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Combined Effects of Chemical Reaction and Wall Slip on MHD Flow in a Vertical Wavy Porous Space with Traveling Thermal Waves

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Abstract

This paper investigates the magnetohydrodynamic (MHD) mixed convective heat and mass transfer flow in a vertical wavy porous space in the presence of a heat source with the combined effects of chemical reaction and wall slip condition. The dimensionless governing equations are perturbed into: mean (zeroth-order) part and a perturbed part, using amplitude as a small parameter. The perturbed quantities are obtained by perturbation series expansion for small wavelength in which terms of exponential order arise. The results obtained show that the velocity, temperature and concentration fields are appreciably influenced by the presence of chemical reaction, magnetic field, porous medium, heat source/sink parameter and wall slip condition. Further, the results of the skin friction and rate of heat and mass transfer at the wall are presented for various values of parameters entering into the problem and discussed with the help of graphs.

Keywords: Mixed convection, wavy walls, porous medium, chemical reaction and wall slip

Introduction

The study of combined free and forced convection flow in vertical channels has received considerable attention because of its wide range of applications, from cooling of electronic devices to that of solar energy collectors. Further, convection problems associated with heat sources within fluid saturated porous media are of great practical significance, for there are a number of practical applications in geophysics and energy related problems, such as recovery of petroleum resources, geophysical flows, cooling of underground electric cables, etc. A comprehensive review of the work on mixed convection can be found in [1-10]. Eldabe *et al.* [8] discussed the problem of mixed convective heat and mass transfer in a non-Newtonian fluid at a peristaltic surface with temperature dependent viscosity. They considered the peristaltic flow between 2 vertical walls, one of which is deformed in the shape of traveling transversal waves exactly like peristaltic pumping, and the other of which is parallel flat plate wall. Recently, Srinivas and Muthuraj [9] have discussed the effects of thermal radiation and space porosity on magnetohydrodynamic (MHD) mixed convection flow in a vertical channel using homotopy analysis method. More recently, Prathap Kumar *et al.* [10] have studied the problem of fully developed free convective flow of micropolar and viscous fluids in a vertical channel. The analyses of laminar heat transfer in slip-flow regime were first undertaken by Sparrow *et al.* [11] and Inman [12] for tubes with uniform heat flux and a parallel plate channel or a circular tube with uniform wall temperature using continuum theory subject to slip-velocity and temperature-jump boundary conditions. Their works show the Nusselt number decrease in the presence of slip. Lately, there has been an increase of interest in studying fluid problems with slip boundary conditions [13-21]. Ebaid [19] studied the effects of magnetic field and wall slip conditions on the peristaltic transport of a Newtonian fluid in an asymmetric channel. More recently, Srinivas *et al.* [20] have examined the influence of slip conditions, wall properties and heat transfer on MHD peristaltic transport. Srinivas and Muthuraj [21] also analyzed the MHD flow with slip effects and temperature dependent heat source in a vertical wavy porous space.

Mixed convection flows with simultaneous heat and mass transfer under the influence of a magnetic field and chemical reaction arise in many transport processes, both naturally and in many branches of science and engineering applications. They play an important role in many industries viz. in the chemical industry, and the power and cooling industry for drying, chemical vapor deposition on surfaces, cooling of nuclear reactors and MHD power generators [22-26]. Hayat *et al.* [27] have studied the laminar flow problem of convective heat transfer for a second grade fluid over a semi-infinite plate in the presence of species concentration and chemical reaction. Mohamed and Abo-Dahab [28] have presented the effects of chemical reaction and thermal radiation on hydromagnetic free convection heat and mass transfer for a micropolar fluid via a porous medium bounded by a semi-infinite vertical porous plate in the presence of heat generation. Pal and Talukdar [29] have analyzed the unsteady MHD convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction using perturbation technique. More recently, Zueco and Ahmed [30] have presented an exact and a numerical solution to the problem of a steady mixed convective MHD flow of an incompressible viscous electrically conducting fluid past an infinite vertical porous plate with combined heat and mass transfer. Several investigators are now engaged in finding the analytical or numerical solutions for highly non-linear equations using different methods [31-37].

The information available indicates that no investigation has been made to analyze the influence of chemical reaction and wall slip on MHD flow with heat and mass transfer in a vertical channel. With the above discussion in mind and motivated by the earlier studies, an attempt has been made to understand the combined effects of chemical reaction and wall slip on MHD flow in a vertical wavy porous space with traveling thermal waves. As the problem is highly nonlinear, it is solved by a perturbation technique wherein the solution is assumed to be made up of 2 parts: a mean part corresponding to the fully developed mean flow, and a small perturbed part. The mean part, the perturbed part, and the total solution of the problem are evaluated numerically for various values of the pertinent parameters entering into the problem. The paper has been organized as follows: in Section 2, the mathematical formulation of the problem is developed. The solution of the problem is presented in the Section 3. In Section 4, the numerical results and discussion are presented, while the concluding remarks are found in Section 5.

Formulation of the problem

Consider the unsteady, mixed convective heat and mass transfer MHD flow in a viscous fluid confined to the vertical wavy walls embedded in a porous medium. We consider the wavy wall in which the x axis is taken vertically upward, and parallel to the direction of buoyancy, and the y axis is normal to it (**Figure 1**). A uniform magnetic field is applied normal to the flow direction. The wavy walls are represented by $y = d + a\cos\lambda x$ and $y = -d + a\cos(\lambda x + \theta)$. The governing equations for this problem are based on the balance laws of mass, linear momentum and energy modified to account for the presence of the magnetic field, thermal buoyancy and heat generation or absorbing effects. These can be written as;

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 . \tag{1}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{\mu\varphi}{k}u - \sigma B_0^2 u + \rho g\beta_t(T - T_1') + \rho g\beta_c(C - C_1') \tag{2}$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\mu\varphi}{k}v \tag{3}$$

