WALAILAK JOURNAL

Natural Sciences

http://wjst.wu.ac.th https://doi.org/10.48048/wjst.2021.13398

An EOQ Model of Selling-Price-Dependent Demand for Non-Instantaneous Deteriorating Items during the Pandemic COVID-19

Susanta Kumar INDRAJITSINGHA[1,*], Padmini RAULA[2], Padmanava SAMANTA[3], Umakanta MISRA[4] and Lakshmi Kanta RAJU[4]

[1]*Department of Mathematics, Saraswati Degree VidyaMandir, Neelakantha Nagar, Berhampurr-760 002, Odisha, India*
[2]*Department of Mathematics, Chikiti Mahavidyalaya, Chikiti, Odisha, India*
[3]*Department of Mathematics, Berhampur University, Berhampur, Bhanja Bihar-760007, Odisha, India*
[4]*NIST, Palur Hills, Berhampur- 761 008, Odisha, India*

([*]Corresponding author's e-mail: ski.rs.math@buodisha.edu.in)

Received: 5 October 2020, Revised: 16 March 2021, Accepted: 23 March 2021

Abstract

A pricing factor plays a dominant role in consumer behavior in most countries affected by the COVID19 pandemic. People have lost their job while others renegotiated for low-paying jobs during this pandemic. Thus, this article aims to develop a viable model to consider various aspects of the COVID19 pandemic. Here, we develop an optimal ordering quantity inventory model of deteriorating items, which are still in demand depending upon the selling price of the product. The items are assumed to be non-instantaneous deteriorating. The shortage is allowed in lead time and is partially backlogged. A solution procedure is presented to determine an optimal cycle, order quantity, and total average cost. A realistic numerical example is given to validate the proposed model by changing different systems of parameters, where sensitivity analysis has been carried out. The effectiveness of the system has been observed through graphical representation.

Keywords: Deterioration, Shortage, Partially-backlogged, Selling-price-dependent demand, Pandemic, COVID-19, EOQ model

Introduction

Nowadays, the business environment is in a havoc situation caused by the COVID19 pandemic. The medium and small-scale businesses of different small cities of low-income countries, like India, Pakistan, Afghanistan, Bangladesh, Nepal, Bhutan, Maldives, Thailand, Myanmar, and the Maldives are going into financial crises due to the shutdown and lockdown implemented by their respective administrations. These businesses must have collected non-instantaneous products like vegetables, fish, meats from small farmers in different villages. Due to the lockdown and shutdown phases, the market is only open five days a week with 11 to 12 working hours per day. Also, the boundaries of different cities are closed by the government for safety purposes. Meanwhile, vegetables, bakery, fish, and meats start to deteriorate due to this lockdown. Therefore, small and medium-scale entrepreneurs need the most affordable EOQ inventory model for items that are non-instantaneous to sustain their business. For efficient management, the inventory system has been the most vital job for any business firm irrespective of small- or large-scale industries.

To relax some particular boundaries of the product's specification, several academicians and researchers developed many models on EOQ. Harris [1] made the 1st attempt to develop an EOQ model in 1913. Within [2], developed an inventory system for un-conventional items, i.e., the items will become old-fashioned periodically after a specific period of time. However, the main goal to develop EOQ/EPQ models was to control defective products' capacity. Rosenblatt and Lee [3] demonstrated a model in which, after a certain time of continuous manufacturing, the system becomes out of control and starts producing imperfect- items. Hayek and Salameh [4] have taken a proper investigation to work on defective produced items in the previous EOQ models. Goyal *et al*. [5] developed a model by taking the ordering policy as a stock level and price under partially backlogging conditions. Singh *et al*. [6] proposed an inventory model on EOQ for deteriorating items where preservation technology is applied to reduce the amount of deterioration. Due to the high stock level in a supermarket shelf attracting maximum customers to purchase the product, they considered stock-dependent-demand with trade credit policy. Tiwari *et al*. [7] developed a model for non-instantaneous deteriorating -items with selling- price-dependent- demand under a fuzzy environment where green technology is used to reduce carbon emissions affecting the total profit. Henceforth many researchers are working on EOQ based models by analyzing the developed theory to get more quality items. Some of the works were developed by Manna *et al.* [8], Mukhopadhaya and Goswami [9], Pasandideh *et al.* [10], Nobil *et al.* [11], Cardenas-Barron *et al*. [12], and many more.

Deterioration is a process of reducing the quality from higher to lower levels. It suggests change, obsolescence, spoilage, loss of utility, or loss of minor estimation of merchandise, resulting in the loss of their value. The deterioration rate is not almost insignificant for items like toys, glassware, hardware, and steel, but it is much more effective for items like vegetables, natural products, drugs, unpredictable fruits, blood, and innovative items. Deteriorated items have different physical highlights in which a large portion of products undergo deterioration over time. This phenomenon is especially valid for items like natural products, like vegetables. Some specific items continuously deteriorate through the mortification process, whereas consumption rates are very large in products like gasoline, alcohol, and turpentine. Hence, deterioration assumes a vital role in the inventory model that should not be ignored. Also, in real life, deterioration is a common phenomenon. In fact, research on deteriorating items is going on continuously since 1963. Share and Schrader [13] have developed a model with a constant deterioration rate, which validated with no shortages. A rapid change in the deterioration like exponential function was introduced by Liao [14] in 2008. Tripathy and Pandey [15] considered an inventory model with deteriorating items under trade credit policy. By assuming the deterioration as constant, Agarwal and Jaggi [16] developed a model.

