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A Modified Weighted Symmetric Estimator for a Gaussian First-Order Autoregressive Model with Additive Outliers

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Abstract

Guttman and Tiao [1], and Chang [2] showed that the effect of outliers may cause serious bias in estimating autocorrelations, partial correlations, and autoregressive moving average parameters (cited in Chang *et al.* [3]). This paper presents a modified weighted symmetric estimator for a Gaussian first-order autoregressive AR(1) model with additive outliers. We apply the recursive median adjustment based on an exponentially weighted moving average (EWMA) to the weighted symmetric estimator of Park and Fuller [4]. We consider the following estimators: the weighted symmetric estimator ($\hat{\rho}_W$), the recursive mean adjusted weighted symmetric estimator ($\hat{\rho}_{RW}$) proposed by Niwitpong [5], the recursive median adjusted weighted symmetric estimator ($\hat{\rho}_{RDW}$) proposed by Panichkitkosolkul [6], and the weighted symmetric estimator using adjusted recursive median based on EWMA ($\hat{\rho}_{RD-EWMA}$). Using Monte Carlo simulations, we compare the mean square error (MSE) of estimators. Simulation results have shown that the proposed estimator, $\hat{\rho}_{RD-EWMA}$, provides a MSE lower than those of $\hat{\rho}_W$, $\hat{\rho}_{RW}$ and $\hat{\rho}_{RDW}$ for almost all situations.

Keywords: Parameter estimation, autoregressive model, recursive median, exponentially weighted moving average, additive outliers

Introduction

Business and economic time series are sometimes influenced by nonrepetitive and interventions such as strikes, outbreaks of war, and sudden changes in the market structure of a commodity, and so forth [7]. Such observations are usually referred to as outliers. Outliers may have a significant impact on the results of standard methodology for time series analysis [8]. Moreover, outliers are known to wreck havoc on the parameter estimation. It is therefore important to have procedures that will deal such outlier effects. The detection of time series outliers was first studied by Fox [9], who introduced two statistical models for times series contaminated by outliers, namely, additive outliers (AO) and innovations outliers (IO). Additive outliers correspond to the situation in which a gross error of observation or recording affects a single observation [9]. An innovations outlier affects not only the particular observation but also subsequent observations [9]. This paper focuses solely on the additive outliers as they are the most common type found in time series due to their association with human error such as typing and recording mistakes [10]. Furthermore, additive outliers are more harmful than innovations outliers [11,12]. A time series that does not contain any outliers is called an outlier-free series.

Suppose an unobservable outlier-free time series $\{X_t\,; t = 2,3,...,n\}$ follows an AR(1) model:

$$X_t = \mu + \rho(X_{t-1} - \mu) + e_t\,, \tag{1}$$

where μ is the population mean, ρ is an autoregressive parameter, $\rho \in (-1,1)$, e_t are unobservable independent errors and identically $N(0, \sigma_e^2)$ distributed. For $|\rho| = 1$, the model (1) is called the random walk model, otherwise it is called a stationary AR(1) model when $|\rho| < 1$. For ρ close to one or near a non-stationary model, the mean and variance of this model change over time. Let the observed time series be denoted by $\{Y_t\}$. In the simple case when $\{X_t\}$ has a single additive outlier at time point T $(1 < T < n)$, model (1) can be modified as

$$Y_t = X_t + \delta I_t^{(T)}, \tag{2}$$

where δ represents the magnitude of the additive outlier effect and $I_t^{(T)}$ is a time indicator signifying the time occurrence of the additive outlier such that