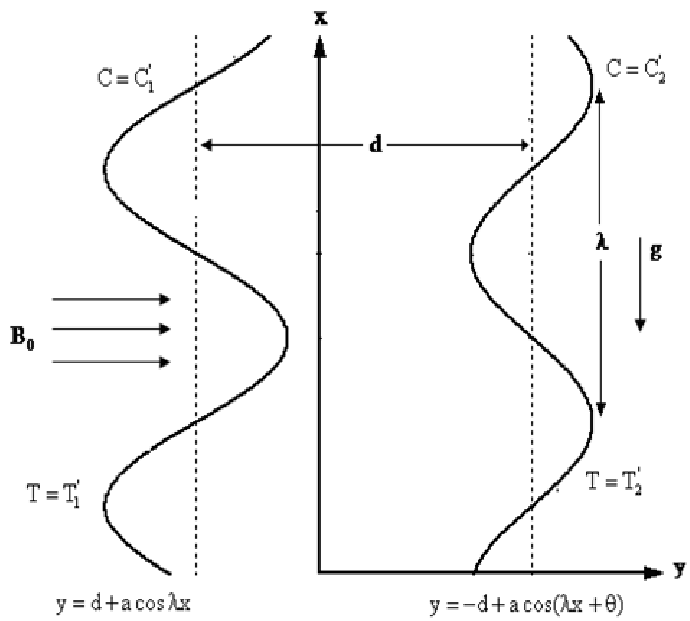

Figure 1 Flow geometry of the problem.

$$\rho C_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=K\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+Q \tag{4}$$

$$\left(\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}\right)=D_m\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)-K_1C\,. \tag{5}$$

The boundary conditions of the problem are [16,21];

$$u=L_1\left(\frac{\partial u}{\partial y}\right),\ v=0,\ T=T_1',\ C=C_1',\quad \text{at}\quad y=d+a\cos\lambda x \tag{6}$$

$$u=-L_1\left(\frac{\partial u}{\partial y}\right),\ v=0,\ T=T_2',\ C=C_2',\quad \text{at}\quad y=-d+a\cos(\lambda x+\theta) \tag{7}$$

where $T_1\left[1+\varepsilon\cos(\lambda x+\omega t)\right]=T_1'$, $T_2\left[1+\varepsilon\cos(\lambda x+\omega t)\right]=T_2'$, $C_1\left[1+\varepsilon\cos(\lambda x+\omega t)\right]=C_1'$, $C_2\left[1+\varepsilon\cos(\lambda x+\omega t)\right]=C_2'$, $L_1=L\left[\dfrac{2-m_1}{m_1}\right]$, L is the mean free path, m_1 is the Maxwell's reflexion coefficient, B_0 is the transverse magnetic field, D_m is the coefficient of mass diffusivity, u, v are velocity components, C is the concentration, K is the thermal conductivity of the fluid, Q is the heat source/sink, T is the temperature, p is the pressure, ρ is the density, μ is the dynamic viscosity, ν is the kinematic viscosity, k is the permeability of the medium, σ is the coefficient of electric conductivity, β_c is the concentration expansion coefficient, β_t is the thermal expansion coefficient, g is the gravitational acceleration, ω is the frequency, T_1 and T_2 are the wall temperatures, $\bar{T}$ is the mean value of T_1 and T_2, C_1 and C_2 are the wall concentrations.

We introduce the non-dimensional variables

$$(x^*,y^*)=\frac{1}{d}(x,y)\,,\ t^*=\frac{t\nu}{d^2}\,,\ (u^*,v^*)=\frac{d}{\nu}(u,v)\,,\ p^*=\frac{p}{\rho\left(\frac{\nu}{d}\right)^2}\,,\ T^*=\frac{T-T_1'}{T_2'-T_1'}\,,\ \phi=\frac{C-C_1'}{C_2'-C_1'}\,. \tag{8}$$

Invoking the above non-dimensional variables, the basic field Eqs. (1) - (7) can be expressed in the non-dimensional form, dropping the asterisks,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{9}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-H^2u+G_c\phi+G_rT \tag{10}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{1}{D_a}v \tag{11}$$

$$\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{1}{P_r}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\alpha \tag{12}$$

$$\left(\frac{\partial \phi}{\partial t}+u\frac{\partial \phi}{\partial x}+v\frac{\partial \phi}{\partial y}\right)=\frac{1}{S_c}\left(\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial y^2}\right)-\gamma\phi-C_1^*. \tag{13}$$

The corresponding boundary conditions are

$$u=hu'\,,v=0\,,\ T=0\,,\ \phi=0 \quad \text{at} \quad y=1+\varepsilon\cos\lambda x \tag{14}$$

$$u=-hu'\,,\ v=0\,,\ T=1\,,\ \phi=1 \quad \text{at} \quad y=-1+\varepsilon\cos\left(\lambda x+\theta\right) \tag{15}$$

where $H^2 = M^2 + \frac{1}{D_a}$, $C_1^* = \frac{K_1 d^2 C_1'}{C_2' - C_1'}$, $G_r = \frac{d^3 \beta_t g(T_2' - T_1')}{\nu^2}$ is the Grashof number, $G_c = \frac{d^3 \beta_c g(C_2' - C_1')}{\nu^2}$ is the local mass Grashof number, $M^2 = \frac{\sigma B_0^2 d^2}{\rho \nu}$ is the Hartmann number, $P_r = \frac{\mu C_p}{K}$ is the Prandtl number, $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity, $\lambda (= \lambda^*) = \lambda d$ is the non dimensional wave number, λx is the wall waviness parameter, $\varepsilon = \frac{a}{d}$ $(\varepsilon << 1)$ is the non-dimensional amplitude parameter, $S_c = \frac{\mu}{\rho D_m}$ is the Schmidt number, $D_a = \frac{k}{\varphi d^2}$ is the porosity parameter, $\alpha = \frac{Q d^2}{K(T_2' - T_1')}$ is the heat source/sink parameter, $\gamma = \frac{K_1 d}{\nu}$ is the chemical reaction parameter, $h = \frac{L_1}{d}$ is the slip parameter.

