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A Demonstration of Underwater Bubble Capture by the Fundamental Acoustic Mode in Spherical Geometry

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ABSTRACT

Nowadays, scientific demonstrations have become a crucial part of scientific learning. Acoustic waves are normally demonstrated in air via Kundt's tube, but a physical demonstration for underwater acoustic waves is still lacking. In this paper, we address one of the aspects by demonstrating a way to acoustically-trap gas bubbles in a spherical, water-filled flask resonating at its first fundamental mode. The theory of acoustic waves in a spherical geometry, particularly the fundamental mode, is reviewed. The full description of the experimental setup is expressed both acoustically and electronically. By using this method, we show that a gas bubble can be stabilized in the middle of a flask at an acoustic frequency of 21.16 kHz, the acoustic fundamental frequency of the flask.

Keywords: Standing wave demonstration, first fundamental mode, single-bubble, spherical geometry

INTRODUCTION

Perhaps the most well-known fundamental experiment in acoustics is the resonance of acoustic waves in "Kundt's tube" in which standing acoustic waves are formed in a gas column at different frequencies producing different audible tones. These resonance frequencies depend on the tube's dimension, geometry, and the speed of sound in the medium [1,2]. Several visual demonstrations have been performed for educational purposes in an effort to visualize the standing wave patterns generated by pressure gradients in a gas inside a cylindrical tube which levitate Styrofoam [3,4]. Nevertheless, a standing wave demonstration in a spherical geometry has yet to be performed directly for a similar purpose [5]. Our objective here is to perform such a demonstration in a spherical geometry by using a standing acoustic field capturing an air bubble in a water-filled flask. The procedure for the creation of the demonstration is given in detail.

THEORY

In a standing-wave acoustic field, a bubble in inviscid fluids experiences 3 forces: buoyancy, gravitation, and the primary Bjerknes [6]. Due to the micrometer-size of the bubble, the first 2 forces are so small that the only significant force is the primary Bjerknes force which can be expressed as

$$\langle F \rangle = -\langle V \nabla p \rangle \tag{1}$$

where "$\langle \ \rangle$" indicates the time-average, V is the volume of the bubble, and ∇p is the pressure gradient in the volume caused by the standing-wave acoustic field. The primary Bjerknes force is the radiation force exerted on the bubble in a standing-wave field. In a spherical geometry at mode (1,0,0), the absolute pressure inside the spherical chamber, derived from the time-dependent wave equation [7-9] when r reaches zero, is

$$p(r,t) = p_0 + \left(\frac{\sin kr}{kr} \right) p_a(t) \tag{2}$$

Where, p_0 and $p_a(t)$ are the ambient pressure and the sinusoidal pressure amplitude of the standing-wave field respectively, k is the wave number and r is the spatial variable. The term in parenthesis is the spherical Bessel function, $j_0(kr)$. As r reaches zero, by the power series expansion around $r = 0$, the first 2 terms of the spherical Bessel function are $\left(1 - \frac{(kr)^2}{6} \right)$, giving the value of resulting pressure at the center of the chamber [10]. At the boundary, where $r = a$ ($a =$ the radius of the chamber, ~4.13 cm in our case), the pressure-release surface condition is imposed, the pressure at the boundary, therefore, equals zero, leading to $kr = l\pi$; where l is an integer and equals 1 for the (1,0,0)-mode. It is then possible to define the resonance frequency of the (1,0,0)-mode as

$$f = \frac{c_0}{2a} \tag{3}$$

where c_0 is the speed of sound in water. Eq. (3) is then used to calculate the fundamental resonance frequency of the 250 ml spherical flask. The calculation gives the frequency of 18.16 kHz, where the value for the speed of sound in water at 25 °C [11] is 1,497.4 m/s and a wavelength of approximately 7 cm is obtained.