$$I_t^{(T)} = \begin{cases} 1, & t = T, \\ 0, & t \neq T. \end{cases}$$

It is known that the ordinary least squares estimator of ρ, which is denoted by $\hat{\rho}_{OLS}$, for (1) is biased (see e.g. Shaman and Stine [13]). Therefore, statisticians have suggested methods to reduce the bias. Park and Fuller [4] proposed the weighted symmetric estimator of ρ, which is denoted by $\hat{\rho}_W$. So and Shin [14] applied a recursive mean adjustment to the OLS estimator (abbreviated, ROLS) and they concluded that the mean square error of the ROLS estimator, which is denoted by $\hat{\rho}_{ROLS}$, is smaller than the OLS estimator for $\rho \in (0,1)$. They also showed that the $\hat{\rho}_{ROLS}$ estimator has a coverage probability which is close to the nominal value. Niwitpong [5] applied a recursive mean adjustment to the weighted symmetric estimator of Park and Fuller [4] (abbreviated, RW). Panichkitkosolkul [6] suggested an estimator for an unknown mean Gaussian AR(1) model with additive outliers by applying the recursive median adjustment to the weighted symmetric estimator (abbreviated, RDW). He found that the $\hat{\rho}_{RDW}$ estimator provides a mean square error lower than those of $\hat{\rho}_W$ and $\hat{\rho}_{RW}$ for almost all situations. We, therefore, apply new recursive median adjustment based on an exponentially weighted moving average (see Box *et al.* [15]) to the weighted symmetric estimator (abbreviated, RD-EWMA) for model (1) when there are additive outliers in time series data. Because the outliers do not affect the median values, we replace the recursive mean adjustment by new recursive median adjustment based on an exponentially weighted moving average to the weighted symmetric estimator. Our aim in this paper is to compare four estimators, $\hat{\rho}_W$, $\hat{\rho}_{RW}$, $\hat{\rho}_{RDW}$ and $\hat{\rho}_{RD-EWMA}$, in terms of mean square error (MSE) of estimators.

The paper is organized as follows. In the next section, the details of all estimators $\hat{\rho}_W$, $\hat{\rho}_{RW}$, $\hat{\rho}_{RDW}$ and $\hat{\rho}_{RD-EWMA}$ are described. This is followed by the simulation results obtained from Monte Carlo simulations. A discussion of the results and conclusions are presented in the final section.

Materials and methods

Park and Fuller [4] proposed the weighted symmetric estimator of ρ given by

$$\hat{\rho}_W = \frac{\sum_{t=2}^{n}(Y_t - \bar{Y})(Y_{t-1} - \bar{Y})}{\sum_{t=3}^{n}(Y_{t-1} - \bar{Y})^2 + n^{-1}\sum_{t=1}^{n}(Y_t - \bar{Y})^2}. \tag{3}$$

Niwitpong [5] replaces $\bar{Y}$ by $\bar{Y}_t = \frac{1}{t}\sum_{i=1}^{t} Y_i$ in Eq. (3). The estimator of ρ obtained as a result of this recursive mean adjustment is defined by

$$\hat{\rho}_{RW} = \frac{\sum_{t=2}^{n}(Y_t - \bar{Y}_t)(Y_{t-1} - \bar{Y}_{t-1})}{\sum_{t=3}^{n}(Y_{t-1} - \bar{Y}_{t-1})^2 + n^{-1}\sum_{t=1}^{n}(Y_t - \bar{Y}_t)^2}. \tag{4}$$

When there are outliers in time series data, it affects the recursive mean $\bar{Y}_t$ in Eq. (4). Panichkitkosolkul [6] replaced the recursive mean in Eq. (4) by the recursive median. The estimator of ρ obtained as a result of the recursive median adjustment is

$$\hat{\rho}_{RDW} = \frac{\sum_{t=2}^{n}(Y_t - \tilde{Y}_t)(Y_{t-1} - \tilde{Y}_{t-1})}{\sum_{t=3}^{n}(Y_{t-1} - \tilde{Y}_{t-1})^2 + n^{-1}\sum_{t=1}^{n}(Y_t - \tilde{Y}_t)^2}, \tag{5}$$

where $\tilde{Y}_t = median(Y_1, Y_2, ..., Y_t)$.