Let us introduce the stream function ψ defined by $u = -\frac{\partial \psi}{\partial y}$ and $v = \frac{\partial \psi}{\partial x}$ (16)

Using Eq. (16), Eqs. (10) - (13) become

$$\frac{\partial^3 \psi}{\partial x^2 \partial t} + \frac{\partial^3 \psi}{\partial y^2 \partial t} - \frac{\partial \psi}{\partial y}\left(\frac{\partial^3 \psi}{\partial x^3} + \frac{\partial^3 \psi}{\partial x \partial y^2}\right) + \frac{\partial \psi}{\partial x}\left(\frac{\partial^3 \psi}{\partial x^2 \partial y} + \frac{\partial^3 \psi}{\partial y^3}\right)$$
$$= \frac{\partial^4 \psi}{\partial x^4} + 2\frac{\partial^4 \psi}{\partial x^2 \partial y^2} + \frac{\partial^4 \psi}{\partial y^4} - M^2 \frac{\partial^2 \psi}{\partial y^2} - \frac{1}{D_a}\left(\frac{\partial^2 \psi}{\partial y^2}\right) - G_r \frac{\partial T}{\partial y} - G_c \frac{\partial \phi}{\partial y} \tag{17}$$

$$\frac{\partial T}{\partial t} - \frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} + \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y} = \frac{1}{P_r}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \alpha \tag{18}$$

$$\frac{\partial \phi}{\partial t} - \frac{\partial \psi}{\partial y}\frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial x}\frac{\partial \phi}{\partial y} = \frac{1}{S_c}\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right) - \gamma \phi - C_1^* \tag{19}$$

and the boundary conditions (14) - (15) become

$$\psi_y = h\psi_{yy}, \quad \psi_x = 0, \quad T = 0, \quad \phi = 0 \quad \text{at} \quad y = 1 + \varepsilon \cos \lambda x \tag{20}$$

$$\psi_y = -h\psi_{yy}, \quad \psi_x = 0, \quad T = 1, \quad \phi = 1 \quad \text{at} \quad y = -1 + \varepsilon \cos(\lambda x + \theta). \tag{21}$$

Solution of the problem

In order to solve Eqs. (17) - (19), we assume that the solution consists of a mean part and perturbed part so that the stream function, temperature and concentration distributions are [3,7,8]

$$\psi(x, y, t) = \psi_0(y) + \varepsilon \psi_1(x, y, t) \tag{22}$$

$$T(x, y, t) = T_0(y) + \varepsilon T_1(x, y, t) \tag{23}$$

$$\phi(x, y, t) = \phi_0(y) + \varepsilon \phi_1(x, y, t) \tag{24}$$

where ψ_0, T_0, ϕ_0 are the mean parts and ψ_1, T_1, ϕ_1 are the perturbed parts also, we introduce

$$\psi_1(x,y,t) = e^{i(\lambda x+\omega t)}\bar{\psi}_1(y) \tag{25}$$

$$T_1(x,y,t) = e^{i(\lambda x+\omega t)}\bar{T}_1(y) \tag{26}$$

$$\phi_1(x,y,t) = e^{i(\lambda x+\omega t)}\bar{\phi}_1(y)\,. \tag{27}$$

With the help of Eqs. (22) - (27) the Eqs. (17) - (21) yield

$$\psi_0^{iv} - H^2\psi_0'' - G_r T_0' - G_c\phi_0' = 0 \tag{28}$$

$$T_0'' + \alpha = 0 \tag{29}$$

$$\frac{1}{S_c}\phi_0'' - \gamma\phi_0 - C_1^* = 0\,, \tag{30}$$

together with the boundary conditions

$$\psi_0' = h\psi_0'',\ \psi_0 = 0,\ T_0 = 0,\ \phi_0 = 0 \quad \text{at} \quad y = 1 \tag{31}$$

$$\psi_0' = -h\psi_0'',\ \psi_0 = 0,\ T_0 = 1,\ \phi_0 = 1 \quad \text{at} \quad y = -1 \tag{32}$$

to the zeroth-order, and

$$\bar{\psi}_1^{iv} - i\omega(\bar{\psi}_1'' - \lambda^2\bar{\psi}_1) + i\lambda\psi_0'(\bar{\psi}_1'' - \lambda^2\bar{\psi}_1) - i\lambda\bar{\psi}_1\psi_0''' - 2\lambda^2\bar{\psi}_1'' + \lambda^4\bar{\psi}_1$$

$$-M^2\bar{\psi}_1'' - \frac{1}{D_a}\bar{\psi}_1'' - G_r\bar{T}_1' - G_c\bar{\phi}_1' = 0 \tag{33}$$

$$\bar{T}_1'' - iP_r\omega\bar{T}_1 + iP_r\lambda(\psi_0'\bar{T}_1 - \bar{\psi}_1 T_0') - \lambda^2\bar{T}_1 = 0 \tag{34}$$

$$\bar{\phi}_1'' - iS_c\omega\bar{\phi}_1 + iS_c\lambda(\psi_0'\bar{\phi}_1 - \bar{\psi}_1\phi_0') - \lambda^2\bar{\phi}_1 - \gamma S_c\bar{\phi}_1 = 0\,, \tag{35}$$

together with the boundary conditions

$$\bar{\psi}_1' = h\bar{\psi}_1'' + e^{-i\omega t}(h\psi_0''' - \psi_0''),\quad \bar{\psi}_1 = 0,\ \bar{T}_1 = -e^{-i\omega t}T_0',\quad \bar{\phi}_1 = -e^{-i\omega t}\phi_0' \quad \text{at} \quad y = 1 \tag{36}$$

$$\bar{\psi}_1' = -h\bar{\psi}_1'' - e^{i(\theta-\omega t)}(h\psi_0''' + \psi_0''),\ \bar{\psi}_1 = 0,\ \bar{T}_1 = -e^{i(\theta-\omega t)}T_0',\ \bar{\phi}_1 = -e^{i(\theta-\omega t)}\phi_0' \quad \text{at} \quad y = -1 \tag{37}$$

to the first-order, where a prime denotes differentiation with respect to y.
For small values of λ, we can expand $\bar{\psi}_1$, $\bar{T}_1$ and $\bar{\phi}_1$ in terms of λ so that

$$\bar{\psi}_1(\lambda, y) = \sum_{r=0}^{\infty}\lambda^r\bar{\psi}_{1r},\ \bar{T}_1(\lambda, y) = \sum_{r=0}^{\infty}\lambda^r\bar{T}_{1r},\quad \bar{\phi}_1(\lambda, y) = \sum_{r=0}^{\infty}\lambda^r\bar{\phi}_{1r} \tag{38}$$