In the last two decades, many papers have been published in which demand is price dependent. Chen and Chen [17] proposed an inventory model with finite deteriorating- items over a finite- horizon. Chang *et al.* [18] developed a model for deteriorating items with partial backlogging for the retailer’s optimal pricing and lot-sizing policies. Dye [19] considered a logic over an infinite time horizon with allowable back-ordering. Similar papers were developed by Panda *et al.* [20], Tsao and Sheen [21], Lin *et al.* [22], Dye and Ouyang [23], Wang and Lin [24], Ghoreishi *et al.* [25], Soni and Patel [26], Srivastava and Gupta [27], Wang and Huang [28], Bai *et al.* [29], Bhunia *et al.* [30], Rabbani *et al.* [31], Indrajitsingha *et al.* [32,33]. Recently one work was developed by Rahamann and Duary [34] to handle the fluctuating customer demand (selling price dependent). As per the present situation, the foods having perishable nature are decaying due to uncertainty caused by the lockdown and shutdown during the COVID-19 pandemic.

In the present study, a model is developed for non-instantaneous deterioration products with selling-price-dependent demand rate and shortages under partially backlogged by making different combinations of these above system parameters. The study’s main objective is to optimize by minimizing the total-average- cost per unit time.

The present paper is organized as follows: In section 2, we provide the notations and assumptions which are considered to develop the model. In section 3, a mathematical model is established with a valid solution procedure. Numerical examples are given in section 4. In section 5, the optimal solution's sensitivity analysis and managerial insights with respect to major parameters are carried out by using Mathematica 11.1 software. Lastly, in section 6, the article ends with some concluding remarks and suggestions for future research.

Materials and methods

Notations and assumptions:

The following notations and assumptions are considered throughout the paper:

Notations

α	:	Demand coefficient
γ	:	Demand constant, $\gamma \geq 1$
s	:	Selling-price (in ₹/unit)
m	:	Time during which there is no deterioration
h	:	Holding cost (in ₹/unit)
θ	:	Deterioration coefficient, $0 \leq \theta \leq 1$
v	:	The time at which the inventory level reaches to zero
ξ	:	Lost-sale cost (in ₹/unit)
R	:	Ordering-cost (in ₹/order)
p	:	Purchasing-cost (in ₹/unit)
T	:	Length of cycle
Q	:	Ordering quantity
q_i	:	On-hand inventory
q_s	:	Back-order inventory
r	:	Shortage-cost (in ₹/unit)
η	:	Rate of backlogging
$I_1(t)$	:	Inventory level, at any time t, during $[0, m]$
$I_2(t)$	:	Inventory level, at any time t, during $[m, v]$
$I_3(t)$	:	Inventory during the shortage period $[v, T]$
$K(v)$	:	Total average inventory cost (in ₹)

Assumptions

(i) The market demand depends upon the selling-price of the non-instantaneous deteriorating products, and it is of the form $D(s) = \frac{\alpha}{s^\gamma}, s > 0, \gamma \geq 1$. As the items are non-instantaneous deteriorating in nature, the selling-price depends upon the quality of the product.
(ii) There is no deterioration during the time interval $[0, m]$ and after that period deterioration starts at a rate θ.
(iii) No replenishment is required during the deterioration time.
(iv) The lead time is assumed to be zero.
(v) Shortages, allowed and partially backlogged.

Model formulation

In this section, we have described the development of an economic order quantity model of non-instantaneous deteriorating products which starts after some time interval. It is assumed that the experiment is started with q_i units of in-hand inventory and q_s units of backorder inventory and assumed to be at the beginning of each cycle. Let T be the length of each cycle. Let in the time interval $[0, m]$, there will be no deterioration of the product. The deterioration starts at $t = m$ and at $t = v$, there will no inventory. Hence for $t > v$, shortages appears and it happens up to the end of the cycle. To fulfill this shortage partially backlogging is taken with a rate η. Let $I_1(t)$ and $I_2(t)$ be the inventory level at any time t in the time interval $[0, m]$ and $[m, v]$, respectively. The behavior of the model is shown in **Figure 1**.

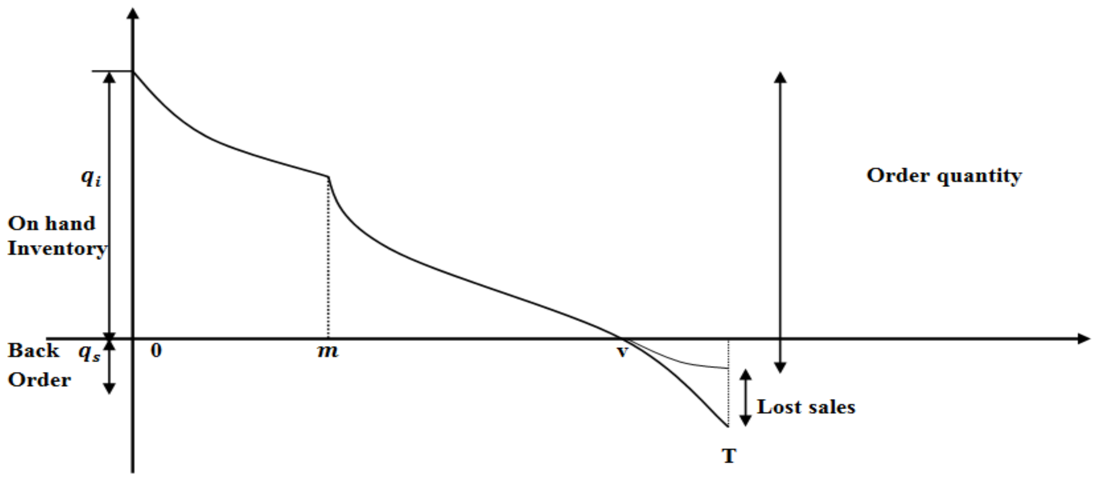

Figure 1 Graphical representation of inventory system: Inventory versus time.

