Direction of Arrival Estimation using MUSIC, ESPRIT and Maximum-Likelihood Algorithms for Antenna Arrays

Mohammed Amine IHEDRANE[1,2,*] and Seddik BRI[1,2]

[1] *Materials and Instrumentations, Department of Electrical Engineering, High School of Technology, Moulay Ismail University, Meknes, Morocco*
[2] *Spectrometry Laboratory of Materials and Archaeomaterials, Faculty of Science, Moulay Ismail University, Meknes, Morocco*

([*]Corresponding author's e-mail: amine.ihedrane@gmail.com)

Received: 14 July 2015, Revised: 19 November 2015, Accepted: 20 December 2015

Abstract

The main objective of this article is to compare the performance of 3 famous Eigen structure algorithms, known as the Multiple Signal Classification (MUSIC), the Estimation of Signal Parameter via Rotational Invariance Techniques (ESPRIT), and non-subspace method Maximum-Likelihood Estimation (MLE) for Direction of Arrival (DOA).The performance of this DOA estimation algorithm is based on Uniform Linear Array (ULA). A number of simulation results were carried out using MATLAB and were compared with experimental ones. The comparison shows that the MUSIC algorithm is more accurate and stable compared to the ESPRIT and MLE algorithms.

Keywords: ULA, MUSIC, ESPRIT, MLE, DOA

Introduction

In the last few decades, accurate determination of direction of arrival (DOA) from a signal source has received a lot of attention in military communication, radar systems, and commercial applications. Wireless communication, radio astronomy, sonar, radar, navigation, and the tracking of various objects are a few examples of the many applications. One example of a defense application is to identify the direction of possible threats [1].

DOA estimation uses antenna arrays. It is known that antenna radiation main lobe beam width is inversely proportional to the number of elements in the antenna. So, if we consider a single antenna, then the array pattern will be wider, and the resolution will not be good. Instead of using single antenna, an antenna array system is used in DOA estimation, which will improve the resolution of the received signals (resolution in DOA estimation is the ability to distinguish 2 signals arriving at different angles). An array system has multiple elements distributed in space.

There are various methods available to use to estimate the angle of arrival (DOA) of radio signals on an antenna array. DOA estimation techniques can be broadly divided into 3 different categories, namely, conventional methods, subspace based methods, and maximum likelihood methods. Convolution methods are based on the concepts of beam forming and null steering, but require a large number of elements to provide high resolution. Examples of this method are delay and sum and Capon's minimum variance method [2].

One major limitation of this method is poor resolution in its ability to separate closely spaced signals. Unlike conventional methods, subspace methods exploit the information of the received data, resulting in high resolution. Two main subspace based algorithms are Multiple Signal Classification and Estimation of Signal Parameters via Rotational Invariance Techniques.

The DOA algorithms are classified as quadratic (non subspace) type and subspace type. The Bartlett and Capon (Minimum Variance Distortion less Response) are quadratic type algorithms. Both methods are highly dependent on the physical size of the array aperture, which results in poor resolution and accuracy. Subspace based DOA estimation method is based on the Eigen decomposition. The subspace based DOA estimation algorithms MUSIC and ESPRIT provide high resolution; they are more accurate, and not limited to the physical size of the array aperture [3].

These algorithms give information about the number of incident signals and the DOA of each signal. Maximum Likelihood method is one of the first techniques to be investigated for DOA estimation, but has the drawback of intensive computational complexity [4].

In this paper, we present a DOA estimation procedure for M uncorrelated signals impinging on a uniform linear array of N elements using high resolution 'MUSIC, ESPRIT' subspace methods and non-subspace Maximum Likelihood method. We have analysed the performances of the proposed algorithms (number of antenna elements, number of snapshots and spacing between elements) and compared them with the published measure.

Materials and methods

An array antenna is an essential part of a communication system. It can be used to exploit the spatial and spectral characteristics of incoming signals to provide highly accurate location information. Before implementing such a system, a simulation step should be carried out, in order to optimize its efficiency. In this study, we use an array antenna with a 4 element uniform linear array. **Figure 1** shows the general configuration for an array antenna, having N elements arranged along a straight line with a distance d between sensor elements. The angle of the incoming signal, θ_M, is determined relative to the antenna bore sight.