Substituting (38) into (33) - (37), we get the following sets of ordinary differential equations and boundary conditions, to the order of λ^2

$$\bar{\psi}_{10}^{iv} - i\omega\bar{\psi}_{10}'' - H^2\bar{\psi}_{10}'' - G_r\bar{T}_{10}' - G_c\bar{\phi}_{10}' = 0 \tag{39}$$

$$\bar{T}_{10}'' - P_r i\omega \bar{T}_{10} = 0 \tag{40}$$

$$\bar{\phi}_{10}'' - S_c i\omega \bar{\phi}_{10} - \gamma S_c \phi_{10} = 0 \tag{41}$$

$$\bar{\psi}_{11}^{iv} - i\omega \bar{\psi}_{11}'' + i(\psi_0' \bar{\psi}_{10}'' - \bar{\psi}_{10}\psi_0''') - H^2\, \bar{\psi}_{11}'' - G_r \bar{T}_{11}' - G_c \bar{\phi}_{11}' = 0 \tag{42}$$

$$\bar{T}_{11}'' + P_r i\omega \bar{T}_{11} + P_r i(\psi_0' \bar{T}_{10} - \bar{\psi}_{10} T_0') = 0 \tag{43}$$

$$\bar{\phi}_{11}'' - S_c i\omega \bar{\phi}_{11} + S_c i(\psi_0' \bar{\phi}_{10} - \bar{\psi}_{10}\phi_0') - \gamma S_c \phi_{11} = 0 \tag{44}$$

$$\bar{\psi}_{10}' = h\psi_{10}'' + e^{-i\omega t}(h\psi_0''' - \psi_0''),\quad \bar{\psi}_{10} = 0,\ \bar{T}_{10} = -e^{-i\omega t} T_0',\quad \bar{\phi}_{10} = -e^{-i\omega t}\phi_0' \quad \text{at} \quad y = 1 \tag{45}$$

$$\bar{\psi}_{10}' = -h\psi_{10}'' - e^{-i\omega t}(h\psi_0''' + \psi_0''),\ \bar{\psi}_{10} = 0,\ \bar{T}_{10} = -e^{i(\theta-\omega t)} T_0',\ \bar{\phi}_{10} = -e^{i(\theta-\omega t)}\phi_0' \ \text{at} \quad y = -1 \tag{46}$$

$$\left.\begin{array}{lllllll} \bar{\psi}_{1r}' = 0, & \bar{\psi}_{1r} = 0 & \bar{T}_{1r} = 0, & \bar{\phi}_{1r} = 0, & \text{at} & y = 1 \\ \bar{\psi}_{1r}' = 0, & \bar{\psi}_{1r} = 0 & \bar{T}_{1r} = 0, & \bar{\phi}_{1r} = 0, & \text{at} & y = -1 \end{array}\right\} r \geq 1. \tag{47}$$

Zeroth-order solution

Eqs. (28) - (30) subject to the boundary conditions (31) - (32) the solution are

$$\psi_0(y) = A_3 + B_3 y + C_3 \cosh Hy + D_3 \sinh Hy + T_5 y^2 + T_6 y^3 + T_7 \sinh \beta y + T_8 \cosh \beta y \tag{48}$$

$$T_0(y) = A_1 + B_1 y - \left(\frac{\alpha}{2}\right) y^2 \tag{49}$$

$$\phi_0(y) = A_2 \cosh \beta y - B_2 \sinh \beta y - \frac{c}{\beta^2} \tag{50}$$

where, $\beta = \sqrt{\gamma S_c}$; $c = C_1^* S_c$; $H = \sqrt{M^2 + \frac{1}{D_a}}$; $A_1 = \frac{1+\alpha}{2}$; $B_1 = -\frac{1}{2}$; $A_2 = \frac{c}{\beta^2 \cosh \beta} + \frac{1}{2\cosh \beta}$;

$B_2 = -\frac{1}{2\sinh \beta}$; $A_3 = \left(\frac{T_{14} + T_{15}}{2}\right) - C_3 \cosh H$; $B_3 = \frac{1}{2}\left(T_{14} - T_{15} - 2D_3 \sinh H\right)$; $C_3 = \frac{T_{11} - T_{13}}{T_9 - T_{12}}$;

$D_3 = \frac{T_{11} + T_{13} - T_{14} + T_{15} - C_3\left(T_9 + T_{12}\right)}{2(T_{10} - \sinh H)}$; $T_1 = B_1 G_r$; $T_2 = -\alpha G_r$; $T_3 = A_2 G_c \beta$; $T_4 = B_2 G_c \beta$;

$T_5 = -\frac{T_1^2}{2H^2}$; $T_6 = -\frac{T_2}{6H^2}$; $T_7 = \frac{T_3}{\beta^2(\beta^2 - H^2)}$; $T_8 = \frac{T_4}{\beta^2(\beta^2 - H^2)}$; $T_9 = H \sinh H - hH^2 \cosh H$;

$T_{10} = H \cosh H - hH^2 \sinh H$;

$T_{11} = 2T_5(h-1) + 3T_6(2h-1) + T_7\beta(h\beta \sinh \beta - \cosh \beta) + T_8\beta(h\beta \cosh \beta - \sinh \beta)$;

$T_{12} = -H \sinh H + hH^2 \cosh H$;

$T_{13} = 2T_5(1-h) + 3T_6(2h-1) + T_7\beta(h\beta \sinh \beta - \cosh \beta) + T_8\beta(\sinh \beta - h\beta \cosh \beta)$;

$T_{14} = -T_5 - T_6 - T_7 \sinh\beta - T_8 \cosh\beta$; $T_{15} = -T_5 + T_6 + T_7 \sinh \beta - T_8 \cosh \beta$;

First order solution

The solutions of Eqs. (39) - (41), subject to the conditions (45) and (46), are

$$\psi_{10} = A_6 + B_6 y + C_6 \cosh H_1 y + D_6 \sinh H_1 y + T_{20} \sinh \alpha_1 y + T_{21} \cosh \alpha_1 y + T_{22} \sinh \beta_1 y + T_{23} \cosh \beta_1 y \tag{51}$$

$$T_{10} = A_4 \cosh \alpha_1 y + B_4 \sinh \alpha_1 y \tag{52}$$

$$\phi_{10} = A_5 \cosh \beta_1 y + B_5 \sinh \beta_1 y \tag{53}$$

where, $\alpha_1 = \sqrt{iP_r\omega};\ \beta_1 = \sqrt{S_c(i\omega+\gamma)};\ H_1 = \sqrt{i\omega + H^2}$; $A_4 = \dfrac{e^{-i\omega t}}{2\cosh\alpha_1}\left[\left(\alpha+\dfrac{1}{2}\right) - e^{i\theta}\left(\alpha - \dfrac{1}{2}\right)\right]$;

