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Analytical Analysis of Peristaltic Flow of a 6 Constant Jeffreys Model of Fluid in an Inclined Planar Channel

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Abstract

In this article we have explored the peristaltic flow of an incompressible six constant Jeffreys model of fluid in an inclined planar channel. The flow is examined in a wave frame of reference moving with the velocity of the wave. The governing equations of 6 constant Jeffreys model of fluid for 2 dimensional flows are first modeled and then simplified under the assumptions of the lubrication approach. The solutions of highly non linear equations are calculated using analytical and numerical techniques. Numerical integration is carried out to calculate the expression of pressure rise and pressure gradient. The graphical results are presented to see the effects of various emerging parameters of interest.

Keywords: Peristaltic flow, six constant Jeffreys model of fluid, inclined planar channel, analytical and numerical solution

Introduction

The study of peristaltic motion has received substantial attention in the last few decades, mainly because of its applications in physiological and engineering fields. In these fields this motion occurs in transport of urine in the ureter, chyme movement in the gastrointestinal tract, movement of ovum in the fallopian tube, in roller and finger pumps, blood pumps in heart lung machines and many others situations. The peristaltic phenomenon was first discussed by Latham [1]. Based on his experimental theory many researchers have examined the phenomenon of peristaltic transport under various conjectures [2-13].

Nowadays, non-Newtonian fluids have become increasingly significant due to their applications in biology, physiology, industry and technology. Some famous examples of non-Newtonian fluids are flows of blood, plasma, nuclear fuel slurries, liquid metals and alloys, mercury amalgams and lubrication with heavy oils and greases etc. There are many models for non-Newtonian fluids, which are classified as rate type and differential type fluids. Due to the intricacy of non-Newtonian fluids, many researchers have discussed the multiplicity of non-Newtonian fluid models for different geometries. Mention may be made in the current works done in peristaltic applications [14-22]. Aristov and Skul'skii [23] discussed the effects of six constant Jeffreys model of fluid for a unidirectional flow. According to them, Maxwell, Dewitt and White-Metzner models as well as three and four constant Oldroyd models can be treated as a special case of the generalized six constant Jeffreys model.

Motivated by the above analyses the aim of the present paper is to discuss the peristaltic flow of a non-Newtonian fluid such as the six constant Jeffreys model of fluid in an inclined planar channel. In a Cartesian coordinate system the governing equations for two dimensional flows are first modeled and then simplified under the assumptions of long wavelength and low Reynolds number approximation. The simplified equations are then solved using analytical techniques. The expression for the pressure rise is computed numerically. The physical features of various physical parameters are discussed and shown graphically.

Mathematical formulation

Let us consider an incompressible 6 constant Jeffreys model of fluid in a 2 dimensional channel of uniform thickness 2a. In the horizontal direction the channel walls are inclined at an angle, Θ. Let the sinusoidal wave promulgates along the channel walls with the constant speed c. For the current flow dilemma we consider the wave shape as follows;

$$Y = H(X,t) = a + b\sin\left[\frac{2\pi}{\lambda}(X - ct)\right] \tag{1}$$

where b is the amplitude of the wave, λ is the wavelength, a is the half width of the channel, c is the velocity of propagation, t is the time and X is the direction of wave propagation. Let the velocity components in a fixed frame of reference (X, Y) be (U,V). Introducing a wave frame (x, y) moving with velocity c away from the fixed frame (X,Y) by the transformation;

$$\mathrm{x = X - ct,\ y = Y,\ u = U - c,\ v = V, and\ \ p(x) = P(X,t)} \cdot \tag{2}$$

The extra stress tensor $\mathbf{S}$ for six constant Jeffreys model of fluid is given by [23];

$$\begin{aligned}&\mathbf{S} + \lambda_1\left[\frac{d\mathbf{S}}{dt} - W\mathbf{S} + \mathbf{S}W + \breve{a}(\mathbf{S}D + D\mathbf{S}) + \breve{b}\mathbf{S} : D\mathbf{I} + \breve{c}Dtr\mathbf{S}\right]\\ &= 2\mu\left[D + \lambda_2\left(\frac{dD}{dt} - WD + DW + 2\breve{a}DD + \breve{b}D : D\mathbf{I}\right)\right]\cdot\end{aligned} \tag{3}$$

In above equation λ_1 is the relaxation time, $D = (\nabla v^t + \nabla v)/2$ is the symmetric part of ∇v, $W = (\nabla v - \nabla v^t)/2$ is the antisymmetric part of ∇v, $\breve{a}, \breve{b}, \breve{c}$ are arbitrary material constants and λ_2 is the delay time. With the help of Eq. (2), the equations governing the flow in the wave frame of reference are given by;

$$\frac{\partial u}{\partial x} + \frac{\partial \mathrm{v}}{\partial y} = 0,$$

$$\rho\left(u\frac{\partial u}{\partial x} + \mathrm{v}\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}(S_{xx}) + \frac{\partial}{\partial y}(S_{xy}) + \rho g\sin[\Theta], \tag{4}$$

$$\rho\left(u\frac{\partial \mathrm{v}}{\partial x} + \mathrm{v}\frac{\partial \mathrm{v}}{\partial y}\right) = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}(S_{yx}) + \frac{\partial}{\partial y}(S_{yy}) - \rho g\cos[\Theta],$$

where ρ is constant density, g is the gravity and P is the pressure.

The components forms of the extra stress tensor are obtained as follows;