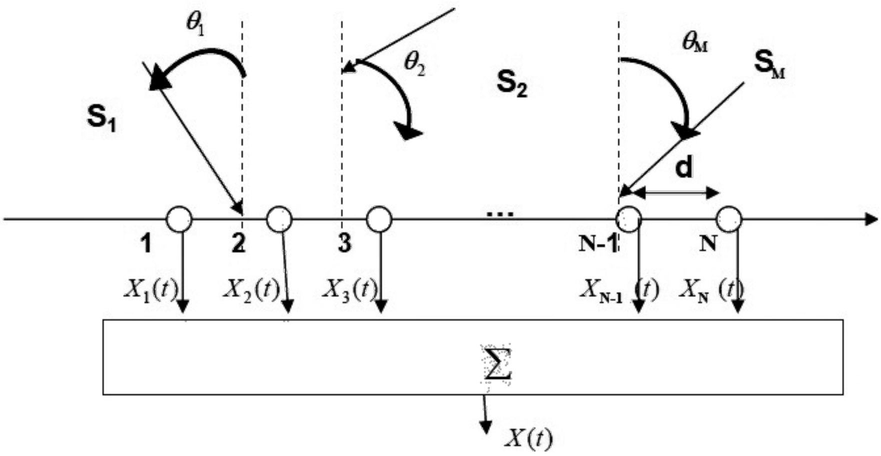

Figure 1 N linear element array with M signals.

Mathematical model for MUSIC algorithm

Multiple Signal Classification (MUSIC) method [5] is widely used in signal processing applications for DOA [6]. In estimation, it is applied to only narrow band signal sources, i.e., frequencies of interest are narrowband [3-7]. Consider M number of narrow band signal sources arriving from different angles θ_i = 1, 2…M, impinging on a uniform linear array of N equispaced array elements (where N > M), as shown in **Figure 1**. At different instances of time t, t = 1, 2 … K, where K is the number of snapshots, the array output will consist of a signal, along with noise components [5].

We choose a signal source S(t) impinging on the array with an angle θ. If the received signal at the first element is $x_1(t) = s(t)$, then the delay at element i is;

$$\Delta i \quad = \frac{(i-1)d \sin\theta}{c} \tag{1}$$

The received signal at sensor i is;

$$xi(t) = e^{-j\omega\Delta i} S1(t) = e^{-j\omega\Delta i}\, S1(t) = e^{-\frac{j\omega(i-1)d \sin\theta}{c}} S1(t) \tag{2}$$

The received signal at N elements due to a single source is;

$$X(t) = \left[1, e^{-\frac{j\omega d \sin\theta}{c}}, e^{-\frac{j\omega 2d \sin\theta}{c}}, \ldots, e^{-\frac{j\omega(N-1)d \sin\theta}{c}}\right] S(t) = \, a(\theta)S(t) \tag{3}$$

If there are M sources, the signals received at the array is given by;

$$X = AS + W \tag{4}$$

$$A = [a(\theta 1), a(\theta 2), \ldots a(\theta M)] \tag{5}$$

$$S = [s1(t), s2(t), \ldots sM(t)]^T \tag{6}$$

where *a (θ)* denotes a steering vector and $[\ ]^T$... denote Transposition of matrix *S*. *A* is an (N×M) matrix of the *M* steering vectors, and *S* is an (N×M) matrix of the *M* signal source vector.

The correlation matrix of received vectors can be written as;

$$R = E[XX^H] \tag{7}$$

$$= E[ASS^H A^H] + E[WW^H] \tag{8}$$

$$= \, AVA^H + \sigma^2 \tag{9}$$

where σ^2 is the variance of white Gaussian noise vector *W*, $[\ ]^H$ denote Hermitian matrix (conjugate transposition of noise vector *W* and signal vector *X)*, V is the covariance matrix of signal vector (S), which is a full rank matrix of order M×M, given by;

$$V \; = E[SS^H] \tag{10}$$

$$= \begin{bmatrix} E[|S_1|^2] & \cdots & 0 \\ 0 & E[|S_2|^2] & 0 \\ 0 & 0 & E[|S_M|^2] \end{bmatrix} \tag{11}$$

where the statistical expectation is denoted by E [ ], and R_S is a signal covariance matrix of order (N×N), with rank M given by;

$$R_s \; = \begin{bmatrix} E[|S_1|^2] & \cdots & \ldots & 0 & \ldots & 0 \\ 0 & E[|S_2|^2] & \ldots & 0 & \ldots & 0 \\ \vdots & \ddots & \ldots & \vdots & .. & 0 \\ 0 & 0 & \ldots & E[|S_M|^2] & .. & 0 \\ 0 & 0 & \ldots & 0 & \ldots & 0 \end{bmatrix} \tag{12}$$