To mitigate the effects of outliers on the recursive median values, those recursive median values can be weighted differently. A common weighting scheme that puts more weight on the most recent observations is based on exponentially declining weights and the resulting weighted moving average is called an exponentially weighted moving average (EWMA). Namely, the EWMA gives more weight to recent events and progressively forgets the past [16]. This weighting structure seems reasonable in that the recent past would be a better guide to the immediate future than would the distant past [17]. Hence, we use the concept of the EWMA in order to construct a new recursive median. The improved recursive median values by using EWMA are derived from computing EWMA of the recursive median. There are two steps for computing the new recursive median. Firstly, we compute the recursive median ($\tilde{Y}_t$) by using time series data Y_t. Secondly, we calculate EWMA by using the recursive median obtained from the first step. Therefore, the recursive median in Eq. (5) is replaced by a new recursive median. The new estimator of ρ obtained as a result of this new recursive median adjustment is given by

$$\hat{\rho}_{RD-EWMA} = \frac{\sum_{t=2}^{n}(Y_t - \ddot{Y}_t)(Y_{t-1} - \ddot{Y}_{t-1})}{\sum_{t=3}^{n}(Y_{t-1} - \ddot{Y}_{t-1})^2 + n^{-1}\sum_{t=1}^{n}(Y_t - \ddot{Y}_t)^2}, \quad (6)$$

where

$\ddot{Y}_t = \lambda\sum_{j=1}^{t}(1-\lambda)^{j-1}\tilde{Y}_{t-j+1}$, $\tilde{Y}_t = median(Y_1, Y_2, ..., Y_t)$,

and λ denotes the smoothing constant, $0 < \lambda < 1$. The values of the smoothing constant λ depends on the properties of the given time series. Values between 0.1 and 0.3 are commonly used [18].

In the next section, we present the Monte Carlo simulation results from estimating the mean square error (MSE) of these estimators, $\hat{\rho}_W$, $\hat{\rho}_{RW}$, $\hat{\rho}_{RDW}$ and $\hat{\rho}_{RD-EWMA}$.

Results and discussion

In this section we examine, via Monte Carlo simulations, the performance of the proposed estimator for a Gaussian AR(1) model with additive outliers, with particular emphasis on comparisons between the new and existing approaches. We random the times of occurrence of the additive outlier; T $(1 < T < n)$. Hence, the additive outliers occurred randomly. In addition, the number of additive outliers is equal to $[np]$ where $[\cdot]$ is the nearest integer function. Data are generated from a Gaussian AR(1) model with additive outliers by using Eq. (2). The following parameter values were used; (μ, σ_e^2) = (0, 1); ρ = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9; sample sizes n = 25, 50, 100 and 250; the magnitude of the AOs effect $\delta = 3\sigma_e$ and $5\sigma_e$; the percentage of additive outliers p = 5 and 10 %; the smoothing constant $\lambda = 0.2$. All simulations were performed using programs written in the R statistical software [19-21] with the number of simulation runs, $M = 10,000$.

Our simulation results are shown in **Tables 1** and **2**. The following information is presented here: the estimated mean square error (MSE) of all estimators, $\hat{\rho}_W$, $\hat{\rho}_{RW}$, $\hat{\rho}_{RDW}$ and $\hat{\rho}_{RD-EWMA}$. As can be seen from **Tables 1** and **2**, the MSE of $\hat{\rho}_W$ is larger than the MSEs of the other estimators for all situations especially when ρ is close to one and for small sample sizes. These values decrease as sample sizes get larger. The $\hat{\rho}_W$ performs well for $n \geq 50$. On the other hand, the new estimator, $\hat{\rho}_{RD-EWMA}$, provides the lowest MSE in all scenarios that we considered except when the parameter ρ is small $(\rho = 0.1)$ for small sample sizes ($n = 25$ and/or $n = 50$). Additionally, $\hat{\rho}_{RD-EWMA}$ performs very well with respect to the other three estimators. The proposed estimator, $\hat{\rho}_{RD-EWMA}$ in (6), dominates all estimators since the MSE of the proposed estimator is the lowest for almost all cases. For the rest, the MSE of $\hat{\rho}_{RDW}$ is less than that of $\hat{\rho}_{RW}$ and $\hat{\rho}_W$ for all situations. The $\hat{\rho}_{RDW}$ often ranks the second best following the proposed estimator. Furthermore, the MSEs shown in **Table 1** are less than those reported in **Table 2** because the time series data of **Table 1** have less outliers shown in **Figure 1**.