MATERIALS AND METHODS

The basic experimental setup is shown in **Figure 1**. The experiment consists of a 250 ml flask with two 2.01 cm hollow transducers and a 0.69 cm disk transducer symmetrically attached to the sides and to the bottom of the flask with epoxy. The hollow transducers are driven in parallel by a power amplifier (Yamaha model P200S) that receives an input signal from a function generator (HP model 33220A). The acoustic signal generated by the 2 hollow transducers is detected by the disk transducer located at the bottom of the flask that is connected to a computer-interfaced lock-in amplifier (model SR830 DSP). The first acoustic resonant frequency of the water-filled flask is determined by scanning the frequency of the driven signal. The first resonant frequency is found to be 21.16 kHz by the disc transducer, as shown in **Figure 2**. The negative and positive resonant peaks occur due to the relative phases between the voltages across the inductor and capacitors. At the resonance, this phase difference is π radians analogous to that of acoustic resonance. It is also possible to avoid confusion of the phase by plotting the average power of the circuit. Due to the significant impedance mismatch of the output of the power amplifier and the driving transducers, there are losses in the driving power which decrease the signal transmission. A series resonance circuit, in which the capacitive and inductive reactance cancel one another out, is employed to correct this mishap. The circuit is designed so that its resonance frequency is identical to that of the acoustic resonance as shown in **Figure 3** [12].

The transducers act as capacitors, each of which has a capacitance of approximately 0.78 nF. The inductor is made of a coil with a ferrite core to conform to the resonance frequency of the acoustic field. The inductance is simply calculated from $f_{resonant} = \frac{1}{2\pi\sqrt{LC}}$, where L and C are the inductance and capacitance, respectively. The calculation gives an inductance of 99 mH. A 1-ohm high-wattage resistor is used to complete the circuit. It serves, if needed, as a monitoring point for the current and prevents a short circuit at the resonance frequency.

The water used throughout the demonstration is tap water. Bubbles are introduced to the water by a syringe. The demonstration is performed at ambient conditions and the acoustic signal inside the flask is monitored for the entire period of the experiment by the disc transducer.

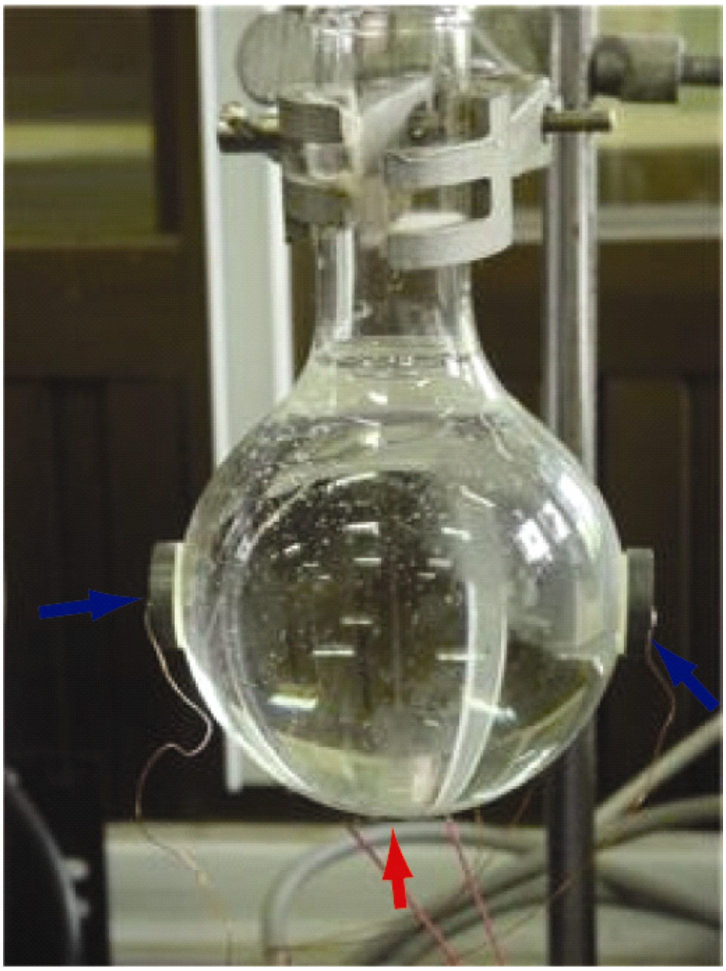

Figure 1 The locations where the transducers were attached to the flask. Blue and red arrows indicate the locations of the hollow and disc transducers respectively.