$B_4 = \dfrac{e^{-i\omega t}}{2\sinh\alpha_1}\left[\left(\alpha+\dfrac{1}{2}\right) + e^{i\theta}\left(\alpha - \dfrac{1}{2}\right)\right]$; $A_5 = \dfrac{T_{24}+T_{25}}{2\cosh\beta_1}$; $B_5 = \dfrac{T_{24}-T_{25}}{2\sinh\beta_1}$; $A_6 = \dfrac{T_{27}+T_{29}-2C_6\cosh H_1}{2}$

; $B_6 = \dfrac{T_{27}-T_{29}-2D_6\sinh H_1}{2}$; $C_6 = \dfrac{T_{26} - T_{28}}{T_{30} - T_{32}}$; $D_6 = \dfrac{T_{26}+T_{28}-T_{27}+T_{29}-C_6\left(T_{30}+T_{32}\right)}{2\left(T_{31}-\sinh H_1\right)}$;

$T_{16} = \alpha_1 G_r A_4$; $T_{17} = \alpha_1 G_r B_4$; $T_{18} = \beta_1 G_c A_5$; $T_{19} = \beta_1 G_c B_5$; $T_{20} = \dfrac{T_{16}}{\alpha_1^2(\alpha_1^2 - H_1^2)}$; $T_{21} = \dfrac{T_{17}}{\alpha_1^2(\alpha_1^2 - H_1^2)}$;

$T_{22} = \dfrac{T_{18}}{\beta_1^2(\beta_1^2 - H_1^2)}$; $T_{23} = \dfrac{T_{19}}{\beta_1^2(\beta_1^2 - H_1^2)}$; $T_{24} = -e^{-i\omega t}\left(\dfrac{c\sinh\beta}{\beta\cosh\beta} + \dfrac{\beta}{2}\left(\dfrac{\sinh\beta}{\cosh\beta} - \dfrac{\cosh\beta}{\sinh\beta}\right)\right)$;

$T_{25} = -e^{i(\theta-\omega t)}\left(-\dfrac{c\sinh\beta}{\beta\cosh\beta} + \dfrac{\beta}{2}\left(-\dfrac{\sinh\beta}{\cosh\beta} - \dfrac{\cosh\beta}{\sinh\beta}\right)\right)$;

$$\begin{aligned} T_{26} = {} & -T_{20}(\alpha_1\cosh\alpha_1 - h\alpha_1^2\sinh\alpha_1) - T_{21}(\alpha_1\sinh\alpha_1 - h\alpha_1^2\cosh\alpha_1) - T_{22}(\beta_1\cosh\beta_1 - h\beta_1^2\sinh\beta_1) \\ & -T_{23}\left(\beta_1\sinh\beta_1 - h\beta_1^2\cosh\beta_1\right) + C_3 e^{-i\omega t}\left(hH^3\sinh H - H^2\cosh H\right) \\ & +D_3 e^{-i\omega t}\left(hH^3\cosh H - H^2\sinh H\right) + e^{-i\omega t}(6T_6 h - 2T_5) + 6e^{-i\omega t}T_6 \\ & +e^{-i\omega t}\sinh\beta(\beta^3 T_8 h - \beta^2 T_7) + e^{-i\omega t}\cosh\beta(\beta^3 T_7 h - \beta^2 T_8); \end{aligned}$$

$T_{27} = -T_{20}\sinh\alpha_1 - T_{21}\cosh\alpha_1 - T_{22}\sinh\beta_1 - T_{23}\cosh\beta_1$;

$$\begin{aligned} T_{28} = {} & -T_{20}(\alpha_1\cosh\alpha_1 - h\alpha_1^2\sinh\alpha_1) - T_{21}(-\alpha_1\sinh\alpha_1 + h\alpha_1^2\cosh\alpha_1) - T_{22}(\beta_1\cosh\beta_1 - h\beta_1^2\sinh\beta_1) \\ & -T_{23}\left(-\beta_1\sinh\beta_1 + h\beta_1^2\cosh\beta_1\right) - C_3 e^{-i\omega t}\left(-hH^3\sinh H + H^2\cosh H\right) \\ & -D_3 e^{-i\omega t}\left(hH^3\cosh H - H^2\sinh H\right) - e^{-i\omega t}(6T_6 h + 2T_5) + 6e^{-i\omega t}T_6 \\ & +e^{-i\omega t}\sinh\beta(\beta^3 T_8 h + \beta^2 T_7) - e^{-i\omega t}\cosh\beta(\beta^3 T_7 h + \beta^2 T_8); \end{aligned}$$

$T_{29} = T_{20}\sinh\alpha_1 - T_{21}\cosh\alpha_1 + T_{22}\sinh\beta_1 - T_{23}\cosh\beta_1$; $T_{30} = H_1\sinh H_1 - hH_1^2\cosh H_1$;

$T_{31} = H_1\cosh H_1 - hH_1^2\sinh H_1$; $T_{32} = -H_1\sinh H_1 + hH_1^2\cosh H_1$.

The shear stress at any point in the fluid is given by $\overline{\tau}_{xy} = \mu\left(\dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x}\right)$. (54)

In non dimensionless form $\tau = \left(\frac{d^2}{\rho\upsilon^2}\right), \bar{\tau}_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$. (55)

The heat transfer coefficient, characterized by Nusselt number (Nu) on the tube boundary is

$$h = -K\frac{\partial T}{\partial y}. \quad (56)$$

The dimensionless mass transfer number corresponding to the Nusselt number is the Sherwood number, written as