$$
\begin{aligned}
&S_{xx}+\lambda_1\left[\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)S_{xx}+S_{xy}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+\breve{a}\left\{2\frac{\partial u}{\partial x}S_{xx}+S_{xy}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right\}\right.\\
&\left.+\breve{b}\left\{\frac{\partial u}{\partial x}S_{xx}+S_{xy}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)+S_{yy}\frac{\partial v}{\partial y}\right\}+\breve{c}\left(S_{xx}+S_{yy}\right)\frac{\partial u}{\partial x}\right]\\
&=2\mu\left[\frac{\partial u}{\partial x}+\lambda_2\left[\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\frac{\partial u}{\partial x}+\frac{1}{2}\left(\left(\frac{\partial v}{\partial x}\right)^2-\left(\frac{\partial u}{\partial y}\right)^2\right)\right.\right.\\
&\left.\left.+2\breve{a}\left\{\left(\frac{\partial u}{\partial x}\right)^2+\frac{1}{4}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2\right\}+\breve{b}\left\{\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\frac{1}{2}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right\}\right]\right],
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
&S_{xy}+\lambda_1\left[\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)S_{xy}+\frac{1}{2}\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)\left(S_{xx}-S_{yy}\right)+\frac{\breve{c}}{2}\left(S_{xx}+S_{yy}\right)\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right.\\
&\left.+\breve{a}\left\{\frac{1}{2}S_{xx}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+S_{xy}\left(\frac{\partial v}{\partial y}+\frac{\partial u}{\partial x}\right)+\frac{1}{2}S_{yy}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right\}\right]\\
&=2\mu\left[\frac{1}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+\lambda_2\left[\frac{1}{2}\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+\frac{1}{2}\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)\left(\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}\right)\right.\right.\\
&\left.\left.+\breve{a}\left\{\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right\}\right]\right],
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
&S_{yy}+\lambda_1\left[\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)S_{yy}+\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)S_{xy}+\breve{a}\left\{S_{xy}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+2S_{yy}\frac{\partial v}{\partial y}\right\}\right.\\
&\left.+\breve{b}\left\{S_{xx}\frac{\partial u}{\partial x}+S_{yx}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+S_{yy}\frac{\partial v}{\partial y}\right\}+\breve{c}(S_{xx}+S_{yy})\frac{\partial v}{\partial y}\right]\\
&=2\mu\left[\frac{\partial v}{\partial y}+\lambda_2\left[\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\frac{\partial v}{\partial y}+\frac{1}{2}\left(\left(\frac{\partial u}{\partial y}\right)^2-\left(\frac{\partial v}{\partial x}\right)^2\right)\right.\right.\\
&\left.\left.+2\breve{a}\left\{\left(\frac{\partial v}{\partial y}\right)^2+\frac{1}{4}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2\right\}+\breve{b}\left\{\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\frac{1}{2}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right\}\right]\right],
\end{aligned}
\tag{7}
$$

Defining the following non dimensional quantities as;

$$\bar{x}=\frac{x}{\lambda},\ \bar{y}=\frac{y}{a},\ \bar{u}=\frac{u}{c},\ \bar{v}=\frac{v}{c\delta},\ \delta=\frac{a}{\lambda},\ \bar{p}=\frac{a^2 p}{\mu c\lambda},\ \bar{t}=\frac{ct}{\lambda},$$
$$h=\frac{H}{a},\ \mathrm{Re}=\frac{\rho ca}{\mu},\ \bar{\Psi}=\frac{\Psi}{cd_1},\ \bar{\lambda}_1=\frac{\lambda_1 c}{d_1},\ \bar{\lambda}_2=\frac{\lambda_2 c}{d_1}, \tag{8}$$
$$\bar{S}_{xx}=\frac{S_{xx}\lambda}{\mu c},\ \ \bar{S}_{xy}=\frac{S_{xy}a}{\mu c},\ \ \bar{S}_{yy}=\frac{S_{yy}\lambda}{\mu c}.$$

With the help of above non-dimensional quantities and the assumption of long wavelength and low Reynolds number the consequential equations in terms of stream function Ψ (dropping the bars, $u=\frac{\partial\Psi}{\partial y}, v=-\frac{\partial\Psi}{\partial x}$) can be written as;

$$\frac{\partial S_{xy}}{\partial y}=\frac{\partial p}{\partial x}, \tag{9}$$

$$\frac{\partial p}{\partial y}=0, \tag{10}$$

$$S_{xy}=\frac{\Psi_{yy}\left(1-\frac{\lambda_1\lambda_2}{2}\alpha\left(\Psi_{yy}\right)^2\right)}{\left(1-\frac{\lambda_1^2}{2}\alpha\left(\Psi_{yy}\right)^2\right)}, \tag{11}$$

where

$$\alpha=-2\left(1-\left(\breve{a}+\breve{c}\right)\left(\breve{a}+\breve{b}\right)\right)\cdot$$

Eliminating pressure from Eq. (9) and (10) we get;

$$\frac{\partial^2 S_{xy}}{\partial y^2}=0\cdot \tag{12}$$

Making use of Eq. (11), Eqs. (9) and (12) this becomes;

$$\frac{\partial^2}{\partial y^2}\left[\frac{\Psi_{yy}\left(1-\frac{\lambda_1\lambda_2}{2}\alpha\left(\Psi_{yy}\right)^2\right)}{\left(1-\frac{\lambda_1^2}{2}\alpha\left(\Psi_{yy}\right)^2\right)}\right]=0, \tag{13}$$

$$\frac{dp}{dx}=\frac{\partial}{\partial y}\left[\frac{\Psi_{yy}\left(1-\frac{\lambda_1\lambda_2}{2}\alpha\left(\Psi_{yy}\right)^2\right)}{\left(1-\frac{\lambda_1^2}{2}\alpha\left(\Psi_{yy}\right)^2\right)}\right]. \tag{14}$$

Applying the binomial expansion with α small, Eqs. (13) and (14) reduce to;

$$\frac{\partial^4\Psi}{\partial y^4}+\alpha A\frac{\partial^2}{\partial y^2}\left(\Psi_{yy}\right)^3+\alpha^2\frac{\partial^2}{\partial y^2}\left(\Psi_{yy}\right)^5 B=0, \tag{15}$$

$$\frac{dp}{dx}=\frac{\partial}{\partial y}\left(\Psi_{yy}+\alpha A\left(\Psi_{yy}\right)^3+\alpha^2\left(\Psi_{yy}\right)^5 B\right), \tag{16}$$

where

$$A=\left(\frac{\lambda_1^2}{2}-\frac{\lambda_1\lambda_2}{2}\right),\quad B=-\frac{\lambda_1^3\lambda_2}{4}\cdot$$

The boundary conditions in terms of stream functions are defined as;

$$\begin{aligned}&\Psi=0,\quad \frac{\partial^2\Psi}{\partial y^2}=0\quad \text{at}\quad y=0,\\ &\Psi=F,\quad \frac{\partial\Psi}{\partial y}=-1\quad \text{at}\quad y=h(x)\cdot\end{aligned} \tag{17}$$

The dimensionless mean flow in laboratory Q and wave F frames are related by the following expressions;

$$Q=F+1, \tag{18}$$

in which

$$F=\int_0^{h(x)}\frac{\partial\Psi}{\partial y}dy=\Psi\left(h(x)\right)-\Psi\left(0\right), \tag{19}$$

where

$$h(x)=1+\phi\sin 2\pi x\cdot \tag{20}$$

Solution of the problem

We employ the well known regular perturbation method to determine the solution of the highly non-linear equation Eq. (15). For the perturbation solution, we expand Ψ, F and p as;

$$\Psi = \Psi_0 + \alpha\Psi_1 + \alpha^2\Psi_2 + O(\alpha^3), \tag{21}$$

$$F = F_0 + \alpha F_1 + \alpha^2 F_2 + O(\alpha^3), \tag{22}$$

$$p = p_0 + \alpha p_1 + \alpha^2 p_2 + O(\alpha^3)\cdot \tag{23}$$

Substituting Eqs. (21) - (23) into Eqs. (15), (16) and the boundary conditions (17), we get the following system.