So, R_S, has N-M eigenvectors, corresponding to zero eigenvalues. We know that steering vector $a(\theta 1)$, which is in the signal subspace, is orthogonal to noise subspace; let Q_n be such an eigenvector.

$$R_S\, Q_n = AVA^H Q_n = 0 \tag{13}$$

$$Q_n{}^H AVA^H Q_n = 0 \tag{14}$$

Since V is a positive definite matrix;

$$AVA^H Q_n = 0 \tag{15}$$

$$a^H(\theta_i) Q_n = 0 \tag{16}$$

This implies that signal steering vectors are orthogonal to eigenvectors corresponding to noise subspace. So, the MUSIC algorithm searches through all angles, and plots the spatial spectrum.

$$P_{MUSIC}(\theta) = \frac{1}{a^H(\theta_i) Q_n} \tag{17}$$

Assume the number of signals, M, is known. Given the data set X (k), k = 1, 2. . . K, the MUSIC algorithm proceeds as per the following steps shown in **Figure 2**.

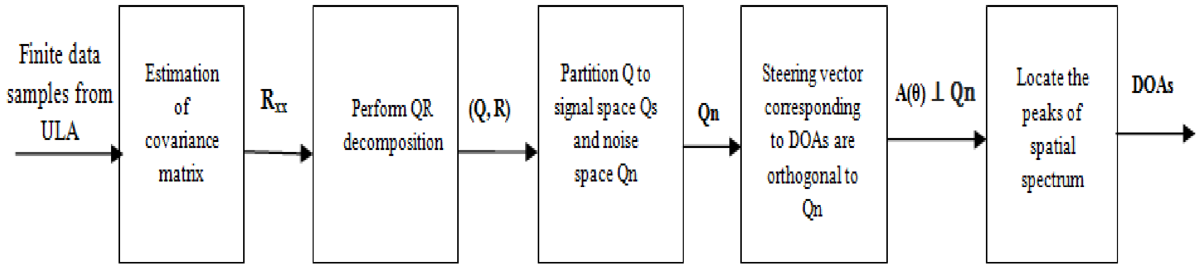

Figure 2 Steps for MUSIC algorithm.

Mathematical model for ESPRIT algorithm

ESPRIT's acronym stands for Estimation of Signal Parameter via Rotational Invariance Technique. This algorithm is more robust with respect to array imperfections than MUSIC [8-10]. Computation complexity and storage requirements are lower than MUSIC, as it does not involve extensive searching throughout all possible steering vectors. However, it explores the rotational invariance property in the signal subspace created by 2 subarrays derived from the original array with a translation invariance structure [11].

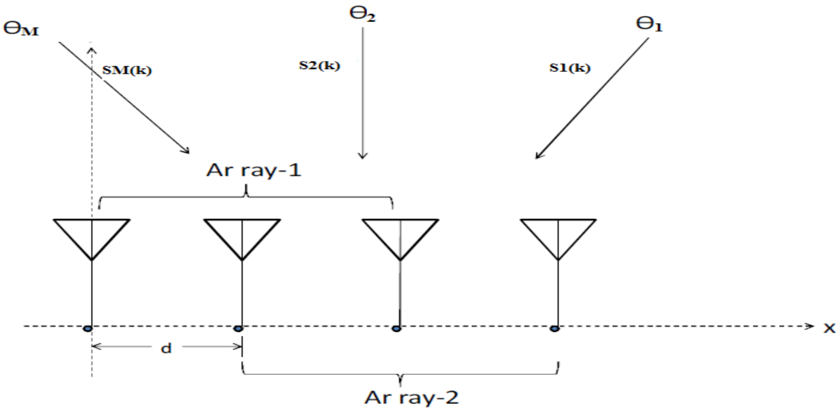

Figure 3 Uniform linear antenna arrays with M incident signals.

It is based on the array elements placed in identical displacement forming matched pairs, with N array elements, resulting in m = N/2 array pairs called "doublets" show in **Figure 3** [12].

Computation of signal subspace for the 2 subarrays, Sub_array-1 and Sub_array-2, are displaced by distance d. The signals induced on each of the arrays are given by;

$$x_1(k) = A_1 * s(k) + n_1(k) \tag{18}$$

$$x_2(k) = A_1 * \Lambda * s(k) + n_2(k) \tag{19}$$

where $\Lambda = \text{diag}[e^{jkd\sin(\theta_1)}, e^{jkd\sin(\theta_2)}, \dots, e^{jkd\sin(\theta_D)}]$
(D×D) diagonal unitary matrix with phase shifts between doublets for DOA.