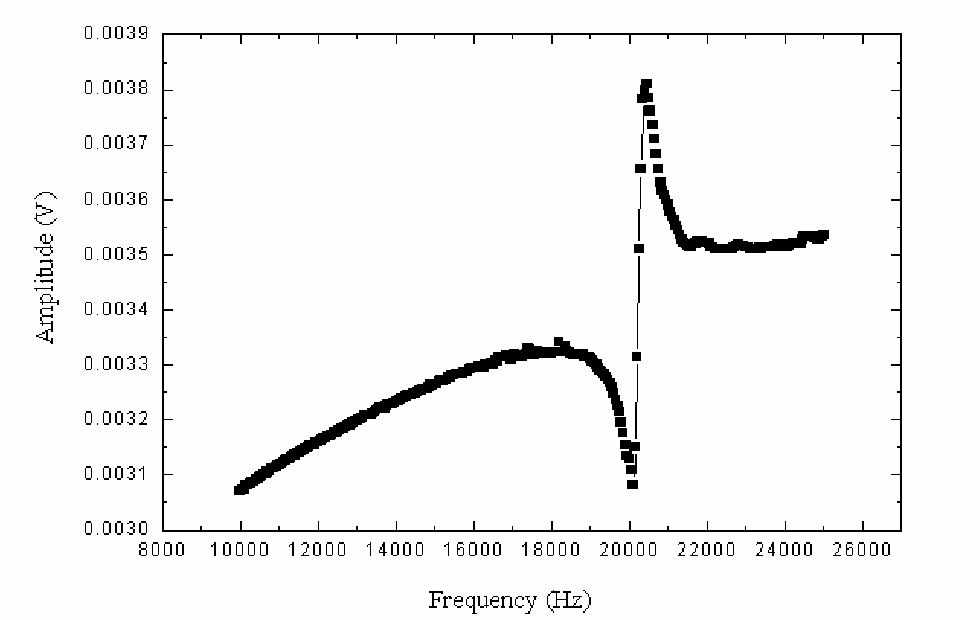

Figure 2 The resonant frequency of the system obtained from the lock-in amplifier by sweeping the driving frequency on the hollow transducers and measuring at the bottom of the flask by the disc transducer.

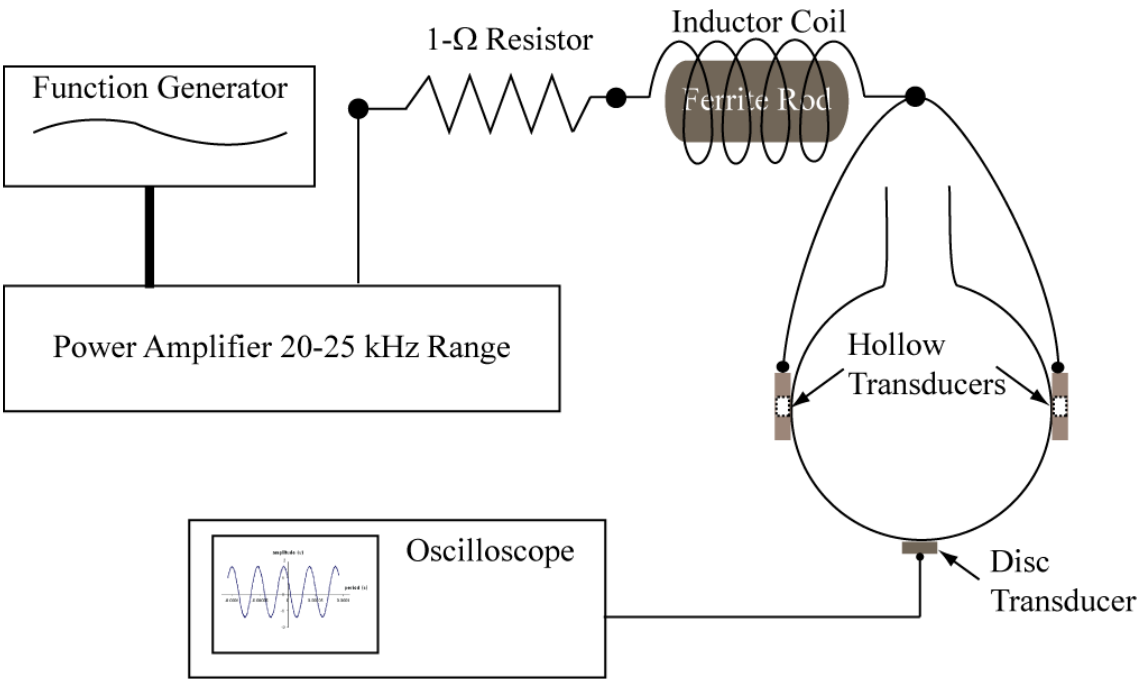

Figure 3 Diagram of the experiment forming the RLC circuit in which the hollow transducers act as capacitors connected in parallel.