System of order α^0

$$\frac{\partial^4\Psi_0}{\partial y^4} = 0, \tag{24}$$

$$\frac{\partial p_0}{\partial x} = \frac{\partial^3\Psi_0}{\partial y^3} + \frac{\mathrm{Re}}{Fr}\sin[\Theta], \tag{25}$$

$$\Psi_0 = 0, \quad \frac{\partial^2\Psi_0}{\partial y^2} = 0 \quad \text{at} \quad y = 0, \tag{26}$$

$$\Psi_0 = F_0, \quad \frac{\partial\Psi_0}{\partial y} = -1 \quad \text{at} \quad y = h(x)\cdot \tag{27}$$

System of order α^1

$$\frac{\partial^4\Psi_1}{\partial y^4} = -A\frac{\partial^2}{\partial y^2}\left(\frac{\partial^2\Psi_0}{\partial y^2}\right)^3, \tag{28}$$

$$\frac{\partial p_1}{\partial x} = \frac{\partial^3\Psi_1}{\partial y^3} + A\frac{\partial}{\partial y}\left(\frac{\partial^2\Psi_0}{\partial y^2}\right)^3, \tag{29}$$

$$\Psi_1 = 0, \quad \frac{\partial^2\Psi_1}{\partial y^2} = 0 \quad \text{at} \quad y = 0,, \tag{30}$$

$$\Psi_1 = F_1, \quad \frac{\partial\Psi_1}{\partial y} = 0 \quad \text{at} \quad y = h(x). \tag{31}$$

System of order α^2

$$\frac{\partial^4\Psi_2}{\partial y^4} = -B\frac{\partial^2}{\partial y^2}\left(\frac{\partial^2\Psi_0}{\partial y^2}\right)^5 - 3A\frac{\partial^2}{\partial y^2}\left(\frac{\partial^2\Psi_1}{\partial y^2}\left(\frac{\partial^2\Psi_0}{\partial y^2}\right)^2\right),, \tag{32}$$

$$\frac{\partial p_2}{\partial x} = \frac{\partial^3\Psi_2}{\partial y^3} + 3A\frac{\partial}{\partial y}\left(\frac{\partial^2\Psi_1}{\partial y^2}\left(\frac{\partial^2\Psi_0}{\partial y^2}\right)^2\right) + B\frac{\partial}{\partial y}\left(\frac{\partial^2\Psi_0}{\partial y^2}\right)^5, \tag{33}$$

$$\Psi_2 = \frac{F_2}{2}, \quad \frac{\partial\Psi_2}{\partial y} = 0 \quad on \quad y = h_1(x), \tag{34}$$

$$\Psi_2 = -\frac{F_2}{2}, \quad \frac{\partial\Psi_2}{\partial y} = 0 \quad on \quad y = h_2(x). \tag{35}$$

Solution for system of order α^0

The solution of Eq. (24) satisfying the boundary conditions (26) and (27) can be written as;

$$\Psi_0 = \frac{h^2(3F_0 + h)y - (F_0 + h)y^2}{2h^3}. \tag{36}$$

The axial pressure gradient is defined as;

$$\frac{dp_0}{dx} = -\frac{3(F_0 + h)}{h^3} + \frac{\mathrm{Re}}{Fr}\sin[\Theta]. \tag{37}$$

For one wavelength the integration of Eq. (37), yields;

$$\Delta p_0 = \int_0^1 \frac{dp_0}{dx}dx. \tag{38}$$

Solution for system of order α^1

Substituting the zeroth-order solution (36) into Eq. (28), the solution of the resulting problem satisfying the boundary conditions takes the following form;

$$\Psi_1 = \frac{y(27A(F_0 + h)^3(h^2 - y^2)^2 + 10F_1h^6(3h^2 - y^2))}{20h^9}. \tag{39}$$

The axial pressure gradient is defined as;

$$\frac{dp_1}{dx} = -\frac{3(5F_1h^6 + 27A(F_0 + h)^3(h^2 - 5y^2))}{5h^9} - A\left(\frac{81(F_0 + h)^3 y^2}{h^9}\right). \tag{40}$$

Integrating Eq. (40) over one wavelength we obtain;

$$\Delta p_1 = \int_0^1 \frac{dp_1}{dx}\Big|_{y=0} dx, \tag{41}$$

Solution for system of order α^2

Substituting Eqs. (36) and (39) into Eq. (32), the solution of the resulting problem satisfying the boundary conditions takes the following form;

$$\begin{aligned}\Psi_2 = \frac{-1}{700h^{15}}(y(2835AF_1h^6(F_0+h)^2(h^2-y^2)^2 - 243A^2(F_0+h)^5(h^2-y^2)^2(37h^2+50y^2)\\ +50(7F_2h^{12}(3h^2-y^2)+81B(F_0+h)^5(2h^6-3h^4y^2+y^6)))),\end{aligned} \tag{42}$$

The axial pressure gradient for this order is defined as;

$$\begin{aligned}\frac{dp_2}{dx} = \frac{1}{175h^{15}}(3(-2835AF_1h^6(F_0+h)^2(h^2-5y^2)+243A^2(F_0+h)^5(12h^4+315h^2y^2-875y^4)\\ +25(-7F_2h^{12}+81B(F_0+h)^5(-3h^4+35y^4))))\\ -3A\left(\frac{81(F_0+h)^2y^2(5F_1h^6+3A(F_0+h)^3(9h^2-25y^2))}{5h^{15}}\right)-B\left(\frac{1215(F_0+h)^5y^4}{h^{15}}\right).\end{aligned} \tag{43}$$