Conclusions

A modified weighted symmetric estimator for a Gaussian AR(1) model with additive outliers has been proposed in this paper. This proposed estimator of ρ is obtained by applying the recursive median adjustment based on an exponentially weighted moving average (EWMA) to the weighted symmetric estimator. New adjusted recursive median values are derived from computing the EWMA of the recursive median. Furthermore, the weighted symmetric estimator ($\hat{\rho}_W$), the recursive mean adjusted weighted symmetric estimator ($\hat{\rho}_{RW}$), the recursive median adjusted weighted symmetric estimator ($\hat{\rho}_{RDW}$) and the new estimator ($\hat{\rho}_{RD-EWMA}$) are compared in this study. The new estimator, $\hat{\rho}_{RD-EWMA}$, performs better than $\hat{\rho}_W$, $\hat{\rho}_{RW}$, and $\hat{\rho}_{RDW}$ in terms of the MSE for almost all scenarios. One reason behind this is that the additive outliers do not affect the median and EWMA values. Moreover, the adjusted recursive median values applied in the formula for $\hat{\rho}_{RD-EWMA}$ in Eq. (6) can also reduce the mean square error (MSE) of the estimator. Therefore, the proposed estimator ($\hat{\rho}_{RD-EWMA}$) which is based on the recursive median adjusted by EWMA is superior to the existing estimators.

Finally, let us mention a problem for further research, which goes beyond the scope of the present paper but is of practical interest. In practice, a statistician or an economist has one time series set that is contaminated by various kinds of outliers (i.e., additive outliers (AO) and innovations outliers (IO)). Thus, it would be interesting to see whether, in this context, the proposed approach still maintains an edge over the other methodologies. Moreover, this paper concentrates on a point estimator. Therefore, it would be interesting to study the confidence intervals for the parameter of a Gaussian AR(1) model with additive outliers, and this is left as a topic for future work.

Table 1 The estimated mean square error (MSE) of $\hat{\rho}_W$, $\hat{\rho}_{RW}$, $\hat{\rho}_{RDW}$ and $\hat{\rho}_{RD-EWMA}$ when the percentage of additive outliers; p = 5 %.