Creating the signal subspace for the 2 subarrays results in 2 matrices, V1 and V2. Since the arrays are translationally related, the subspaces of the eigenvectors are related by a unique non-singular transformation matrixφ, such that [6];

$$V_1\varphi = V_2 \tag{20}$$

There must also exist a unique non-singular transformation matrix T, such as;

$$V_1 = AT \text{ and } V_2 = A\Lambda T \tag{21}$$

And, finally, we can derive;

$$T\varphi T^{-1} = \Lambda \tag{22}$$

Thus, the eigenvalues of φ must be equal to the diagonal elements of Λ, such that;

$$\lambda 1 = e^{jkd\sin(\theta_1)}, \lambda 2 = e^{jkd\sin(\theta_2)}, \dots \dots, \lambda M = e^{jkd\sin(\theta_M)}$$

Once the eigenvalues of ϕ, λ1, λ2... λ_M are calculated, we can estimate the angles of arrivals as;

$$\theta_i = \sin^{-1}\left(\frac{\arg(\lambda_i)}{kd}\right) \tag{23}$$

Clearly the ESPRIT eliminates the search procedure and produces the DOA estimation directly in terms of the eigenvalues without many computational and storage requirements. This Eigen structure method has shown excellent accuracy and resolution in many experimental and theoretical studies. The ESPRIT algorithm proceeds as per the following steps shown in **Figure 4**.

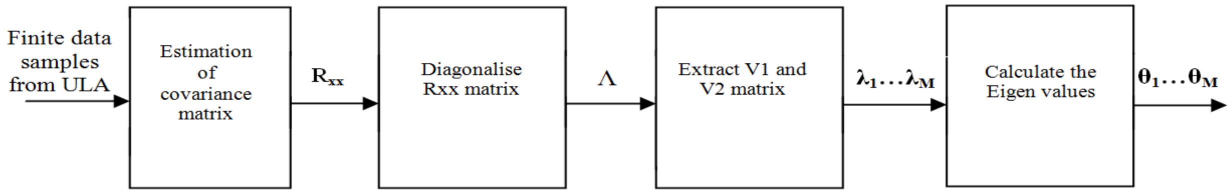

Figure 4 Steps for ESPRIT algorithm.

Mathematical Model for Maximum-Likelihood Estimation Algorithm

This method depends on spatial spectrum [13]. DOAs are obtained as locations of peaks in the spectrum. The concept of localisation is simple, but offers modest or poor performance in terms of resolution [14]. One of the main advantages of these techniques is that it can be used in situations where we lack information about properties of the signal [15].

The estimate is derived by finding the steering vector A, which minimizes the beam energy AVA^H subject to the constraint $EA^H = 1$.

$$F = AVA^H + \alpha(EA^H - 1) \tag{24}$$

When the gradients of A and A^H are evaluated, they are found to be complex conjugates of each other. Setting one of them to zero results in the solution;

$$A = -\alpha V^{-1}/2 \tag{25}$$

The quantity α is determined from the constraint $EA^H = 1$. Hence;

$$A = R^{-1}E(E^H V^{-1} E)^{-1} \tag{26}$$

Thus, the power spectrum in the beam is given by;

$$P(\theta) = AVA^H \tag{27}$$

$$= (E^H V^{-1} E)^{-1} \tag{28}$$

As expected, the peaks of P(θ) correspond to the direction of arrival of the given signal. Hence, the following algorithm steps:

- Collect the data samples X
- Estimate the correlation matrix R
- Estimate the number of signals
- Evaluate P (θ).

Results and discussion

A comparative study [16-18] has been made between MUSIC, ESPRIT and MLE algorithms for DOA estimation, using the MATLAB software tool. We analyzed the performance of these algorithms by varying a number of parameters relating to antenna arrays, such as the number of array elements N, spacing between the array elements d, and the number of snapshots taken at any time. In this simulation, we have considered the M number of stationary signal sources impinging on the number of uniform linear array elements, which are equispaced with a separation of λ/2. We also considered the randomly generated symbols for each of the signal sources with equal magnitudes. The noise is assumed to be additive white Gaussian having unit variance. Simulations have been done for 3 signals arriving from different angles $\theta_1 = -30°$, $\theta_2 = 30°$, and $\theta_3 = 60°$, and our algorithm spatially searched through angles from –90° to 90°.

Table 1 DOA estimation (k = 1024, d = 0.5λ).