Thus, we have the following differential equations for the proposed inventory system;

$$\frac{dI_1(t)}{dt} = -\frac{\alpha}{s^{\gamma}},\ 0 \le t \le m \tag{1}$$

$$\frac{dI_2(t)}{dt} = -\theta t I_2(t) - \frac{\alpha}{s^{\gamma}},\ m \le t \le v \tag{2}$$

$$\frac{dI_3(t)}{dt} = -\frac{\alpha}{s^{\gamma}},\ v \le t \le T \tag{3}$$

with boundary conditions;

$$I_3(v) = 0, I_2(v) = 0, I_1(m) = I_2(m) \tag{4}$$

The solutions of the above Eqs. (1) - (3) with the help of boundary condition (4) are given by;

$$I_1(t) = \frac{\alpha}{s^{\gamma}}(m-t) + \frac{\alpha}{s^{\gamma}}\left\{(v-m) + \frac{\theta}{6}(v^3 - m^3)\right\}e^{-\frac{\theta m^2}{2}},\ 0 \le t \le m \tag{5}$$

$$I_2(t) = \frac{\alpha}{s^{\gamma}}\left\{(v-t) + \frac{\theta}{6}(v^3 - t^3)\right\}e^{-\frac{\theta t^2}{2}},\ m \le t \le v \tag{6}$$

$$I_3(t) = \frac{\alpha}{s^{\gamma}}(v-t), v \le t \le T \tag{7}$$

Using the initial condition $I_1(0) = q_i$, we have;

$$q_i = \frac{\alpha}{s^{\gamma}}m + \frac{\alpha}{s^{\gamma}}\left\{(v-m) + \frac{\theta}{6}(v^3 - m^3)\right\}e^{-\frac{\theta m^2}{2}} \tag{8}$$

Total average cost per unit time for the model during a cycle is given by;

$$K(v) = \frac{1}{T}\begin{bmatrix} Buyingcost + Inventoryholdingcost + Shortagecost \\ +Lostsalecost + Orderingcost \end{bmatrix} \tag{9}$$

Now we estimate the following costs;

(i). Buying cost (BC):

We have the total ordering quantity $Q$is the sum of on-hand inventory and back-order inventory which is $Q = q_i + q_s$.

Hence the total buying cost can be formulated as;

$$BC = Qp = [q_i + q_s]p$$

Here, we have;

$$q_i = \frac{\alpha}{s^\gamma} m + \frac{\alpha}{s^\gamma}\left\{(v-m) + \frac{\theta}{6}(v^3 - m^3)\right\} e^{-\frac{\theta m^2}{2}}$$

$$q_s = \int_v^T \frac{\alpha}{s^\gamma}\theta dt = \frac{\alpha}{s^\gamma}\theta(T - v)$$

Therefore;

$$BC = \left[\frac{\alpha}{s^\gamma} m + \frac{\alpha}{s^\gamma}\left\{(v-m) + \frac{\theta}{6}(v^3 - m^3)\right\} e^{-\frac{\theta m^2}{2}} + \frac{\alpha}{s^\gamma}\theta(T - v)\right] p \tag{10}$$

(ii). Inventory holding-cost (IHC);

We have;

$$IHC = h\left[\int_0^m I_1(t)dt + \int_m^v I_2(t)dt\right]$$

$$= h\begin{bmatrix} \frac{\alpha}{s^\gamma}\frac{t^2}{2} + \frac{\alpha}{s^\gamma}\begin{Bmatrix} \left(vm - \frac{m^2}{2}\right) \\ + \frac{\theta}{6}(v^3 - m^3)m - \frac{\theta}{2}(vm^2 - m^3)m \end{Bmatrix} \\ + \frac{\alpha}{s^\gamma}\begin{Bmatrix} \frac{v^2}{2} + \frac{\theta v^4}{8} - \frac{\theta v^3}{24} - \left(vm - \frac{m^2}{2}\right) \\ -\frac{\theta}{6}\left(v^3 m - \frac{m^4}{4}\right) + \frac{\theta}{6}\left(\frac{vm^3}{3} - \frac{m^4}{4}\right) \end{Bmatrix} \end{bmatrix} \tag{11}$$

(iii). Shortage-cost (ISC):

The shortage-cost is given by;

$$ISC = r\int_v^T \frac{\alpha}{s^\gamma} dt = \frac{\alpha}{s^\gamma} r(T - v) \tag{12}$$

(iv). Lost-sale-cost (LSC):

We have the Lost-sale-cost as;

$$LSC = \int_v^T (1-\eta)\frac{\alpha}{s^\gamma}\xi dt = (1-\eta)\frac{\alpha}{s^\gamma}\xi(T - v) \tag{13}$$

(v). Ordering-cost (OC):

$$OC = R \tag{14}$$

Hence the total average cost of the system per unit time is given by;

$$K(v) = \frac{1}{T}\left[\left(\begin{array}{c}\frac{\alpha}{s^\gamma}m \\ +\frac{\alpha}{s^\gamma}\left\{(v-m)+\frac{\theta}{6}(v^3-m^3)\right\}e^{-\frac{\theta m^2}{2}} \\ +\frac{\alpha}{s^\gamma}\theta(T-v)\end{array}\right)p + \left(\begin{array}{c}\frac{\alpha}{s^\gamma}\frac{t^2}{2}+\frac{\alpha}{s^\gamma}\left\{\begin{array}{c}\left(vm-\frac{m^2}{2}\right) \\ +\frac{\theta}{6}(v^3-m^3)m-\frac{\theta}{2}(vm^2-m^3)m\end{array}\right\} \\ +\frac{\alpha}{s^\gamma}\left\{\begin{array}{c}\frac{v^2}{2}+\frac{\theta v^4}{8}-\frac{\theta v^3}{24}-\left(vm-\frac{m^2}{2}\right) \\ -\frac{\theta}{6}\left(v^3m-\frac{m^4}{4}\right)+\frac{\theta}{6}\left(\frac{vm^3}{3}-\frac{m^4}{4}\right)\end{array}\right\}\end{array}\right)h \\ +\frac{\alpha}{s^\gamma}r(T-v)+(1-\eta)\frac{\alpha}{s^\gamma}\xi(T-v)+R\right] \tag{15}$$