Integrating Eq. (43) over one wavelength we obtain;

$$\Delta p_2 = \int_0^1 \frac{dp_2}{dx}\Big|_{y=0} dx, \tag{44}$$

Summarizing the perturbation results up to second order for $\Psi, dp/dx$, and Δp;

$$\Psi = \Psi_0 + \alpha\Psi_1 + \alpha^2\Psi_2, \tag{45}$$

$$\frac{dp}{dx} = \frac{dp_0}{dx} + \alpha\frac{dp_1}{dx} + \alpha^2\frac{dp_2}{dx}, \tag{46}$$

$$\Delta p = \Delta p_0 + \alpha\Delta p_1 + \alpha^2\Delta p_2, \tag{47}$$

Defining;

$$F = F_0 + \alpha F_1 + \alpha^2 F_2. \tag{48}$$

Inserting $F_0 = F - \alpha F_1 - \alpha^2 F_2$ and then neglecting the terms greater than $O(\alpha^2)$, the results given by Eq. (45) to Eq. (47) are expressed up to α^2 in which all Ψ_0 to Ψ_2, $\frac{dp_0}{dx}$ to $\frac{dp_2}{dx}$ and Δp_0 to Δp_2 are as defined previously.

Graphical results and discussion

In this segment we present the graphical results of the proposed problem. Mathematica is used to calculate the expression of the pressure rise and pressure gradient numerically. **Figures 1 - 4** are plotted in order to see the behavior of pressure rise Δp with volume flow rate Q for different values of Θ, α, λ_1 and λ_2. It is observed from **Figure 1** that the pressure rise increases in retrograde pumping $(\Delta p > 0$ and $Q < 0)$, free pumping and peristaltic pumping $(\Delta p > 0$ and $Q > 0)$ regions with an increase in Θ. It is depicted in **Figure 2** that there is no variation of pressure rise in peristaltic pumping $(\Delta p > 0$ and $Q > 0)$, retrograde pumping $(\Delta p > 0$ and $Q > 0)$ and free pumping regions, while in the augmented pumping $(\Delta p < 0$ and $Q > 0)$ region pressure rise increases with an increase in α. **Figure 3** shows the variation of pressure rise with volume flow rate Q for different values of λ_1. It is observed from **Figure 3** that with an increase in λ_1 the pressure rise decreases in retrograde pumping ($\Delta p > 0$ and $Q > 0)$ and increases in the augmented pumping region, while there is no variation in peristaltic $(\Delta p > 0$ and $Q > 0)$ and free pumping region. It is observed from **Figure 4** that pressure rises in the peristaltic pumping $(\Delta p > 0$ and $Q > 0)$, retrograde pumping $(\Delta p > 0$ and $Q < 0)$ and free pumping regions, while in the augmented pumping $(\Delta p < 0$ and $Q < 0)$ region the pressure rise increases with an increase in λ_2. The pressure gradient for different values of α, Θ, λ_1 and λ_2 are shown in **Figures 5 - 8**. It is seen from the figures that for $x \in [0, 0.2]$ and $x \in [0.8, 1]$, the pressure rise is small i.e. the flow can easily pass without the imposition of a large pressure gradient, while in the narrow part of the channel $x \in [0.2, 0.8]$, to retain the same flux a large pressure gradient is required. Moreover in the narrow part of the channel, the pressure gradient increases with an increase in α, Θ, λ_1 and λ_2. In order to see the behavior of velocity for different values of α, relaxation time λ_1, delay time λ_2, and volume flow rate Q **Figures 9 - 12** are plotted. It is observed from **Figures 9 - 11** that as $y \in [0, 0.6]$ and $[0.6, 1]$ the magnitude value of the velocity profile increases with an increase in α, λ_1 and λ_2. It is also observed that near the channel walls the velocity profile increases whereas the maximum velocity occurs at the center of the channel. From **Figure 12** it is observed that with an increase in volume flow rate Q the velocity profile increases.

Trapping phenomena

The trapping phenomena for different values of α, λ_1 and λ_2 are shown in **Figures 13 - 15**. It is observed from **Figures 13** and **14** that the size of the trapped bolus increases with an increase in α and λ_1. It is depicted in **Figure 15** that with an increase in λ_2, the size of the trapped bolus decreases.

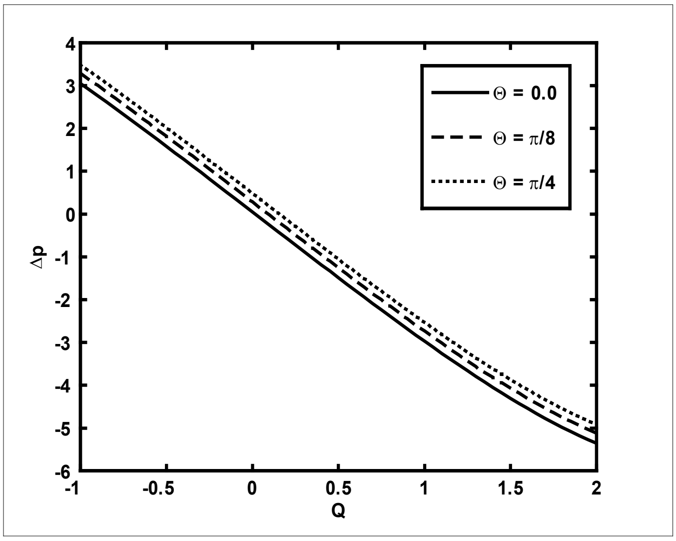


Figure 1 Variation of Δp with Q for different values of Θ at $Fr = 0.8$, $\mathrm{Re} = 0.5$, $\phi = 0.1$, $\alpha = 0.04$, $\lambda_1 = 0.5$, $\lambda_2 = 0.9$.