N	θ_{in}(°)	θ_{MUSIC}°	Δ_{MUSIC}	θ_{ESPRIT}°	Δ_{ESPRIT}	θ_{MLE}°	Δ_{MLE}
4	-30	-30.2	-0.2	-29.8	+0.2	-29.5	+0.5
	30	30.2	0.2	29.6	-0.4	31.6	+1.6
	60	59.9	-0.1	59.4	-0.8	55.4	-4.6
6	-30	-30.1	-0.1	-29.9	+0.1	-29.9	+0.1
	30	30	0	30.1	0.1	30	0
	60	59.8	-0.2	60.1	-0.2	60.4	+0.4
8	-30	-29.9	+0.1	-29.8	+0.2	-30.1	-0.1
	30	30	0	30.2	0.2	29.9	0.1
	60	60	0	60	0	60	0
10	-30	-30	0	-30	0	-30	0
	30	30	0	29.8	-0.2	30.2	+0.2
	60	60	0	59.9	-0.1	60	0
12	-30	-30	0	-29.8	+0.2	-30	0
	30	30	0	30.1	0.1	29.9	-0.1
	60	60	0	59.9	-0.1	60	0

Table 2 DOA estimation (k = 128, d = 0.5λ).

N	θ_{in}(°)	θ_{MUSIC}(°)	Δ_{MUSIC}	θ_{ESPRIT}(°)	Δ_{ESPRIT}	θ_{MLE}(°)	Δ_{MLE}
4	-30	-30.6	-0.6	-30.8	-0.8	-30.4	-0.4
	30	29.4	-0.6	31.5	+1.5	32.6	+2.6
	60	58.6	-1.4	55.1	-5.9	56	-4
6	-30	-29.9	+0.1	-30.9	-0.9	-30.1	-0.1
	30	30	0	28.8	-1.2	30.1	+0.1
	60	59.9	-0.1	61	+1	60.1	+0.1
8	-30	-30	0	-30.1	-0.1	-30.2	-0.2
	30	30.2	0.2	30.1	+0.1	30	0
	60	59.6	-0.4	59.7	-0.3	60.1	0.1
10	-30	-30	0	-29.9	+0.1	-29.9	+0.1
	30	30.1	0.1	30	0	30.1	+0.1
	60	59.9	-0.1	60.5	+0.5	60	0
12	-30	-30	0	-30.2	-0.2	-30.1	-0.1
	30	30	0	30	0	30	0
	60	59.9	-0.1	60.6	+0.6	59.9	-0.1

Table 3 DOA estimation (k = 1024, d = 0.25λ).

N	$\theta_{in}(°)$	$\theta_{MUSIC}(°)$	Δ_{MUSIC}	$\theta_{ESPRIT}(°)$	Δ_{ESPRIT}	$\theta_{MLE}(°)$	Δ_{MLE}
4	-30	-30.6	-0.6	-31.9	-1.9	-29.5	+0.5
	30	31	1	40.2	+10.2	32.6	+2.6
	60	64.4	+4.4	55.1	-5.1	42.6	-12.6
6	-30	-29.8	+0.2	-29.6	+0.4	-30.3	-0.3
	30	30	0	30.4	+1.4	30.1	+0.1
	60	60.2	+0.2	61	+1	42.6	-12.6
8	-30	-30.1	-0.1	-30.2	-0.2	-30.1	-0.1
	30	30	0	30.1	+0.1	31.3	1.3
	60	59.9	-0.1	60.1	+0.1	56.7	-4.3
10	-30	-30	0	-29.9	+0.1	-30	0
	30	30	0	29.9	-0.1	30.4	+0.4
	60	59.9	-0.1	59.8	-0.2	59.4	-0.6
12	-30	-30	0	-30	0	-30.2	-0.2
	30	30	0	30.1	0.1	29.9	-0.1
	60	59.9	-0.1	60.5	+0.5	60.2	+0.2

Table 4 DOA estimation (k = 128, d = 0.25λ).

N	$\theta_{in}(°)$	$\theta_{MUSIC}°$	Δ_{MUSIC}	$\theta_{ESPRIT}°$	Δ_{ESPRIT}	$\theta_{MLE}°$	Δ_{MLE}
4	-30	-29.9	+0.1	-28.9	+1.1	-31.6	-1.6
	30	33.7	3.3	26.5	-4.5	22.6	-7.4
	60	60.4	+0.4	45.1	-15.9	44.5	-16.5
6	-30	-30.4	-0.4	-31.9	-1.9	-30.2	-0.2
	30	29.9	-0.1	32.8	2.8	30.1	+0.1
	60	59.1	-0.9	57.5	-3.5	42.2	-17.8
8	-30	-29.9	0.1	-29.5	+0.5	-28.6	+1.4
	30	30.1	0.1	30.7	+0.7	30.4	+0.4
	60	59.9	-0.1	61	+0.1	60.1	+0.1
10	-30	-29.7	0.3	-30.5	-0.5	-29.9	+0.1
	30	27.7	-2.3	30.3	-30.3	30.6	+0.6
	60	59.9	-0.1	60.5	+0.5	58.9	-1.1
12	-30	-30	0	-30.8	-0.8	-29.7	+0.3
	30	30	0	29.3	0.7	30.1	+0.1
	60	59.9	-0.1	59.2	-0.8	60.1	+0.1

Table 5 DOA estimation (k = 1024, d = 0.75λ).