$$Sh = \frac{\partial \phi}{\partial y}. \quad (57)$$

Results and discussion

Graphical representation of results is very useful to discuss the physical features presented by the solution. Therefore, the non dimensional velocity, temperature and concentration fields are plotted and carried out for several values of Hartmann number (M), frequency parameter (ω), permeability parameter (D_a), Prandtl number (P_r), Grashof number (G_r), local Grashof number (G_c), chemical reaction parameter (γ), slip parameter (h), heat source/sink parameter (α) and Schmidt number (S_c). **Figure 2** describes the behavior of the velocity for various values of M, γ, D_a, α and G_r. The effect of magnetic field on velocity is depicted in **Figure 2a**. It is observed that the effect of magnetic field is to decrease the value of velocity, because the presence of magnetic field in an electrically conducting fluid introduces a force called the Lorentz force, which acts against the flow if the magnetic field is applied in the normal direction, as in the present problem. Also, it is noted that when h increases from 0 to 0.1 there is a nearly 13 % increase in the velocity value [21]. **Figure 2b** displays the influence of chemical reaction parameter with fixed values of other parameters. It shows that the effect of increasing γ leads to a decrease in fluid velocity. The effect of the permeability parameter on u is illustrated in **Figure 2c**. As anticipated, the increase of permeability parameter reduces the drag force and hence causes the flow velocity to increase. **Figure 2d** displays that with increasing α there is an increase in velocity field. **Figure 2e** displays the effect of the Grashof number (G_r) on the velocity u. It is found that the effect of increasing G_r is to enhance the velocity field as expected. An increase in the Grashof number physically means an increase of the buoyancy force, which supports the flow. The cross velocity v is plotted in **Figure 3** for different values of h and α. It shows that cross velocity increases with an increase of h and α. In **Figure 4** the temperature profile is drawn for different values of α. It is clear that in the presence of heat sources/sinks the temperature profiles are parabolic in nature and also note that the fluid temperature increases with increasing α. **Figure 5** illustrates the behavior of fluid concentration for different values of S_c and γ. **Figure 5a** depicts the behavior of the concentration distribution (ϕ) against y for various values of S_c (= 0.5, 0.6, 0.78, 1 and 2, which corresponds to Hydrogen gas, water vapor, ammonia, carbon dioxide at 25 °C, and ethyl benzene in air, respectively). We find that ϕ is positive and decreases significantly with both S_c and y. The opposite result can be observed in **Figure 5b**, if S_c is replaced by γ. **Figure 6** shows the skin friction profile for different values of S_c, γ and h at both the walls. **Figure 6a** displays the effects of Schmidt number for 2 different values of slip parameter (h = 0 and 0.1).

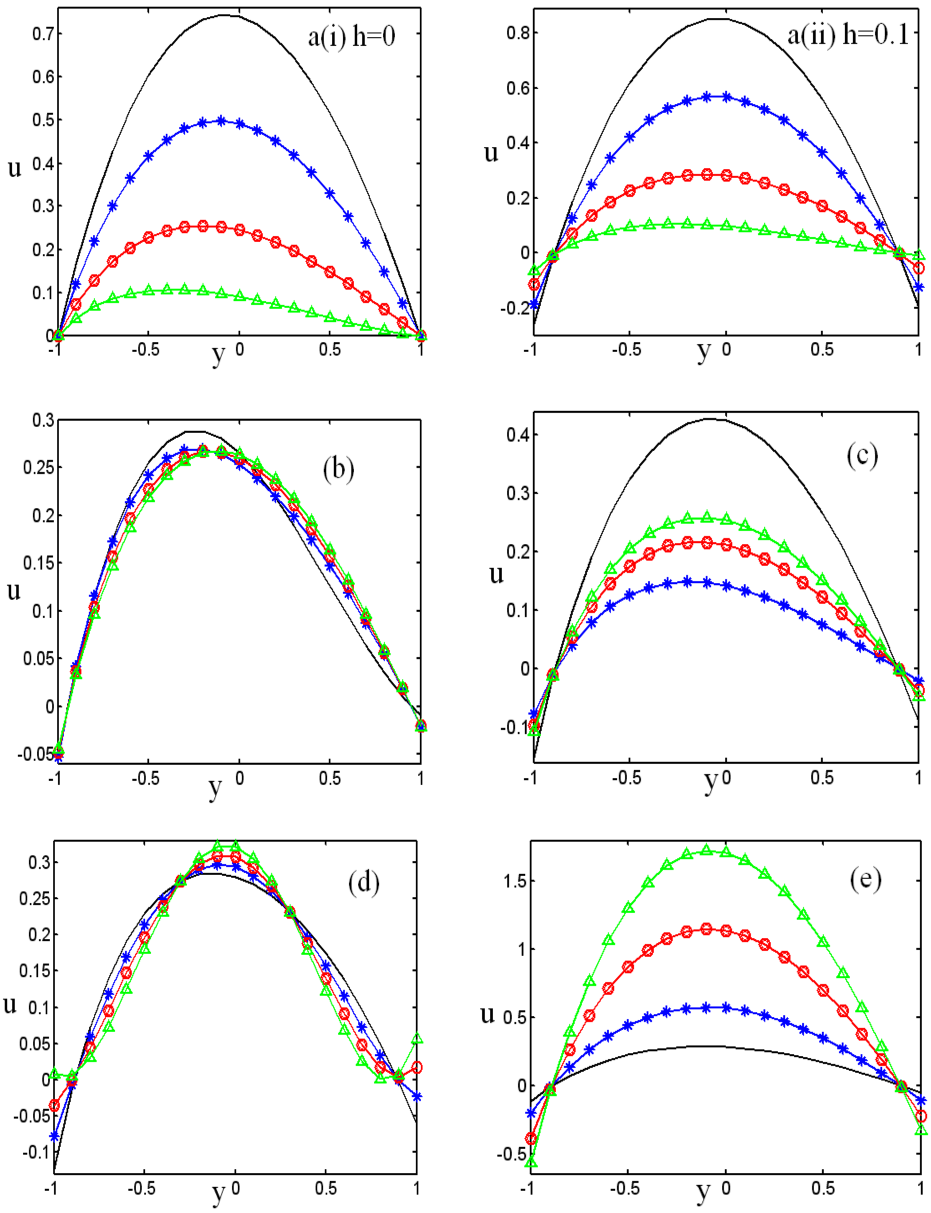

Figure 2 Velocity distribution ($C_1 = 1$, $G_c = 1$, $S_c = 2$, $h = 0.1$, $\theta = \pi/2$, $\lambda = 0.2$, $P_r = 0.71$, $x = 1$, $\omega = 1$, $t = 1$, $\varepsilon = 0.001$)

a: _M = 0, *_M = 0.5, o_M = 1, ^_M = 2, $\gamma = 5$, $D_a = 2$, $\alpha = 0.5$, $G_r = 1$

b: _ $\gamma = -2$, *_ $\gamma = 0$, o_ $\gamma = 5$, ^_ $\gamma = 20$, M = 1, $D_a = 2$, $\alpha = 0.5$, $G_r = 1$

c: _$D_a = \infty$, *_$D_a = 0.5$, o_$D_a = 1$, ^_$D_a = 1.5$, $\gamma = 5$, M = 1, $\alpha = 0.5$, $G_r = 1$

d: _$\alpha = 0$, *_$\alpha = 5$, o_$\alpha = 10$, ^_$\alpha = 15$, $\gamma = 5$, M = 1, $D_a = 2$, $G_r = 1$

e: _ $G_r = 0$, *_ $G_r = 2$, o_ $G_r = 4$, ^_ $G_r = 6$, $\gamma = 5$, M= 1, $D_a = 2$, $\alpha = 0.5$