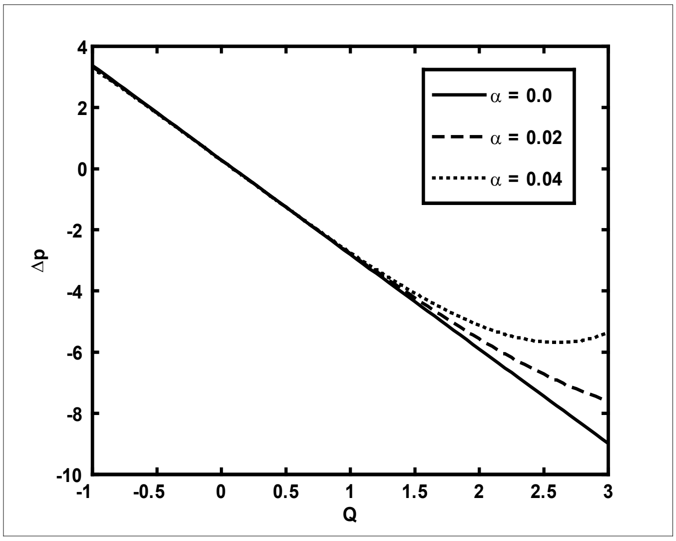


Figure 2 Variation of Δp with Q for different values of α at $Fr = 0.8$, $\mathrm{Re} = 0.5$, $\phi = 0.1$, $\lambda_1 = 0.5$, $\lambda_2 = 0.9$, $\Theta = \frac{\pi}{8}$.

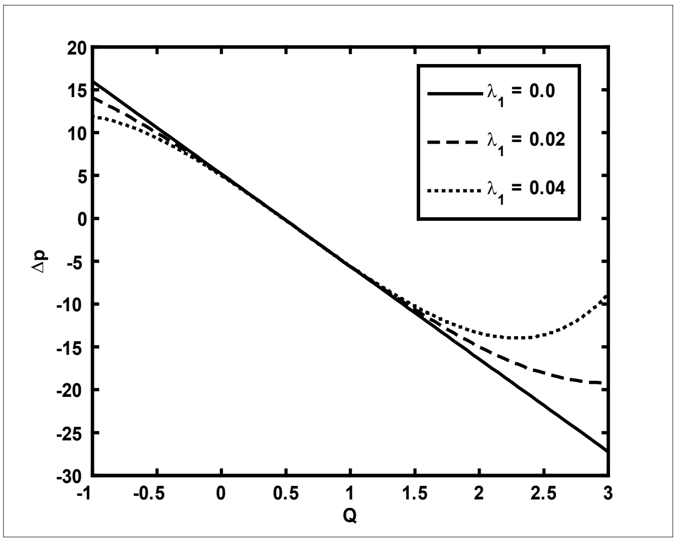


Figure 3 Variation of Δp with Q for different values of λ_1 at $Fr = 0.8$, $\mathrm{Re} = 0.5$, $\phi = 0.6$, $\alpha = 0.04$, $\lambda_2 = 0.9$, $\Theta = \frac{\pi}{8}$.

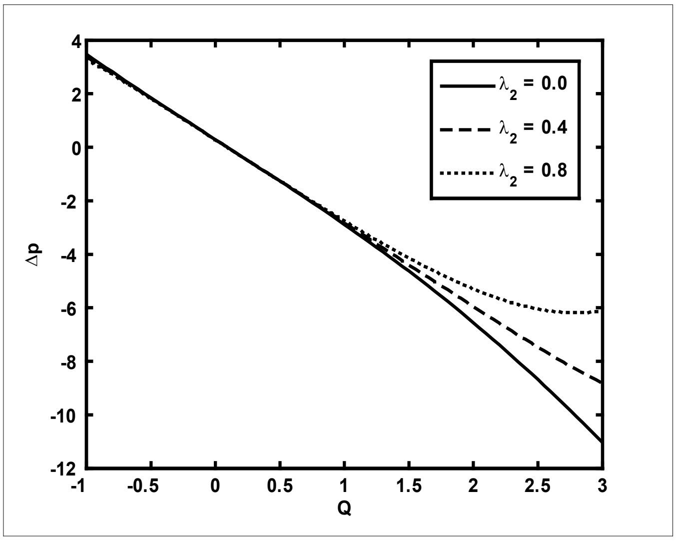


Figure 4 Variation of Δp with Q for different values of λ_2 at $Fr = 0.8$, $\mathrm{Re} = 0.5$, $\phi = 0.1$, $\alpha = 0.04$, $\lambda_1 = 0.5$, $\Theta = \frac{\pi}{8}$.

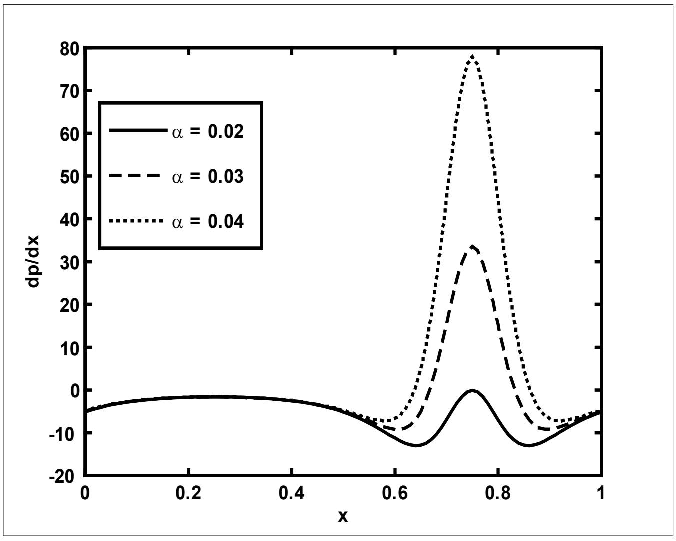


Figure 5 Variation of $\frac{dp}{dx}$ with x for different values of α at $Fr = 0.8$, $\mathrm{Re} = 0.9$, $\phi = 0.5$, $\lambda_1 = 0.5$, $\lambda_2 = 0.9$, $Q = 2$, $\Theta = \frac{\pi}{6}$.

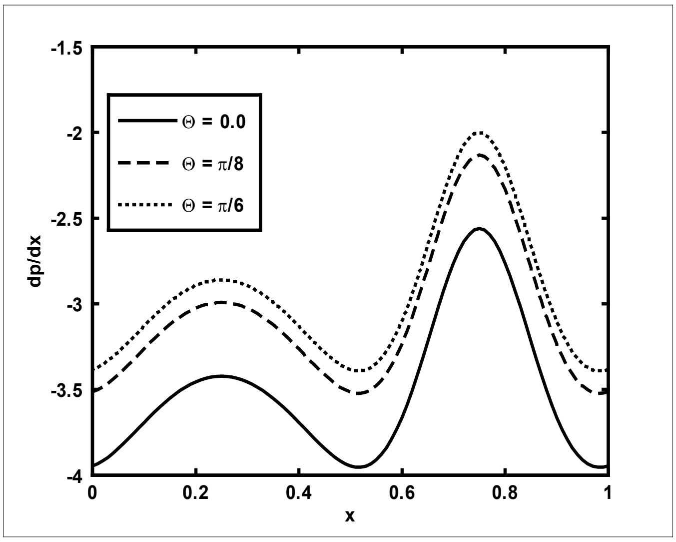


Figure 6 Variation of $\frac{dp}{dx}$ with x for different values of Θ at $Fr = 0.8$, $\mathrm{Re} = 0.9$, $\phi = 0.15$, $\lambda_1 = 0.5$, $\lambda_2 = 0.9$, $Q = 2$, $\alpha = 0.09$.

