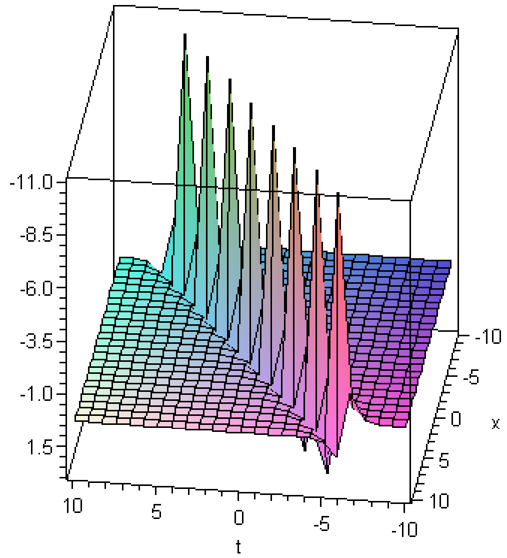

Figure 3 Travelling wave solution of Eq. (17) when $R = 0.5,\ c = -1.5,\ \omega = 1$.

Conclusions

The Tanh-Coth method was successfully used to establish solitary wave solutions of the Zoomeron equation. The performance of the Tanh-Coth method is reliable, effective and hence it may be used to tackle other nonlinear problems of a versatile physical nature.

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