RESULTS AND DISCUSSION

The introduction of bubbles by a syringe creates multiple bubbles in the water. Under the standing-wave sound field, these bubbles come together and fuse into a single bubble at the location where the net time-average radiation force of the sound field or the primary Bjerknes force is zero [13]. In regard to the geometry of the flask used in the experiment, there are 2 important points that need to be made. First of all, the flask can be assumed to be a perfect sphere as long as water is filled up to the bottle neck. The ratio of the angular distance, calculated for a perfect sphere, and the diameter of the free surface at the bottle neck yields a value of 0.996. This ratio approaches unity as the geometry yields a perfect sphere. In addition, the acoustic wavelength generated, ~7 cm, is about 4 orders of magnitude larger than the variation between the angular distance and the diameter of the open flask. The other essential part for this experiment is the boundary condition employed. Here, we use the pressure-release surface where a significant difference in the acoustic impedance of 2 media ideally causes a complete phase-reversed reflection allowing a standing wave to be created. The flask and the water are considered as 1 medium since they vibrate together; providing air as the second medium.

In **Figure 4**, a gas bubble is trapped and stably keeps its position in the center of the spherical flask at the driving frequency of 21.16 kHz, the fundamental resonance frequency. This frequency is obtained by scanning the transducer-driving frequency in the experiment. At this frequency the pressure anti-node is at the center and the pressure gradient vanished following from Eq. (1). Changes in bubble volume are so small that its deviation closes to zero. The resonance frequency obtained from the experiment is higher than that obtained from the Eq. (3). This error mainly comes from the measurement of the flask radius; a millimeter deviation results in a significant 2 kHz difference.

The water used in this demonstration is non-degassed tap water. To avoid surface-clinging bubbles which occur when the water is left over several hours, fresh tap water should be used. The surface-clinging bubbles can greatly alter the applied acoustic field; as a result, the trapped bubble becomes unstable and finally extricated. Due to the micrometer-size of the bubble, a He-Ne laser is used to illuminate the bubble for visibility purposes as shown in **Figure 5**.

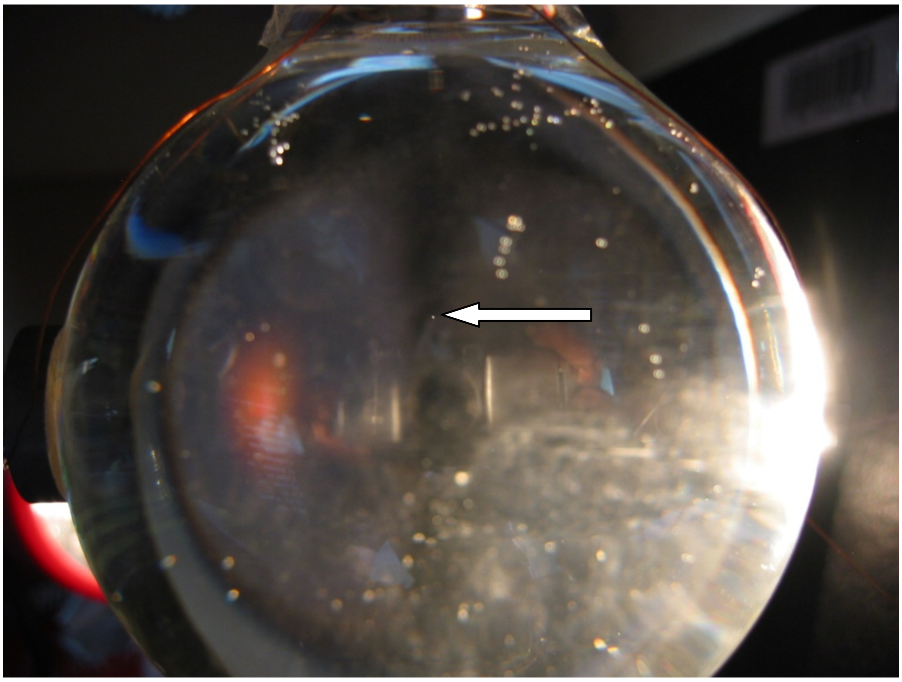

Figure 4 A minuscule gas bubble trapped in the center of the flask.