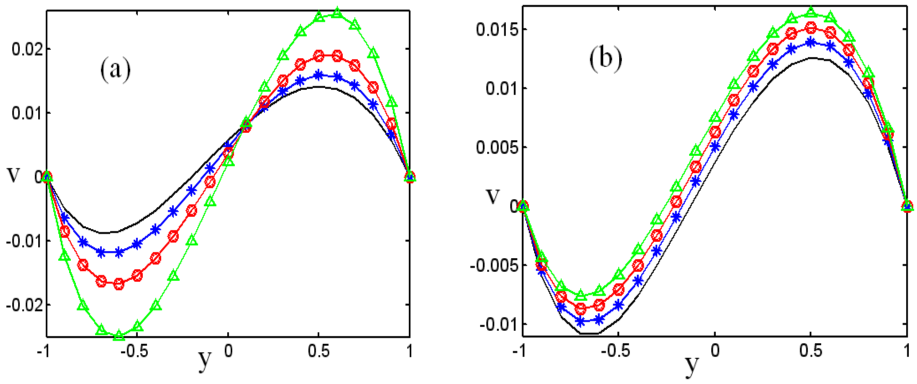

Figure 3 Cross velocity distribution ($\gamma = 0.1,\ C_1 = 1;\ D_a = 0.5,\ G_c = 5;\ G_r = 5, M = 1,\ S_c = 2,\ \theta = \pi/2$, $\lambda = 0,\ P_r = 0.71,\ x = 1,\ \omega = 1,\ t = 1,\ \varepsilon = 0.1$)
a: _h = 0, *_h =0.05, o_h = 0.1, ^_h = 0.15, $\alpha = 2$
b: _$\alpha = -5$, *_$\alpha = 0$, o_$\alpha = 5$, ^_$\alpha = 10$, h = 0.01.

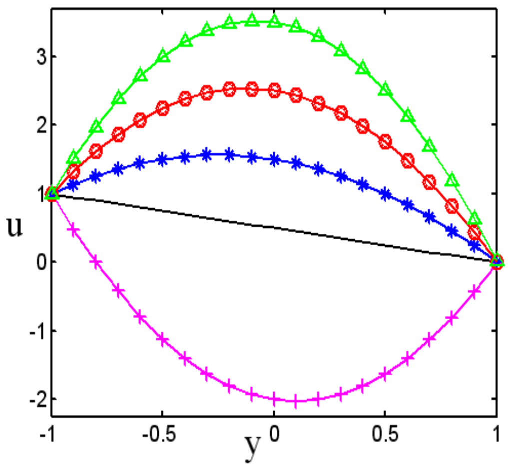

Figure 4 Temperature distribution ($\theta = \pi/2, \lambda = 0.2,\ P_r = 0.71,\ x = 1,\ \omega = 1,\ t = 1,\ \varepsilon = 0.001$)
+_$\alpha = -5$, _$\alpha = 0$, *_$\alpha = 2$, o_$\alpha = 4$, ^_$\alpha = 6$.

From this figure, it is clear that skin friction enhances with an increase of the Schmidt number, while it decreases with an increase of α at the wall $y = -1$ but is reversed at the other wall. Further, it is observed that there is a nearly 45 % increase in skin friction when h rises from 0 to 0.1. The opposite trend is observed for the case of increasing the value of chemical reaction parameter, as shown in **Figure 6b**. **Figure 6c** illustrates the influence of different values of slip parameter. It shows that skin friction decreases by increasing α up to a value (at a constant value of $\alpha = 5$) after which it increases at the wall $y = -1$. The opposite effect can be noticed at the other wall.

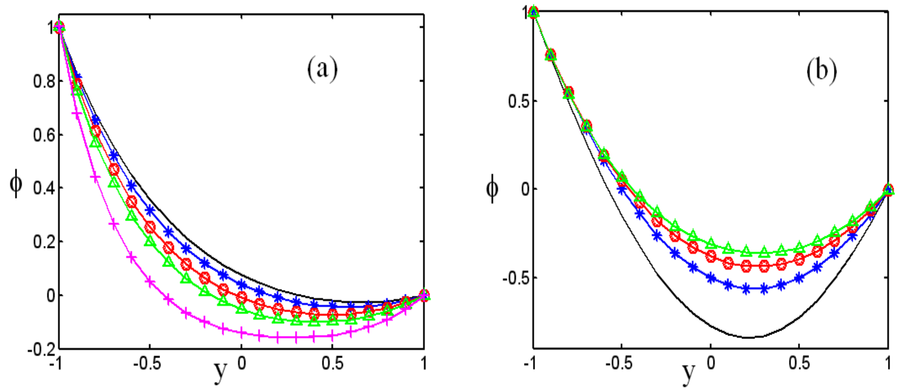

Figure 5 Concentration distribution ($C_1 = 1$, $\theta = 0$, $\lambda = 0.2$, $P_r = 0.71$, $x = 1$, $\omega = 1$, $t = 1$, $\varepsilon = 0.001$)

a: _$S_c = 0.5$, *_$S_c = 0.6$, o_$S_c = 0.78$, ^_$S_c = 1$, +_ $S_c = 2$

b: _$\gamma = -0.5$, *_$\gamma = 0$, o_$\gamma = 0.5$, ^_$\gamma = 1$.