For minimizing the total average cost$K(v)$, value of $v$can be obtained by solving the differential equation;

$$\frac{dK(v)}{dv}=0 \tag{16}$$

Satisfying;

$$\frac{d^2K(v)}{dv^2}>0 \tag{17}$$

Eqs. (16) and (17) are equivalent to;

$$\frac{1}{T}\left[\begin{array}{c}-\frac{\alpha}{s^\gamma}r+p\left(-\frac{\alpha}{s^\gamma}\theta+\frac{\alpha}{s^\gamma}\left(1+\frac{v^2\theta}{2}\right)e^{-\frac{\theta m^2}{2}}\right)-\frac{\alpha}{s^\gamma}\xi(1-\eta) \\ +h\left\{\frac{\alpha}{s^\gamma}\left(m+\frac{v^2m\theta}{2}-\frac{\theta m^3}{2}\right)+\frac{\alpha}{s^\gamma}\left(v-\frac{v^2\theta}{8}+\frac{v^3\theta}{2}-m-\frac{v^2m\theta}{2}+\frac{\theta m^3}{6}\right)\right\}\end{array}\right]=0$$

and

$$\frac{1}{T}\left[\frac{\alpha}{s^\gamma}\frac{v^2\theta}{2}pe^{-\frac{\theta m^2}{2}}+h\left\{\frac{\alpha}{s^\gamma}v\theta m+\frac{\alpha}{s^\gamma}\left(1-\frac{v\theta}{4}+\frac{3v^2\theta}{2}-v\theta m\right)\right\}\right]>0,$$

respectively.

Solution procedure

We have solved the abovementioned problem by using the algorithm given below:

Algorithm

Step-1 : Start

Step-2 : Input the values of the parameters $\alpha, \gamma, s, m, h, \theta, \xi, R, p, T, r$ and η

Step-3 : Find $\frac{dK(v)}{dv}$

Step-4 : Solve the equation $\frac{dK(v)}{dv} = 0$ and find the value of v

Step-5 : Find $\frac{d^2K(v)}{dv^2}$

Step-6 : If the value of $\frac{d^2K(v)}{dv^2} > 0$, then we get minimum solution and go to Step-8

Step-7 : Otherwise, we go to Step-2

Step-8 : Stop

Numerical examples

To validate the result of the model considered the following example:

Example

$\alpha = 1300, \gamma = 2.3, s = 5, m = 5, \theta = 0.02, \xi = 6, h = 0.3, R = 150, \eta = 0.5, r = 8, p = 3$.The values of different parameters considered here are realistic, though these are not taken from any case study. By using Mathematica 11.1 software we get the unique $v = 11.2011$, and $Q = 515.061$ with optimum total average cost $K(v) =$ ₹280.812. The convexity of the function is not possible to prove using mathematical analytic approach, the graphical analytic approach is used in the **Figure 2**.

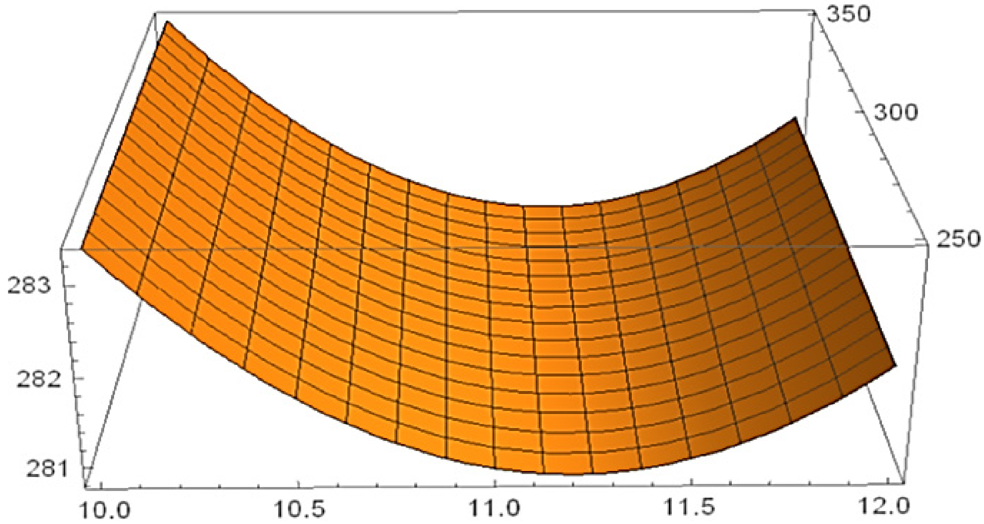

Figure 2 Total average cost versus v and Q .

Results and discussion

Sensitivity analysis

From the numerical example mentioned above, the following sensitivity analysis has been carried out to reveal the following managerial insights by changing the parameters one at a time keeping other unchanged.

Table 1 Sensitivity- analysis for different parameter.

γ	v	$K(v)$	Q	m	v	$K(v)$	Q
2.12	11.2011	372.206	688.135	4	10.8834	284.105	526.331
2.16	11.2011	349.55	645.23	5	11.2011	280.812	515.061
2.18	11.2011	338.757	624.792	6	11.5802	277.419	504.138
2.2	11.2011	328.306	605.001	7	12.0127	274.217	494.251
2.22	11.2011	318.186	585.837	8	12.4896	271.495	486.001
2.3	11.2011	280.812	515.061	9	13.0017	269.526	479.921
S	v	$K(v)$	Q	θ	v	$K(v)$	Q
4.5	11.2011	355.394	656.298	0.01	13.4267	260.926	564.6993
4.7	11.2011	322.409	593.834	0.012	12.7926	266.564	551.859
4.9	11.2011	293.748	539.559	0.016	11.8615	274.893	531.265
5.0	11.2011	280.812	515.061	0.018	11.506	278.08	522.733
5.2	11.2011	257.35	470.633	0.02	11.2011	280.812	515.061
5.4	11.2011	236.687	431.503	0.022	10.9361	283.183	508.092
ξ	v	$K(v)$	Q	h	v	$K(v)$	Q
4.5	10.8037	272.324	499.484	0.22	12.3535	262.479	565.344
5.0	10.9384	275.216	504.667	0.24	12.832	267.518	550.524
5.5	11.0709	278.045	509.862	0.25	11.8805	269.911	543.757
6.0	11.2011	280.812	515.061	0.28	11.4587	276.647	525.629
6.5	11.3239	283.518	520.052	0.3	11.2011	280.812	515.061
7	11.4552	286.163	525.483	0.32	10.9599	284.749	505.503
R	v	$K(v)$	Q	p	v	$K(v)$	Q
150	11.2011	280.812	515.061	2.5	11.5336	265.464	528.773
160	11.2011	281.4	515.061	3.0	11.2011	280.812	515.061
165	11.2011	281.694	515.061	3.5	11.8844	295.774	502.577
170	11.2011	281.988	515.061	4.0	10.5825	310.386	491.185
180	11.2011	282.576	515.061	4.5	10.2943	324.677	480.759
190	11.2011	283.165	515.061	5.0	10.0188	338.674	471.193