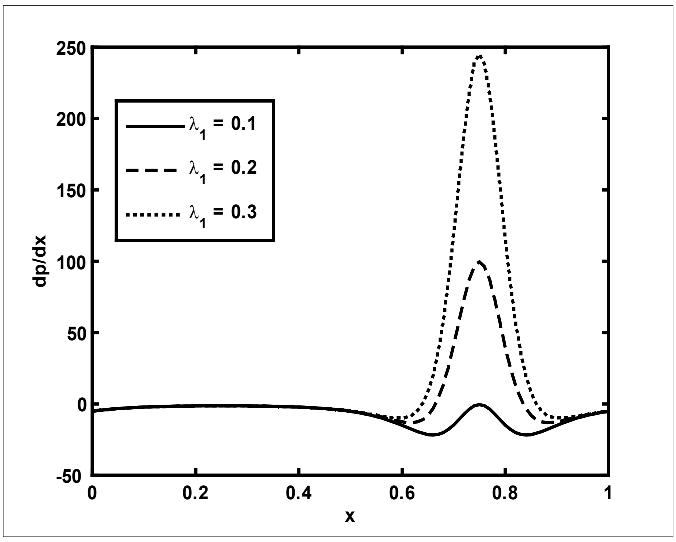


Figure 7 Variation of $\frac{dp}{dx}$ with x for different values of λ_1 at $Fr = 0.8$, $\mathrm{Re} = 0.9$, $\phi = 0.6$, $\Theta = \frac{\pi}{6}$, $\lambda_2 = 0.9$, $Q = 2$, $\alpha = 0.04$.

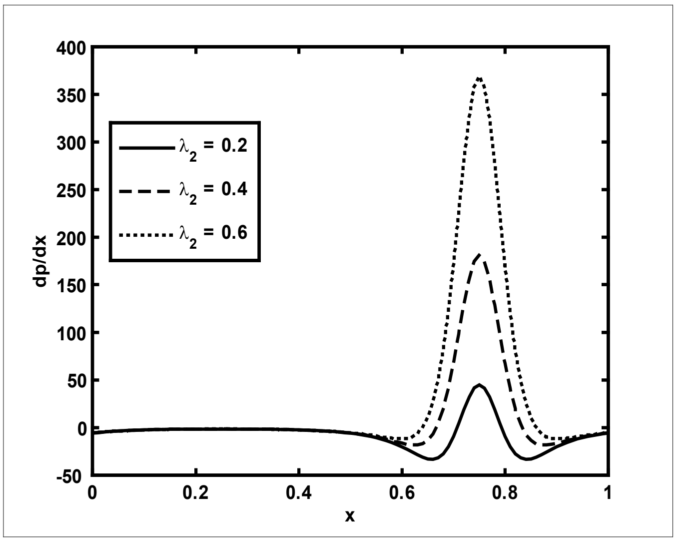


Figure 8 Variation of $\frac{dp}{dx}$ with x for different values of λ_2 at $Fr = 0.8$, $\mathrm{Re} = 0.9$, $\phi = 0.6$, $\Theta = \frac{\pi}{6}$, $\lambda_1 = 0.5$, $Q = 2$, $\alpha = 0.04$.

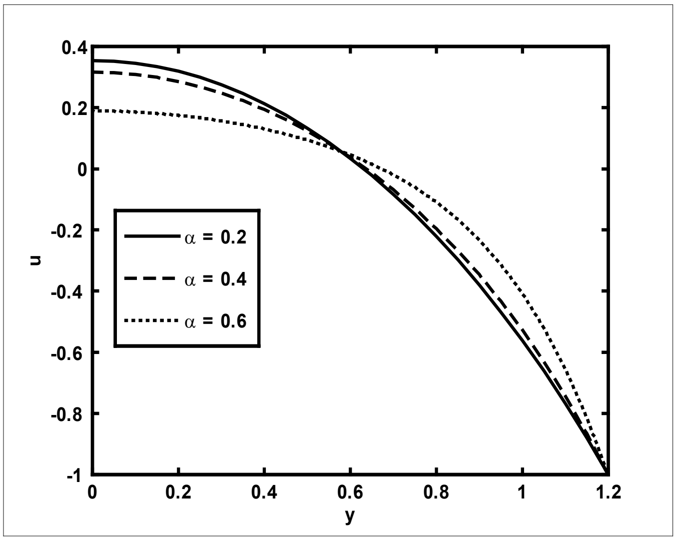


Figure 9 Velocity profile for different values of α at $\phi = 0.2$, $\lambda_1 = 0.6$, $\lambda_2 = 0.9$, $Q = 0.9$, $x = 0.25$.

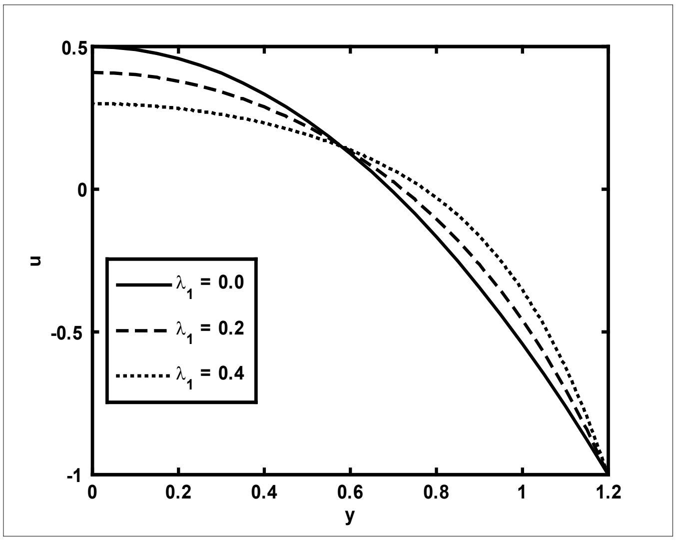


Figure 10 Velocity profile for different values of λ_1 at $\phi = 0.2$, $\alpha = 0.9$, $\lambda_2 = 0.9$, $Q = 1$, $x = 0.25$.