n	ρ	$\delta = 3\sigma_e$				$\delta = 5\sigma_e$			
		W	RW	RDW	RD-EWMA	W	RW	RDW	RD-EWMA
25	0.1	0.0415	0.0369	0.0347	0.0368	0.0403	0.0352	0.0321	0.0327
	0.2	0.0488	0.0419	0.0392	0.0391	0.0551	0.0468	0.0434	0.0409
	0.3	0.0588	0.0492	0.0460	0.0430	0.0777	0.0656	0.0620	0.0552
	0.4	0.0685	0.0568	0.0540	0.0474	0.1019	0.0871	0.0833	0.0714
	0.5	0.0824	0.0692	0.0667	0.0552	0.1320	0.1138	0.1111	0.0934
	0.6	0.0958	0.0811	0.0793	0.0634	0.1615	0.1407	0.1382	0.1132
	0.7	0.1028	0.0877	0.0872	0.0661	0.1907	0.1680	0.1674	0.1336
	0.8	0.1114	0.0970	0.0967	0.0702	0.2114	0.1884	0.1875	0.1451
	0.9	0.1166	0.1042	0.1054	0.0686	0.2180	0.1961	0.1964	0.1394
50	0.1	0.0212	0.0197	0.0190	0.0195	0.0230	0.0211	0.0195	0.0190
	0.2	0.0246	0.0221	0.0208	0.0200	0.0336	0.0301	0.0273	0.0247
	0.3	0.0317	0.0282	0.0262	0.0235	0.0496	0.0447	0.0416	0.0360
	0.4	0.0380	0.0337	0.0316	0.0268	0.0672	0.0608	0.0580	0.0488
	0.5	0.0450	0.0402	0.0385	0.0310	0.0881	0.0807	0.0772	0.0643
	0.6	0.0476	0.0427	0.0412	0.0322	0.1055	0.0970	0.0947	0.0778
	0.7	0.0508	0.0462	0.0452	0.0344	0.1194	0.1108	0.1086	0.0874
	0.8	0.0496	0.0457	0.0448	0.0333	0.1189	0.1110	0.1091	0.0863
	0.9	0.0435	0.0410	0.0406	0.0291	0.1046	0.0985	0.0971	0.0735
100	0.1	0.0115	0.0108	0.0102	0.0100	0.0136	0.0127	0.0112	0.0105
	0.2	0.0155	0.0143	0.0131	0.0119	0.0242	0.0225	0.0195	0.0173
	0.3	0.0200	0.0184	0.0169	0.0144	0.0402	0.0377	0.0335	0.0293
	0.4	0.0257	0.0238	0.0221	0.0183	0.0589	0.0558	0.0514	0.0447
	0.5	0.0313	0.0291	0.0275	0.0223	0.0779	0.0742	0.0698	0.0606
	0.6	0.0353	0.0331	0.0318	0.0255	0.0958	0.0916	0.0877	0.0756
	0.7	0.0356	0.0337	0.0326	0.0257	0.1031	0.0989	0.0958	0.0815
	0.8	0.0323	0.0308	0.0298	0.0232	0.0967	0.0930	0.0904	0.0753
	0.9	0.0226	0.0219	0.0213	0.0163	0.0713	0.0690	0.0671	0.0547
250	0.1	0.0051	0.0049	0.0046	0.0044	0.0074	0.0071	0.0057	0.0053
	0.2	0.0078	0.0074	0.0066	0.0058	0.0163	0.0157	0.0129	0.0117
	0.3	0.0118	0.0112	0.0102	0.0088	0.0297	0.0288	0.0251	0.0228
	0.4	0.0162	0.0156	0.0144	0.0123	0.0453	0.0441	0.0400	0.0365
	0.5	0.0204	0.0196	0.0186	0.0159	0.0627	0.0613	0.0572	0.0524
	0.6	0.0227	0.0220	0.0211	0.0179	0.0748	0.0733	0.0695	0.0632
	0.7	0.0227	0.0221	0.0213	0.0180	0.0797	0.0782	0.0756	0.0682
	0.8	0.0179	0.0175	0.0170	0.0142	0.0695	0.0683	0.0664	0.0591
	0.9	0.0102	0.0101	0.0098	0.0081	0.0407	0.0401	0.0389	0.0340

Table 2 The estimated mean square error (MSE) of $\hat{\rho}_W$, $\hat{\rho}_{RW}$, $\hat{\rho}_{RDW}$ and $\hat{\rho}_{RD-EWMA}$ when the percentage of additive outliers; p = 10 %.