r	v	$K(v)$	Q	η	v	$K(v)$	Q
6.5	10.3845	263.259	483.976	0.3	11.5051	287.205	492.311
7.0	10.6665	269.368	494.306	0.35	11.4302	285.631	497.635
7.5	10.9384	275.216	504.667	0.4	11.3546	284.051	503.199
8.0	11.2011	280.812	515.061	0.45	11.2782	282.442	509.005
8.2	11.3037	282.981	519.225	0.5	11.2011	280.812	515.061
8.5	11.4552	286.163	525.483	0.55	11.1232	279.159	521.367

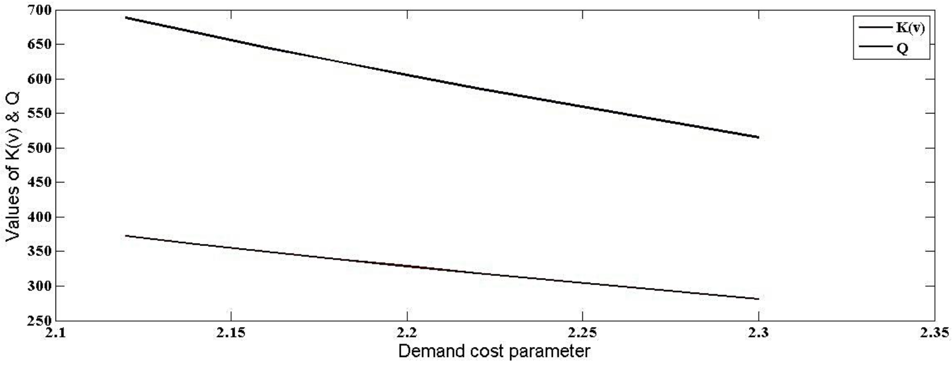

Figure 3 Demand parameter (γ) Vs $K(v)\&Q$.

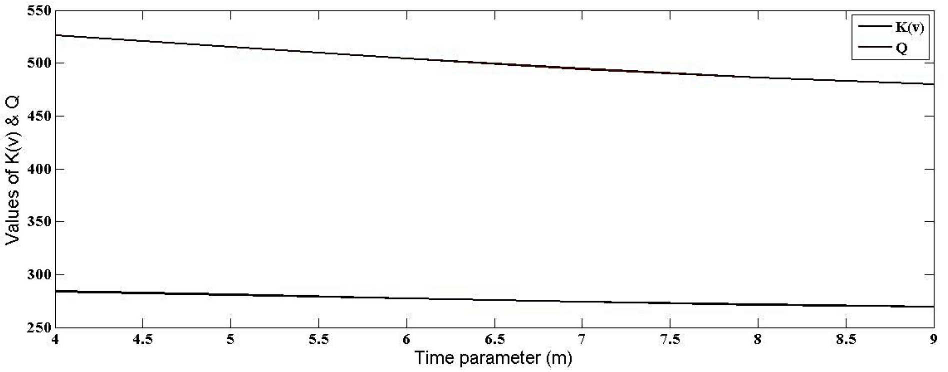

Figure 4 Time parameter (m) Vs $K(v)\&Q$.

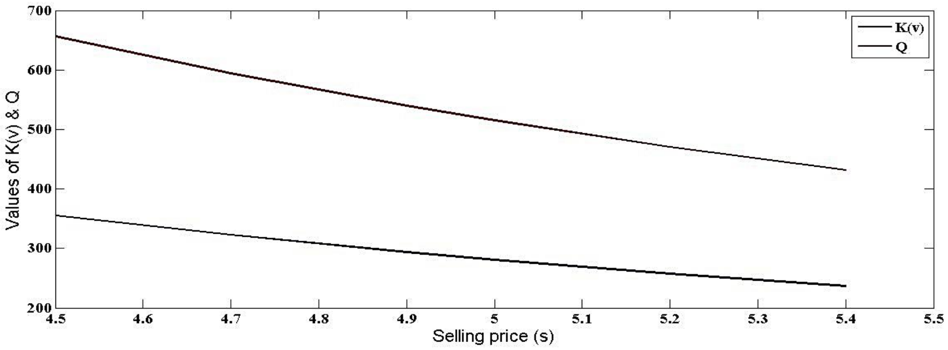

Figure 5 Selling price (s) Vs $K(v)\&Q$.