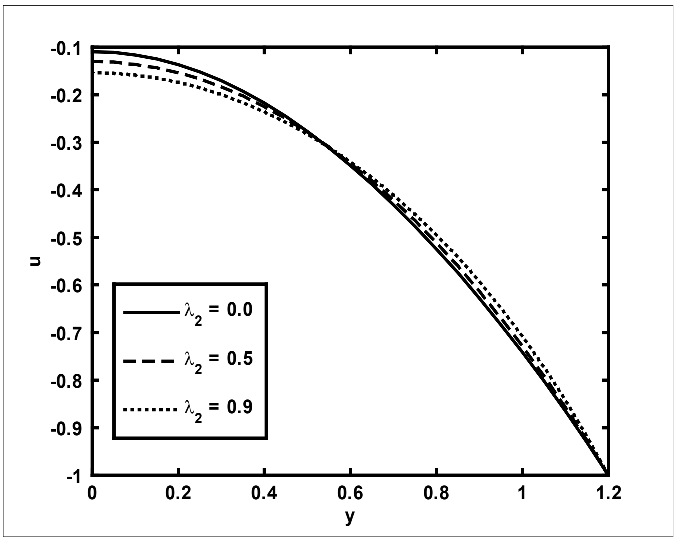


Figure 11 Velocity profile for different values of λ_2 at $\phi = 0.2,\ \ \alpha = 0.9,\ \ \lambda_1 = 0.5,\ \ Q = 0.5,$ $x = 0.25.$

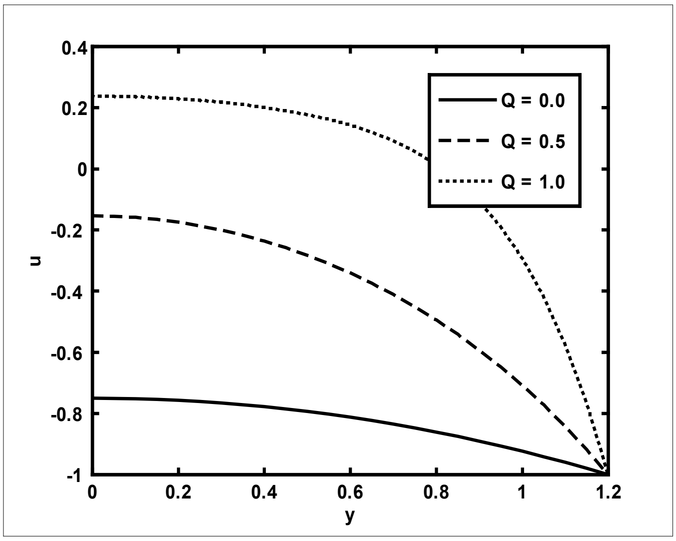
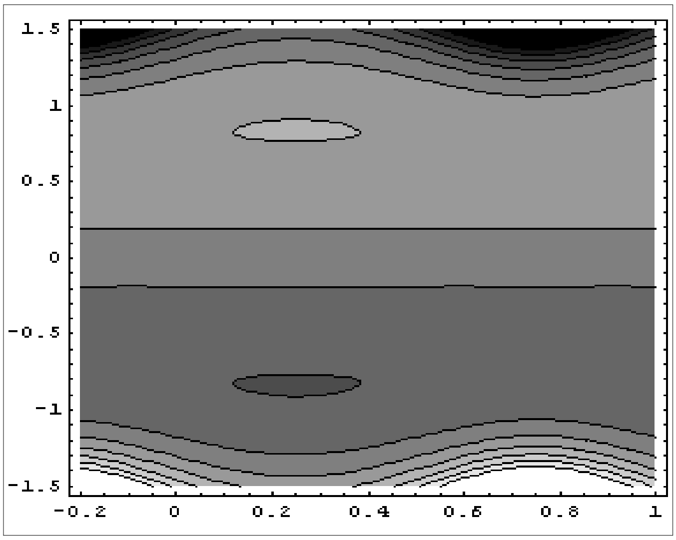
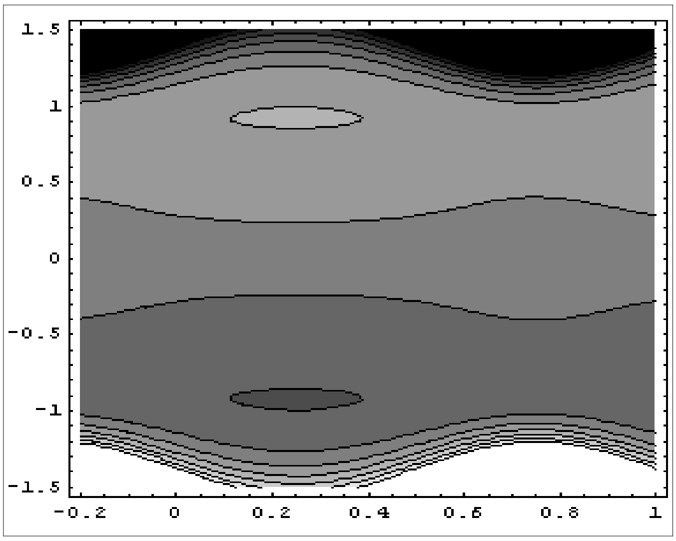


Figure 12 Velocity profile for different values of Q at $\phi = 0.2,\ \ \alpha = 0.9,\ \ \lambda_1 = 0.5,\ \ \lambda_2 = 0.9,$ $x = 0.25.$