n	ρ	$\delta = 3\sigma_e$				$\delta = 5\sigma_e$			
		W	RW	RDW	RD-EWMA	W	RW	RDW	RD-EWMA
25	0.1	0.0434	0.0387	0.0364	0.0371	0.0439	0.0387	0.0337	0.0331
	0.2	0.0552	0.0469	0.0427	0.0410	0.0667	0.0576	0.0496	0.0457
	0.3	0.0727	0.0613	0.0564	0.0503	0.0992	0.0856	0.0764	0.0677
	0.4	0.0925	0.0781	0.0722	0.0625	0.1387	0.1212	0.1110	0.0967
	0.5	0.1109	0.0946	0.0890	0.0741	0.1827	0.1620	0.1506	0.1297
	0.6	0.1364	0.1178	0.1128	0.0919	0.2315	0.2073	0.1965	0.1676
	0.7	0.1536	0.1338	0.1300	0.1027	0.2809	0.2522	0.2429	0.2031
	0.8	0.1667	0.1470	0.1445	0.1096	0.3141	0.2841	0.2731	0.2229
	0.9	0.1764	0.1581	0.1562	0.1084	0.3345	0.3033	0.2950	0.2270
50	0.1	0.0242	0.0221	0.0200	0.0198	0.0269	0.0247	0.0198	0.0189
	0.2	0.0335	0.0299	0.0262	0.0237	0.0447	0.0407	0.0316	0.0284
	0.3	0.0461	0.0412	0.0363	0.0313	0.0732	0.0672	0.0543	0.0478
	0.4	0.0623	0.0563	0.0509	0.0426	0.1086	0.1006	0.0859	0.0754
	0.5	0.0786	0.0716	0.0660	0.0542	0.1487	0.1392	0.1232	0.1078
	0.6	0.0927	0.0850	0.0796	0.0648	0.1908	0.1792	0.1640	0.1423
	0.7	0.1021	0.0944	0.0895	0.0715	0.2238	0.2110	0.1971	0.1693
	0.8	0.1027	0.0957	0.0921	0.0723	0.2379	0.2249	0.2128	0.1793
	0.9	0.0906	0.0854	0.0833	0.0630	0.2188	0.2069	0.1984	0.1614
100	0.1	0.0131	0.0122	0.0110	0.0106	0.0161	0.0151	0.0111	0.0105
	0.2	0.0205	0.0190	0.0162	0.0143	0.0320	0.0300	0.0215	0.0194
	0.3	0.0311	0.0289	0.0252	0.0215	0.0579	0.0550	0.0428	0.0384
	0.4	0.0440	0.0412	0.0371	0.0314	0.0893	0.0855	0.0708	0.0639
	0.5	0.0568	0.0537	0.0492	0.0415	0.1239	0.1193	0.1033	0.0932
	0.6	0.0665	0.0631	0.0592	0.0495	0.1569	0.1516	0.1364	0.1225
	0.7	0.0700	0.0668	0.0639	0.0530	0.1820	0.1761	0.1631	0.1453
	0.8	0.0642	0.0615	0.0593	0.0483	0.1840	0.1781	0.1684	0.1472
	0.9	0.0467	0.0452	0.0438	0.0350	0.1447	0.1401	0.1342	0.1142
250	0.1	0.0063	0.0060	0.0049	0.0046	0.0095	0.0091	0.0052	0.0049
	0.2	0.0128	0.0122	0.0098	0.0087	0.0240	0.0232	0.0148	0.0136
	0.3	0.0222	0.0214	0.0183	0.0162	0.0474	0.0463	0.0342	0.0319
	0.4	0.0334	0.0324	0.0289	0.0258	0.0768	0.0753	0.0606	0.0570
	0.5	0.0442	0.0431	0.0396	0.0354	0.1083	0.1065	0.0910	0.0856
	0.6	0.0517	0.0504	0.0476	0.0425	0.1382	0.1361	0.1217	0.1143
	0.7	0.0526	0.0515	0.0493	0.0437	0.1563	0.1541	0.1423	0.1327
	0.8	0.0448	0.0439	0.0426	0.0372	0.1494	0.1472	0.1399	0.1288
	0.9	0.0252	0.0249	0.0241	0.0208	0.1007	0.0992	0.0954	0.0862