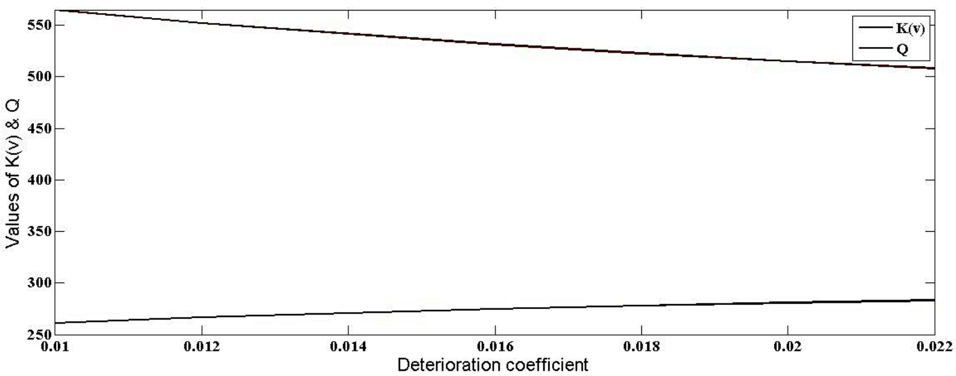

Figure 6 Deterioration coefficient (θ) Vs $K(v)\&Q$.

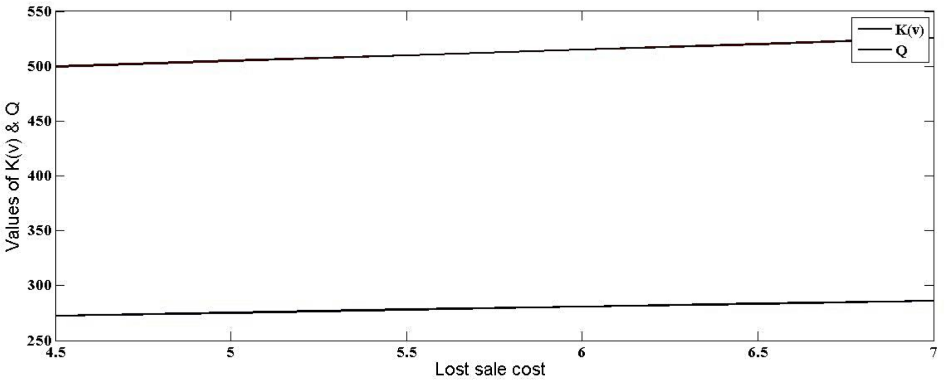

Figure 7 Lost sale cost (ξ) Vs $K(v)\&Q$.

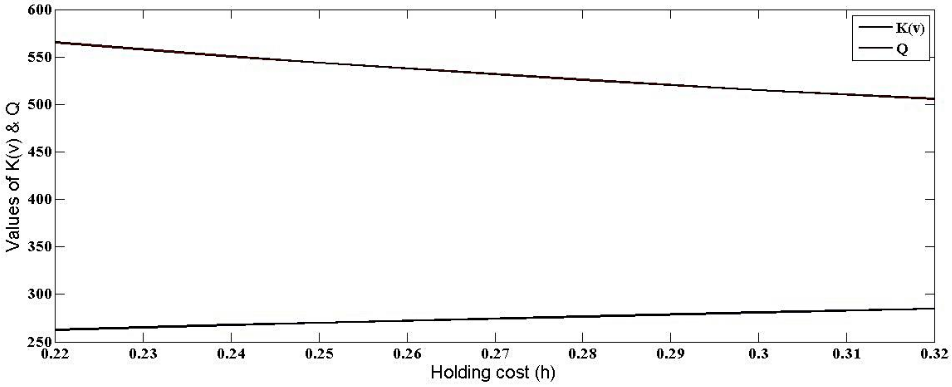

Figure 8 Holding cost (h) Vs $K(v)\&Q$.

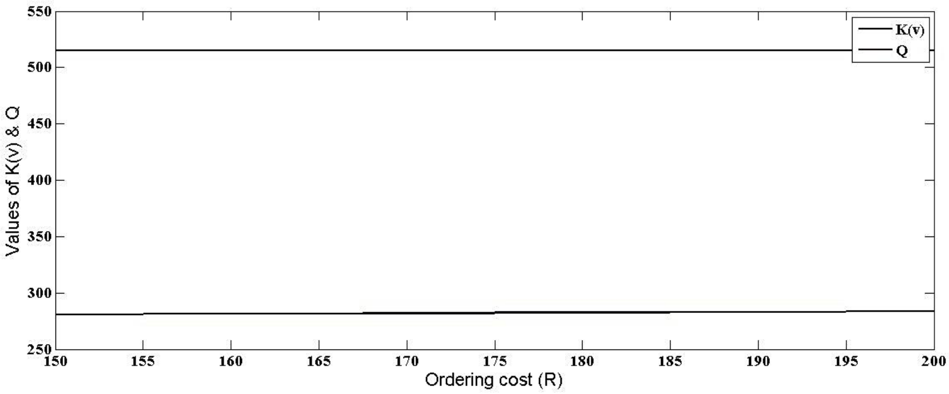

Figure 9 Ordering cost (R) Vs $K(v)\&Q$.

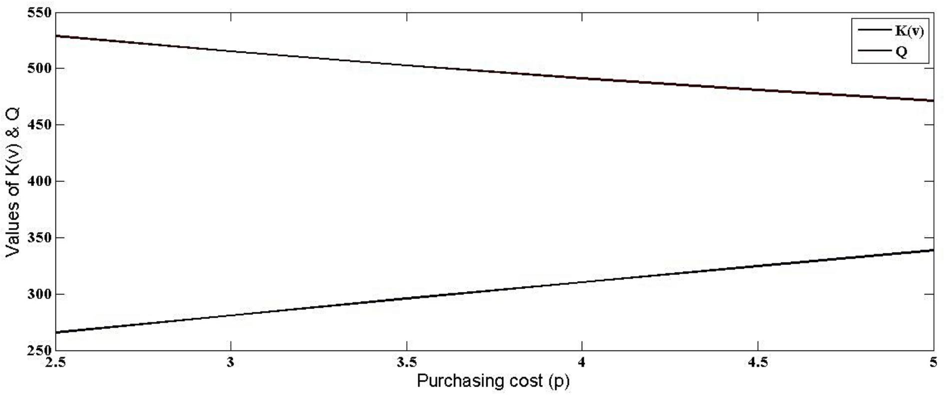

Figure 10 Purchasing cost (p) Vs $K(v)\&Q$.