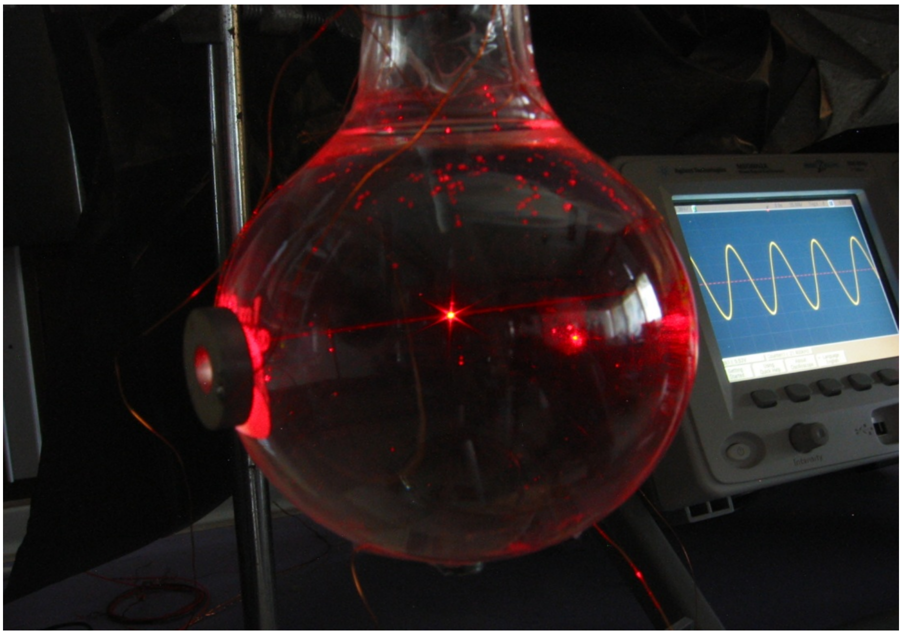

Figure 5 The illuminated bubble using a He-Ne laser.

CONCLUSION

We have elaborately described the proposed demonstration starting from the theory responsible for the bubble levitation, in which the pressure gradient plays a significant role in defining the primary Bjerknes force. The demonstration shows that the bubble is trapped at 21.16 kHz, corresponding to the (1,0,0)-mode as expected. Our work may be developed further with investigations and demonstrations possible for other acoustic modes and geometries. Light scattering experiments can also be performed to determine the size of the trapped bubble. In addition, single-bubble sonoluminescence can be demonstrated by a similar experimental setup.

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บทคัดย่อ

อุมาพร คนธรักษ์ และ สรศักดิ์ ด่านวรพงศ์

การสาธิตการจับฟองอากาศใต้น้ำโดยความถี่มูลฐานของเสียงในทรงกลม

ปัจจุบันการสาธิตทางวิทยาศาสตร์ได้เป็นส่วนที่สำคัญสำหรับการเรียนการสอนวิทยาศาสตร์ การสาธิตปรากฏการณ์คลื่นของเสียงโดยทั่วไปใช้หลอดของคุนด์ แต่การสาธิตของคลื่นเสียงใต้น้ำนั้นยังไม่ปรากฏอย่างแพร่หลาย ในงานวิจัยนี้ผู้วิจัยแสดงถึงอิทธิพลของคลื่นเสียงใต้น้ำโดยการสาธิตการจับฟองอากาศใต้น้ำโดยใช้คลื่นเสียงที่ความถี่มูลฐานในภาชนะรูปทรงกลม บทความนี้จะทบทวนทฤษฎีของคลื่นเสียงในพิกัดทรงกลมโดยเน้นความถี่มูลฐาน และอธิบายขั้นตอนการสาธิตโดยละเอียดทั้งทางด้านคลื่นเสียงและอิเลกทรอนิกส์ จากขั้นตอนดังกล่าวผู้วิจัยสามารถจับฟองอากาศได้ที่ความถี่ 21.16 กิโลเฮิร์ต ซึ่งเป็นความถี่มูลฐานของภาชนะทรงกลมที่ใช้

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