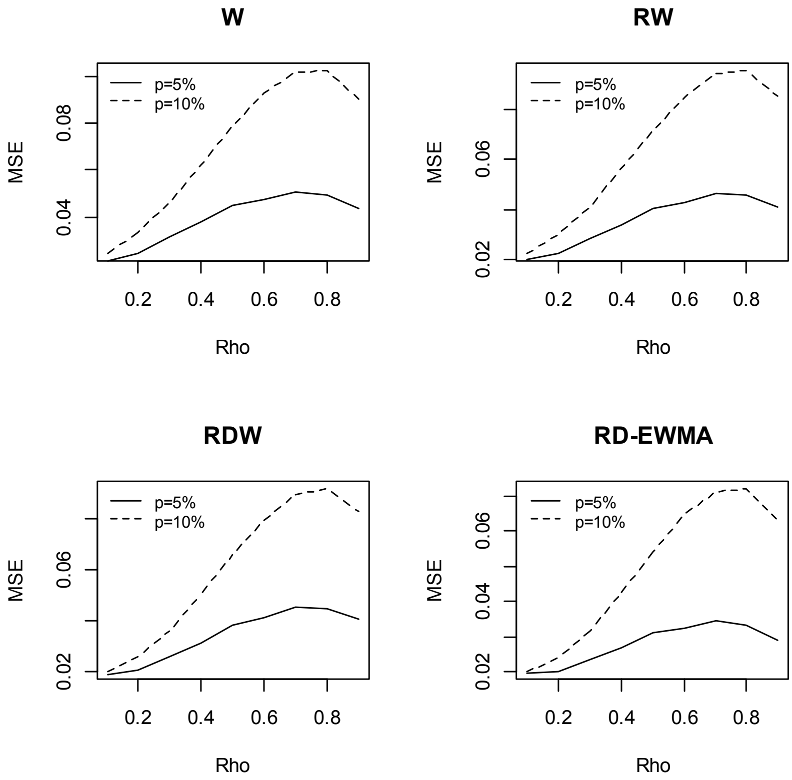

Figure 1 The estimated mean square error (MSE) of $\hat{\rho}_W$, $\hat{\rho}_{RW}$, $\hat{\rho}_{RDW}$ and $\hat{\rho}_{RD-EWMA}$ when $n = 50$ and $\delta = 3\sigma_e$.

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References

[1] I Guttman and GC Tiao. Effect of correlation on the estimation of a mean in the presence of spurious observations. *Can. J. Stat.* 1978; **6**, 229-47.

[2] I Chang. 1982, Outliers in Time Series, Ph. D. Dissertation. University of Wisconsin-Madison, USA.

[3] I Chang, GC Tiao and C Chen. Estimation of time series parameters in the presence of outliers. *Technometrics* 1988; **30**, 193-204.

[4] HJ Park and WA Fuller. Alternative estimators and unit root tests for the autoregressive process. *J. Time Ser. Anal.* 1995; **16**, 415-29.

[5] S Niwitpong. 2007, Predictive Inference for Time Series, Ph. D. Dissertation. La Trobe University, Australia.

[6] W Panichkitkosolkul. New estimator for an unknown mean Gaussian AR(1) process with additive outliers. *Chiang Mai J. Sci.* 2010; **37**, 14-20.

[7] B Abraham and J Ledolter. *Statistical Methods for Forecasting*. John Wiley & Sons, New York, 1983.

[8] H Louni. Outlier detection in ARMA models. *J. Time Ser. Anal.* 2005; **29**, 1057-65.

[9] AJ Fox. Outliers in time series. *J. Roy. Stat. Soc. B.* 1972; **34**, 350-63.

[10] A Zaharim, R Rajali, RM Atok, K Ibrahim and AM Razali. A study on the nature of an additive outlier in ARMA(1,1) models. *Eur. J. Sci. Res.* 2009; **32**, 362-8.

[11] I Chang, and GC Tiao. *Estimation of time series parameters in the presence of outliers*. Technical Report 8, Statistics Research Center, University of Chicago, 1983.
[12] C Chatfield. *Time-Series Forecasting*. Chapman & Hall, New York, 2001.
[13] P Shaman and RA Stine. The bias of autoregressive coefficient estimators. *J. Am. Stat. Assoc*. 1988; **83**, 842-8.
[14] BS So and DW Shin. Recursive mean adjustment in time series inferences. *Stat. Probabil. Lett.* 1999; **43**, 65-73.
[15] GEP Box, GM Jenkins and GC Reinsel. *Time Series Analysis: Forecasting and Control*. Prentice-Hall, New Jersey, 1994.
[16] PM Berthouex and LC Brown. *Statistics for Environmental Engineers*. Lewis Publishers, Florida, 2002.
[17] A Pankratz. *Forecasting with Univariate Box-Jenkins Models: Concepts and Cases*. John Wiley & Sons, New York, 1983.
[18] C Chatfield. *The Analysis of Time Series: An Introduction*. Chapman & Hall, New York, 2003.
[19] R Ihaka and R Gentleman. R: A language for data analysis and graphics. *J. Comput. Graph. Stat.* 1996; **5**, 299-314.
[20] The R Development Core Team. *An Introduction to R*. R Foundation for Statistical Computing, Vienna, 2010a.
[21] The R Development Core Team. *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, 2010b.