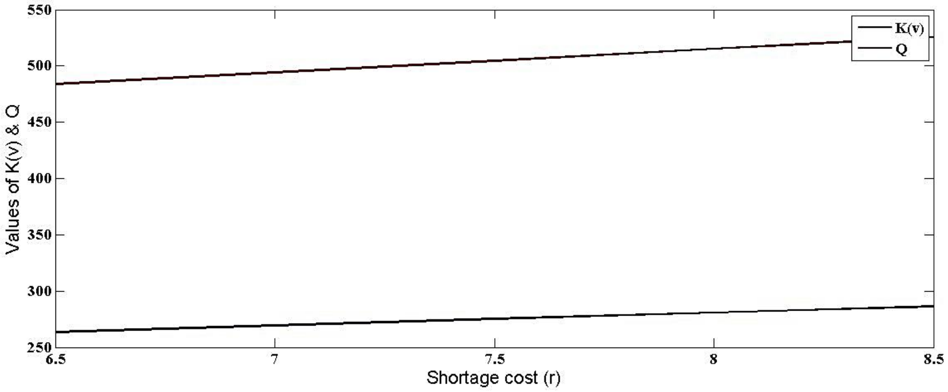

Figure 11 Shortage cost (r) Vs $K(v)\&Q$.

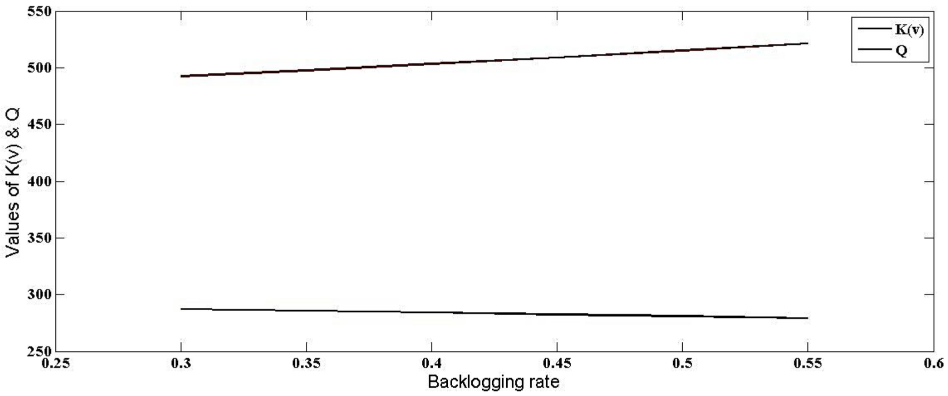

Figure12 Backlogging rate (η) Vs $K(v)\&Q$.

The following observations are found from **Table 1** and from the corresponding **Figure**:

1) When the demand parameter γ increases, then total average cost e increases and ordering quantity Qdecreases with constant v.

2) When the value of the parameter $m$increases, it is observed that $v$increases with decrease of total average cost function$K(v)$ and ordering quantity Q of the products.

3) When then selling price increases, then both the ordering quantity $Q$and the total average cost $K(v)$ decrease with constant $v$of the products. That is, the increase in selling -price means the increase in the total profit.

4) By increasing the value of θ, the total average cost $K(v)$ of the products such as vegetables, bakeries, fishes, meats increases. But$v$, the time at which the inventory level becomes 0, the order quantity also decreases.

5) When the lost sale cost ξ increases, the total average cost $K(v)$ increases with an increase of order -quantity and v.

6) When the holding cost $h$increases, the total average cost $K(v)$ per unit time of the products increases with the decreasein ordering quantity Q and the time at which the inventory level becomes $0(v)$.

7) When the value of the ordering cost Rincreases, it is observed that the total average cost $K(v)$ increases which is obvious in nature with constant ordering quantity Q.

8) When the purchasing cost p increases, it is observed that the total average-cost $K(v)$ increases with decrease of ordering- quantity Q and v.

9) When the shortage cost r increases, it is concluded that $K(v)$ increases with increase of the parameters $Q$and v.

10) If the backlogging- parameter η increases, the ordering-quantity Q increases and $K(v)$ decreases with slightly decrease in the value of the parameter v.

Conclusions

In most countries, the economic situation has greatly changed due to the COVID-19 pandemic. Medium and small-scale entrepreneurs are mostly affected by the pandemic. A country's economy will develop if it focuses on the businessmen, who are the grass root of the economy. Owing to the above discussion, we develop an economic order quantity model of selling- price- dependent demand rate. Since items such as vegetables, bakery, fish, and meats do not deteriorate instantaneously, we consider the products which are non-instantaneous decaying. A procedure for getting the solution is provided to get the optimum total average cost, and practical application examples support the application of the proposed model to support decision-making. It is observed that the present result illustrates the usefulness of the model in determining optimal ordering policy. It can also be used in inventory -control of some non-instantaneous decaying items which exhibit selling price-dependent demand. It is suggested that for further research in this line, one can take the problem to study the impact of time-varying deterioration on optimal policy.

Acknowledgment

The authors are grateful to anonymous reviewers for their encouragement and constructive remarks.