N	$\theta_{in}(°)$	$\theta_{MUSIC}(°)$	Δ_{MUSIC}	$\theta_{ESPRIT}(°)$	Δ_{ESPRIT}	$\theta_{MLE}(°)$	Δ_{MLE}
4	-30	-29.8	+0.2	-26.5	+3.5	-28.8	+1.2
	30	30	0	30.1	+0.1	30	0
	60	56.7	-3.3	62.6	+2.6	58.4	-1.6
6	-30	-28.4	-1.6	-29.9	+0.1	-29	1
	30	30	0	30.1	2.8	29.9	-0.1
	60	59.1	-0.9	56.7	-3.5	57.7	-2.3
8	-30	-29.9	+0.1	-27.8	+0.5	-29	+1
	30	30	0	29.9	+0.7	30.1	+0.1
	60	56.6	-3.4	60.1	+0.1	58.2	-1.8
10	-30	-29.8	0.2	-27.7	+2.5	-28.9	+1.1
	30	30	0	30	0	30.2	+0.2
	60	59.6	-0.4	60.3	+0.3	58.3	-1.7
12	-30	-30	0	-29.9	+0.1	-28.8	+1.2
	30	30	0	30	0	30	+0
	60	59.8	-0.2	56.6	-3.6	58.5	-1.5

Table 6 DOA estimation (k = 1024, d = 0.75λ).

N	$\theta_{in}(°)$	$\theta_{MUSIC}(°)$	Δ_{MUSIC}	$\theta_{ESPRIT}(°)$	Δ_{ESPRIT}	$\theta_{MLE}(°)$	Δ_{MLE}
4	-30	-28.7	+1.3	-28.7	+1.3	-28.9	+1.1
	30	29.6	-0.4	30.3	+0.3	30.6	0.6
	60	58.5	-1.5	58.6	-1.4	58.4	-1.6
6	-30	-29.1	+0.9	-24.6	+5.4	-28.8	+1.2
	30	30.1	0.1	30	0	29.9	-0.1
	60	57.9	-2.1	56.5	-3.5	58.1	-1.9
8	-30	-29.5	+0.5	-28.2	+1.8	-29	+1
	30	29.9	-0.1	30.1	+0.1	30.1	+0.1
	60	57.2	-2.8	59.5	-0.5	58.1	-1.9
10	-30	-29.3	+0.7	-29.8	+0.2	-28.9	+1.1
	30	30	0	30	0	30.1	+0.1
	60	57.6	-2.4	56.9	-3.1	58.3	-1.7
12	-30	-29.7	+0.3	-29.9	+0.1	-28.7	+1.3
	30	30	0	30.1	0.1	30.1	+0.1
	60	58.8	-1.2	60.4	0.4	58.3	-1.7

The simulation results of MUSIC, ESPRIT, and MLE algorithms on 3 signals coming from different angles (-30, 30, 60), shown in **Tables 1 - 6**, indicate clearly that, if array size increases from 4 to 12 elements, the peak spectrum becomes sharp. The resolution capacity increases also if the number of snapshots increases (from 128 to 1024). The 3 signals are clearly identified. We observe also that if the spacing between the antenna array changes from 0.25 λ to 0.75 λ, we get better resolution of estimated peaks, but we also observe some peaks in the case of d = 0.75 λ due to grating lobes.

A comparison can be made between the 3 methods in terms of errors; the tables indicate that MUSIC presents less errors then ESPRIT and MLE. The tables illustrate that, for different numbers of array, values of snapshot, and distances between the elements of array, MUSIC presents a maximal error of 11 % and a minimal error of 0.16 %, compared with ESPRIT with a maximal and minimal error of 33.3 and 0.33 %, respectively, and MLE with a maximal error of 29.66 % and a minimal error of 0.33 %.