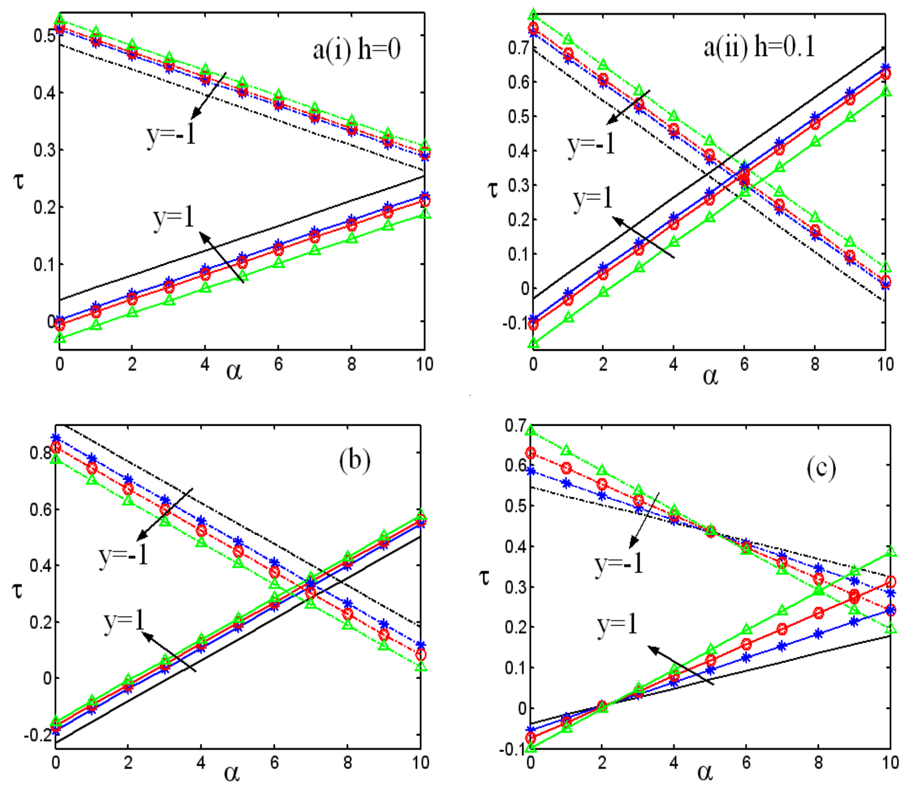

Figure 6 Skin friction distribution ($C_1 = 1$; $D_a = 2$, $G_c = 1$, $G_r = 1$, $M = 2$, $\theta = \pi/2$, $\lambda = 0.2$, $P_r = 0.71$, $x = 1$, $\omega = 1$, $t = 1$, $\varepsilon = 0.001$)

a:_ $S_c = 0.2$, *_ $S_c = 0.78$, o_ $S_c = 1$, ^_ $S_c = 2$, $\gamma = 0.5$, $h = 0.1$

b:_ $\gamma = -0.5$, *_ $\gamma = 0$, o_$\gamma = 0.5$, ^_ $\gamma = 1.5$, $S_c = 2$, $h = 0.1$

c:_$h = 0$, *_$h = 0.02$, o_$h = 0.04$, ^_$h = 0.06$.

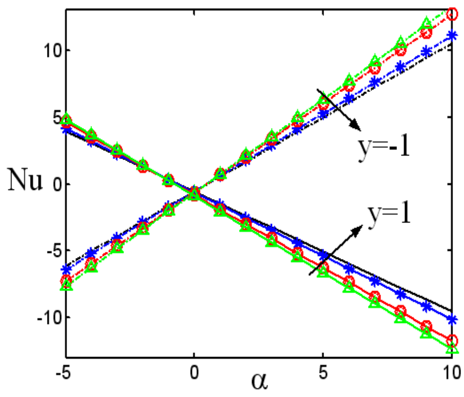

Figure 7 Nusselt number distribution ($\theta = \pi/2$, $\lambda = 0.02$, $x = 1$, $\omega = 5$, $t = 1$, $\varepsilon = 0.1$)
_ $P_r = 0.044$, *_ $P_r = 0.71$, o_ $P_r = 7$, ^_ $P_r = 11.4$.

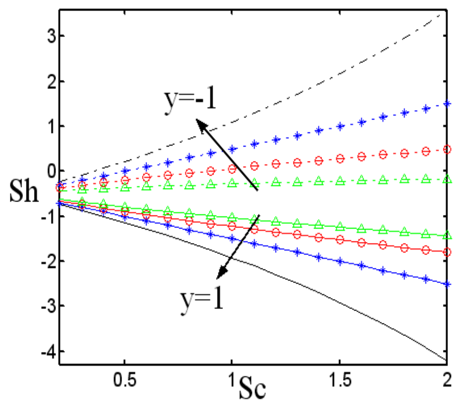

Figure 8 Sherwood number distribution ($\theta = \pi/2$, $\lambda = 0.2$, $P_r = 0.71$, $x = 1$, $\omega = 1$, $t = 1$, $\varepsilon = 0.001$)
_ $\gamma = -0.5$, *_ $\gamma = 0$, o_ $\gamma = 0.5$, ^_ $\gamma = 1$.

Figure 7 depicts the effect various values of P_r (i.e., = 0.044, 0.71, 7 and 11.4, which corresponds to mercury, air, water and water at 4 °C, respectively) on the Nusselt number distribution. It shows that the Nusselt number enhances with an increase in value of P_r on the wall $y = -1$ while it decreases at wall $y = 1$. Also we observe from the same figure that the Nusselt number enhances in the presence of a heat source ($\alpha > 0$) but the opposite is true presence of a heat sink ($\alpha < 0$) [3]. The reverse trend can be observed in the Sherwood number distribution if P_r is replaced by γ (see **Figure 8**).

Conclusions

The problem of MHD mixed convective heat and mass transfer flow in a vertical wavy porous space in the presence of chemical reaction and wall slip with traveling thermal waves has been studied. The dimensionless governing equations are perturbed into a mean (zeroth-order) part and a perturbed part, using amplitude as a small parameter. The perturbed quantities are obtained by perturbation series expansion for small wavelength in which terms of exponential order arise. Analytical solutions have been developed for velocity, temperature and concentration field. The features of the flow characteristics are analyzed by plotting graphs and discussed in detail. The main findings are summarized as follows.

1. The velocity of the fluid increases with an increase of h, D_a, G_r, α while it decreases with S_c, M and γ.

2. Temperature enhances with increasing values of α.

3. Increasing S_c leads to a decrease in the fluid concentration, whereas γ leads to an increase in the fluid concentration.

4. Increasing chemical reaction leads to an increase in the skin friction at the wall y = −1 but the opposite is true at the other wall.

5. The Sherwood number decreases with an increase of S_c at the wall y = −1 while it increases at the other wall.

6. The results of a hydrodynamics case for a non porous space in the absence of chemical reaction can be captured as a limiting case of our analyses by taking M, $\gamma \to 0$ and $D_a \to \infty$.

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