References

[1] FW Harris. How many parts to make at once, *Fact. Mag. Manag.* 1913; **10**, 135-6.
[2] TM Whitin.*T.M.The theory of inventory management*.Vol IX.Princeton University Press, Princeton,1957, p. 347.
[3] MJ Rosenblatt and HL Lee. Economic production cycles with imperfect production processes.*IIE Trans*. 1986; **18**, 48-55.
[4] PA Hayek and MK Salameh. Production lot-sizing with the working of imperfect quality items produced.*Prod. Plan. Control.* 2001; **12**, 584-90.
[5] AK Goyal, AChauhan and SR Singh. An EOQ inventory model with stock and selling price dependent demand rate, partial backlogging and variable ordering cost.*Int. J. Agri. Stat. Sci.* 2015; **11**, 441-7.
[6] SR Singh, D Khurana and S Tayal. An EOQ model for deteriorating products having stock dependent demand with trade credit period and preservation technology.*Uncertain Supply Chain Manag.* 2016; **4**, 29-42.
[7] S Tiwari, W Ahmed and B Sarkar. Sustainable ordering policies for non-instantaneous deteriorating items under carbon emission and multi-trade-credit policies.*J. Clean. Prod.* 2019; **240**, 118183.
[8] AK Manna, JKDey and SK Mondal. Imperfect production inventory model with production rate dependent defective rate and advertisement dependent demand.*Compute. Ind. Eng.* 2017; **104**, 9-22.
[9] A Mukhopadhyay and AGoswami. An inventory model with shortages for imperfect items using substitution of two products*. Int. J. Oper. Res*. 2017;**30**, 193-219.
[10] S Pasandideh, S Niaki, ANobil and LE Cárdenas-Barron. A multiproduct single machine economic production quantity model for an imperfect production system under warehouse construction cost.*Int. J. Prod. Econ*. 2015; **169**, 203-14.
[11] A Nobil, ASedigh and LE Cárdenas-Barrón. A multi-product single machine economic production quantity (EPQ) inventory model with discrete delivery order, joint production policy and budget constraints.*Ann. Oper. Res.* 2017; **286**, 265-301.

[12] LE Cardenas-Barron, AA Shaikh, S Tiwari and GT Garza. An EOQ inventory model with non-linear stock dependent holding cost, non-linear stock dependent demand and trade credit. *Comp. Ind. Eng.* 2020; **139**, 1-13.
[13] PM Ghare and SF Schrader. A model for exponentially decaying inventory.*J. Ind. Eng.* 1963; **14**, 238-43.
[14] JJ Liao. An EOQ model with non-instantaneous receipt and exponentially deteriorating items under two-level trade credit. *Int. J. Prod. Econ.* 2008;**113**, 852-61.
[15] RP Tripathi and HS Pandey. An EOQ model for deteriorating item with Weibull time dependent demand rate under trade credits. *Int. J. Inf. Manag.* 2013; **24**, 329-47.
[16] SP Aggarwal and CK Jaggi. Ordering policies of deteriorating items under permissible delay in payments. *J. Oper. Res. Soc.* 1995; **46**, 658-62.
[17] JM Chen and LT Chen.Pricing and production lot-size/scheduling with finite capacity for a deteriorating item over a finite horizon.*Comput. Oper. Res*. 2005; **32**, 2801-19.
[18] HJ Chang, JT Teng, LY Ouyang and CY Dye. Retailer's optimal pricing and lot-sizing policies for deteriorating items with partial backlogging.*Eur. J. Oper. Res.* 2006; **168**, 51-64.
[19] CY Dye. Joint pricing and ordering policy for a deteriorating inventory with partial backlogging. *Omega* 2007; **35**, 184-9.
[20] S Panda, S Senapati and M Basu. Optimal replenishment policy for perishable seasonal products in a season with ramp-type time dependent demand. *Comput. Ind. Eng*. 2008; **54**, 301-14.
[21] YTsao and G Sheen.Joint pricing and replenishment decisions for a deteriorating item under permissible delay in payments. *Comput.Oper.Res.* 2007; **35**, 3562-80.
[22] Y Lin, J Yu and K Wang. An efficient replenishment model of deteriorating items for a supplier-buyer partnership in hi-tech industry. *Prod. Plan. Control.* 2009; **20**, 431-44.
[23] C Dye and L Ouyang. A particle swarm optimization for solving joint pricing and lot-sizing problem with fluctuating demand and trade creditfinancing. *Comput. Ind. Eng.* 2011; **60**, 127-37.
[24] K Wang and Y Lin. Optimal inventory replenishment strategy for deteriorating items in a demand-declining market with the retailer's price manipulation. *Ann. Oper. Res*. 2012; **201**, 475-94.
[25] M Ghoreishi, AKhamseh and A Mirzazadeh. Joint optimal pricing and inventory control for deteriorating items under inflation and customer returns*. J. Ind. Eng.*2013; **2013**, 1-7.
[26] H Soni and K Patel. Joint pricing and replenishment policies for non-instantaneous deteriorating items with imprecise deterioration free time and credibility constraint.*Comput. Ind. Eng*. 2013; **66**, 944-51.
[27] M Srivastava and R Gupta. An EPQ model for deteriorating items with time and price dependent demand under markdown policy.*Opsearch* 2014; **51**, 148-58.
[28] C Wang and R Huang. Pricing for seasonal deteriorating products with price- and ramp-type time-dependent demand.*Comput. Ind. Eng*. 2014; **77**, 29-34.
[29] Q Bai, X Xu, M Chen and Q Luo. Two-echelon supply chain coordination for deteriorating item with a multi-variable continuous demand function.*Int. J. Syst. Sci. Oper. Logist.* 2015; **2**, 49-62.
[30] AK Bhunia, AA Shaikh, G Sharma and SPareek. A two-storage inventory model for deteriorating items with variable demand and partial backlogging*, J. Ind. Prod. Eng*. 2015; **32**, 263-72.
[31] M Rabbani, NP Zia and H Rafiei. Coordinated replenishment and marketing policies for non-instantaneous stock deterioration problem.*Comput. Ind. Eng*. 2015; **88**, 49-62.
[32] SK Indrajitsingha, PN Samanta and UK Misra. A fuzzy inventory model for deteriorating items with stock dependent demand rate.*Int. J. Logist. Manag.* 2018; **30**, 538-55.
[33] SK Indrajitsingha, PN Samanta and UK Misra. Fuzzy two-warehouse inventory model for single deteriorating item with selling price dependent demand and shortages under partially backlogged condition. *Appl. Math. Comput.*2019; **14**, 511-36.
[34] MS Rahman, ADuary and AA Shaikh.An application of parametric approach for interval differential equation in inventory model for deteriorating items with selling-price-dependent demand.*Neural Comput. Appl.* 2010; **32**, 14069-85.