We have proved that the MUSIC algorithm provided great resolution and accuracy. In previous studies [16], the authors showed that the spectrum does not contain side lobes, but they omitted that if the distance between elements of antenna exceed 0.6 λ the spectrum contain side lobes. The computation complexity and storage requirements for ESPRIT are lower than MUSIC, as it does not involve extensive searching throughout all possible steering, as was presented in other work [17].

The comparison between MUSIC method investigate in this work and the proposed method in [18] show that the performance of MLE degrades by changing the parameters: number of antenna, samples and distance between elements, moreover the results of the MLE and MUSIC show that the MLE algorithm present more errors at the level of angles compared to the MUSIC results.

Table 7 DOA comparison.

d	N	θ_{in}(°)	θ_{MUSIC}[19]	θ_{MUSIC}	θ_{ESPRIT}[19]	θ_{ESPRIT}	θ_{MLE}[19]	θ_{MLE}
$\lambda/2$	2	-10	18	-11.4	-51.8	-15.9	-16.75	-17.6
		-30	-49	-28.8	-44.8	-32.3	-48.75	-40.8
		50	36	53.6	75.42	49.6	35.5	42.9
	4	-10	3	-9.3	-27.1	-13.2	-6.5	-5.3
		-30	-7	-35.1	-34.45	-39.3	-28.25	-20.7
		50	58	49.9	46.9	52.8	57.2	56.6

Table 8 MUSIC comparison.

θ_{in}(°)	θ_{out}[21]	% Error	θ_{out}	% Error
50	55	10	50.4	0.8
60	62	3.33	59.6	0.6

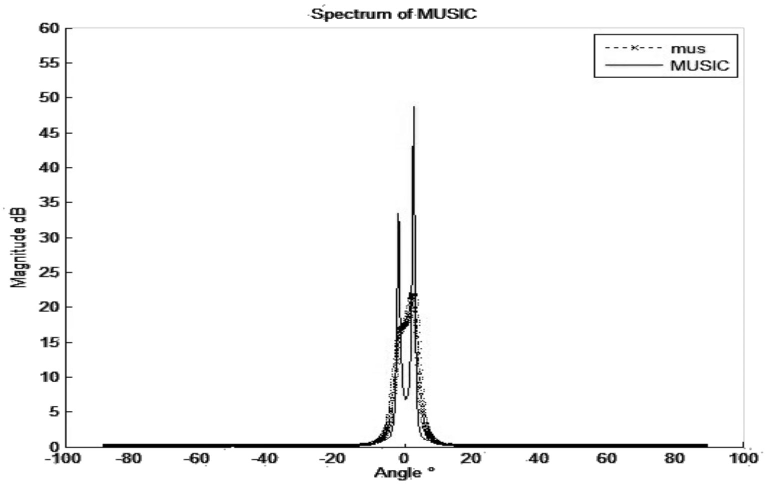

Figure 5 Spectrum of MUSIC and proposed method.

To validate our studies, a comparison was made with experimental results. **Table 7** illustrates the comparison between the MUSIC, ESPRIT, and MLE methods. The authors in [19] indicate that, to decrease the SNR and avoid undesirable lobes, the number of antennas should be increased and the distance between the elements should be limited to λ/2. However, they have omitted 2 essential factors: the signal power and the spectrum of angles that are linked to the number of snapshots. If we increase them, the power increases and the peaks become sharp. The angles [-2 3] are properly determined with precision and with an important magnitude = 49.9 dB, as shown in **Figure 5**.

Table 8 groups the results of a comparison between the MUSIC algorithm [21] and the proposed MUSIC algorithm. This table shows that, for [21], the angles 50° and 60° present an error of 10 and 3.33 %, contrary to the proposed algorithm, which assures a minimum error of 0.8 % for 50° and 0.6 % for 60°. Therefore, we note that the proposed algorithm MUSIC is more robust and precise in detection of the angles.

Conclusions

This article presents the results of direction of arrival estimation using the MUSIC, ESPRIT, and MLE algorithms. The MUSIC and ESPRIT methods have greater resolution and accuracy than MLE and, hence, they are investigated in greater detail. The experimental and simulation results show that performance of MUSIC, ESPRIT, and MLE improves with more elements in the array and with higher number of snapshots of signals. These improvements are displayed in the form of sharper peaks in the MUSIC spectrum and smaller errors in angle detection. The results indicate that, if the number of snapshots, distance, and number of elements of array increases, the errors of angle of arrival decreases. Therefore, the method MUSIC is highly efficient, with an error not exceeding 0.8 %.

Acknowledgements

This work is supported by the University Moulay Ismail, Meknes, Morocco.