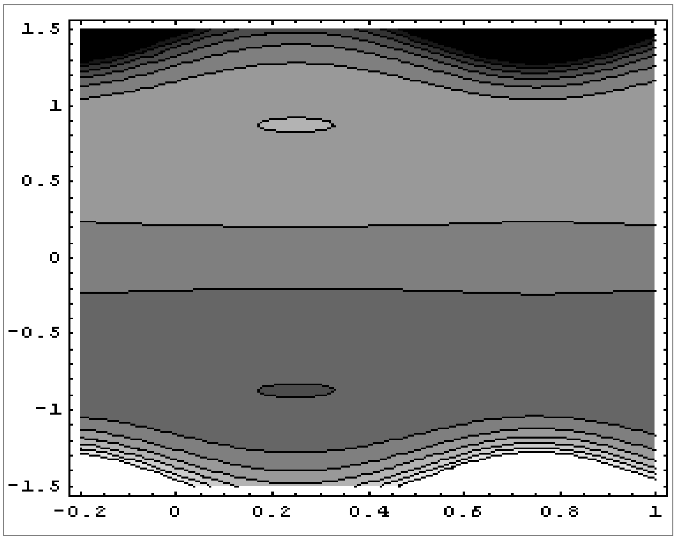


(a)

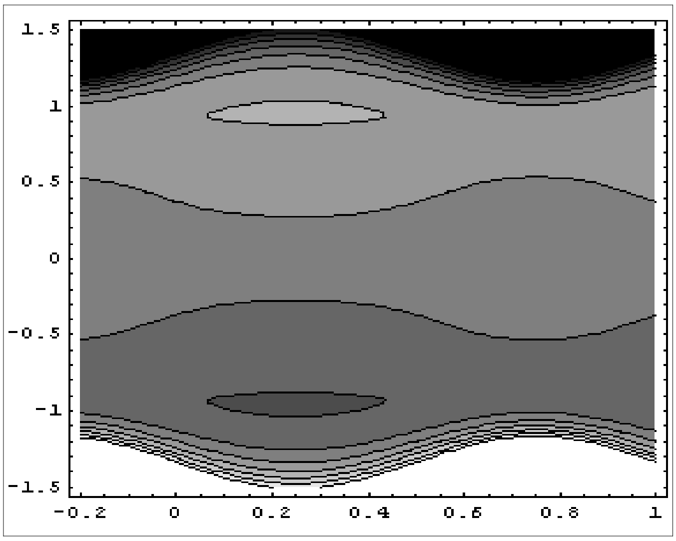


(b)

Figure 13 Stream lines for different values of α. (a) for $\alpha = 0.4,$ (b) for $\alpha = 0.99.$ The other parameters are $\phi = 0.1,\ \lambda_1 = 0.2,\ \lambda_2 = 0.7,\ Q = 1.5.$

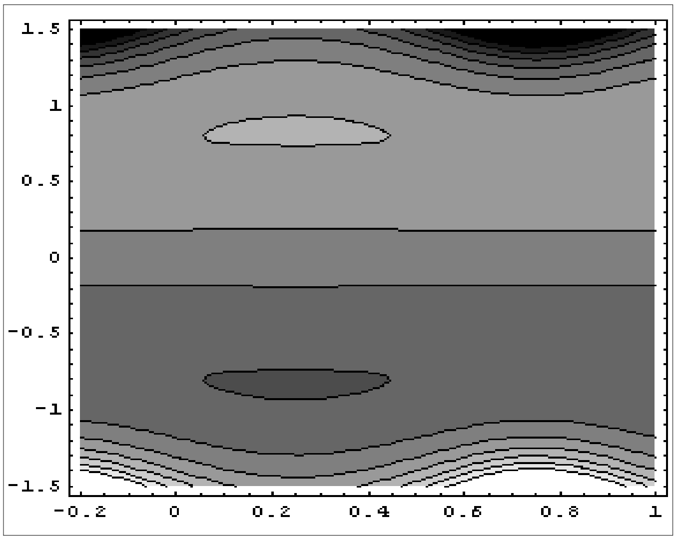


(a)

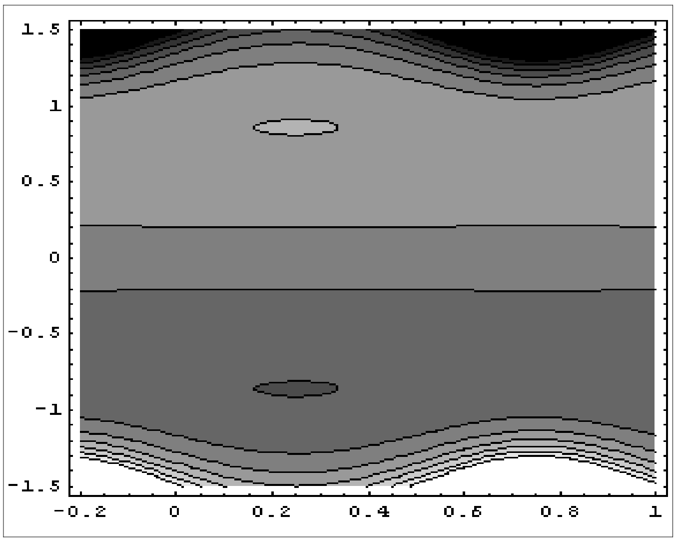


(b)

Figure 14 Stream lines for different values of λ_1. (a) for $\lambda_1 = 0.1,$ (b) for $\lambda_1 = 0.2.$ The other parameters are $\phi = 0.1,\ \alpha = 0.9,\ \lambda_2 = 0.9,\ Q = 1.5.$



(a)



(b)

Figure 15 Stream lines for different values of λ_2. (a) for $\lambda_2 = 0.5,$ (b) for $\lambda_2 = 0.8.$ The other parameters are $\phi = 0.1,\ \alpha = 0.95,\ \lambda_2 = 0.5,\ Q = 1.5.$

Concluding remarks

In the present analysis we have discussed the peristaltic motion of a 6 constant Jeffreys model of fluid in an inclined planar channel. Using the lubrication approach the governing 2 dimensional equations are simplified. An analytical method is used to calculate the solution of the present problem. Graphical results are displayed to see the behavior of various emerging parameters. We conclude with the following interpretations;

1) It is observed that in the peristaltic pumping region the pressure rise increases with an increase in Θ and λ_2, while the behavior is quite opposite in the augmented pumping region.

2) It is observed that there is no variation of pressure rise in the peristaltic pumping, retrograde pumping and free pumping regions, while in the augmented pumping region pressure rise increases with an increase in α.

3) The pressure gradient increases with an increase in α, Θ, λ_1 and λ_2.

4) The magnitude of the velocity profile increases with an increase in α, λ_1, λ_2 and Q.

5) With an increase in α and λ_1, the size of the trapped bolus increases.

6) The size of the trapped bolus decreases with an increase in λ_2.

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