References

[1] LC Godara. Application of antenna arrays to mobile communications part II: Beam-forming and direction of arrival consideration. *Proc. IEEE* 1997, **85**, 1195-245.

[2] AK Shauerman and AA Shauerman. Spectral-based algorithms of direction-of-arrival estimation for adaptive digital antenna arrays. *In*: Proceedings of the 9th International Conference and Seminar on Micro/Nanotechnologies and Electron Devices, Novosibirsk, Russia, 2010, p. 251-5.
[3] B Liao and SC Chan. DOA estimation of coherent signals for uniform linear arrays with mutual coupling. *In*: Proceedings of the IEEE International Symposium on Circuits and Systems, Rio de Janeiro, Brazil, 2011, p. 377-80.
[4] D Zhang, F Wang, R Burgos and D Boroyevich. Common mode circulating current control of interleaved three-phase two-level voltage-source converters with discontinuous space-vector modulation. *In*: Proceedings of IEEE Energy Conversion Congress and Exposition, San Jose, CA, USA, 2009, p. 2801-7.
[5] M Jalali, MN Moghaddasi and A Habibzadeh. Comparing accuracy for ML, MUSIC, ROOT-MUSIC and spatially smoothed algorithms for 2 users. *In*: Proceedings of the IEEE Conference Mediterranean on Microwave Symposium, Tangiers, Morocco, 2009, p. 1-5.
[6] SS Balabadrapatruni. Performance evaluation of direction of arrival estimation using Matlab. *Signal Image Process. Int. J*. 2012; **3**, 57-72.
[7] RO Schidt. Multiple emitter location and signal parameter estimation. *IEEE Trans. Antenn. Propag*. 1986; **34**, 276-80.
[8] A Paulraj, R Roy and T Kailath. A subspace rotation approach to signal parameter estimation. *Proc. IEEE* 1986; **74**, 1044-6.
[9] L Osman, I Sfar and A Gharsallah. Comparative study of high-resolution direction-of-arrival estimation algorithms for array antenna system. *Int. J. Res. Rev. Wireless Comm*. 2012; **2**, 72-7.
[10] R Roy and T Kailath. ESPRIT-estimation of signal parameters via rotational invariance techniques. *IEEE Trans. Acoust. Speech Signal Process*. 1989; **37**, 984-95.
[11] G Mao, B Fridan and B Anderson. Wireless sensor network localization techniques. *Comput. Network*. 2007; **51**, 2529-53.
[12] P Yang, F Yang and ZP Nie. DOA estimation with sub-array divided technique and interpolated esprit algorithm on a cylindrical conformal array antenna. *Progr. Electromagn. Res*. 2010; **103**, 201-16.
[13] F Vincent, O Besson and E Chaumette. Approximate unconditional maximum likelihood direction of arrival estimation for two closely spaced targets. *IEEE Signal Process. Lett*. 2015; **22**, 86-9.
[14] J Xin, N Zheng and A Sano. Simple and efficient nonparametric method for estimating the number of signals without eigen decomposition. *IEEE Trans. Signal Process*. 2007; **55**, 1405-20.
[15] P Stoica and A Nehorai. Music, maximum likelihood, and Cramer-Rao bound. *IEEE Trans. Acoust. Speech Signal Process.* 1990; **37**, 720-41.
[16] Y Khamou, S Safi and M Frikel. Comparative study between several direction of arrival estimation methods. *J. Telecomm. Inform. Tech*. 2014; **1**, 41-8.
[17] CR Dongarsane, AN Jadhav and SM Hirikude. Performance analysis of ESPRIT algorithm for smart antenna system. *Int. J. Comput. Comm. Tech*. 2011; **2**, 51-4.
[18] C Joe, C Ralph, E Hudson and K Yao. Maximum-likelihood source localization and unknown sensor location estimation for wideband signals in the near field. *IEEE Trans. Signal Process.* 2002; **50**, 1843-54.
[19] FA de Leon and JJS Marciano. Application of MUSIC, ESPIRIT and SAGE algorithms for narrowband signal detection and localization. *In*: Proceedings of the IEEE Region 10 Conference TENCON, Hong Kong, China, 2006, p. 1-4.
[20] WJ Si, XY Lan and Y Zou. Novel high-resolution DOA estimation using subspace projection method. *J. China Univ. Post. Telecomm*. 2012; **19**, 110-6.
[21] B Li, YX Zou and YS Zhu. Direction estimation under compressive sensing framework: A review and experimental results. *In*: Proceedings of the IEEE International Conference on Information and Automation, ICIA, Shenzhen, China, 2011, p. 63-8.